If we introduce the new velocity variables

$$v'_x = v_x ,$$

$$v'_y = (\Omega_s/\Omega)[v_y - u(X')] ,$$

$$v'_z = v_z ,$$

the zeroth-order moment of F_0 [chosen to be of the same form as (11)] gives

$$n_0(x) = \int d\vec{v} \left(\frac{m}{2\pi KT}\right)^{3/2} h(X) \exp\left(-\frac{mv^2}{2KT} + \frac{e\Phi(x)}{KT}\right)$$

$$= \frac{\Omega}{\Omega_s} \int d\vec{v}' \left(\frac{m}{2\pi KT}\right)^{3/2} h\left(X' - \frac{u(X')}{\Omega}\right) \times \exp\left(-\frac{mv'^2}{2KT} - \frac{mu^2(X')}{2KT} + \frac{e\Phi(X')}{KT}\right).$$

Hence,

$$n_0^t(k) = (\Omega/\Omega_s) h_a^t(k)\, e^{-k^2 r_c^2/2} , \quad \text{(A3)}$$

where

$$h_a^t(k) = \int_{-\infty}^{\infty} dx\, e^{-ikx} h\left(x - \frac{u(x)}{\Omega}\right) \times \exp\left(-\frac{mu^2(x)}{2KT} + \frac{e\phi(x)}{KT}\right) .$$

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[1]S. J. Buchsbaum and A. Hasegawa, Phys. Rev. Letters 12, 685 (1964).

[2]S. J. Buchsbaum and A. Hasegawa, Phys. Rev. 143, 303 (1966).

[3]G. A. Pearson, Phys. Fluids 9, 2454 (1966).

[4]J. C. Azevedo and M. L. Vianna, Phys. Rev. 177, 300 (1969).

[5]C. L. Hedrick, Ph.D. dissertation (University of California at Los Angeles, 1970) (unpublished).

[6]Y. Shima and T. K. Fowler, Phys. Fluids 8, 2245 (1965).

[7]D. K. Bhadra and L. D. Pearlstein, Phys. Rev. A 1, 898 (1970).

[8]I. B. Bernstein, Phys. Rev. 109, 10 (1958).

[9]A. Sivasubramanian, Ph.D. dissertation (University of Massachusetts, 1971) (unpublished).

[10]V. D. Shafranov, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1967), Vol. 3, p. 78.

[11]J. P. Holdren, Stanford University Institute of Plasma Research Report No. 351, 1970 (unpublished).

[12]I. Stakgold, *Boundary Value Problems of Mathematical Physics* (Macmillan, New York, 1967), Vol. II, p. 42.

[13]N. I. Muskhelishvile, *Singular Integral Equations* (P. Noordhoff, Groningen, The Netherlands, (1953).

[14]N. G. Van Kampen, Physics 21, 949 (1955).

[15]K. M. Case, Ann. Phys. (N.Y.) 7, 349 (1959).

[16]H. L. Berk and D. L. Book, Phys. Fluids 12, 649 (1969).

[17]T. Tang and A. Sivasubramanian, Phys. Fluids 15, 1534 (1972).

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Search for Bose-Einstein Condensation in Superfluid ^{4}He †

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A search was made for Bose–Einstein condensation in superfluid ^{4}He using neutron inelastic scattering. A two-component scattering distribution has been resolved for ^{4}He at 1.2 °K giving direct evidence for the existence of the zero-momentum state. Analysis of the measured data leads to a condensate fraction of $(2.4 \pm 1)\%$, a value much smaller than previously estimated.

London[1] suggested in 1938 that some of the unusual properties of superfluid ^{4}He might be the result of a fraction of the ^{4}He atoms undergoing Bose–Einstein condensation to a zero-momentum state. Hohenberg and Platzman[2] pointed out that this conjecture of a zero-momentum state could be checked by performing a neutron scattering experiment with a momentum transfer sufficiently high that the neutron scattering could be considered to take place from individual helium atoms. In this case the scattered neutrons would be Doppler shifted by the motion of the helium atoms and the measured neutron energy distribution would reflect the momentum distribution of the helium. The scattering law for such a process is given by

$$S(\vec{Q}, \omega) = \sum_{\vec{p}} n(\vec{p})\delta(\hbar\omega - (\hbar^2/2M)(Q^2 + 2\vec{Q}\cdot\vec{p})) , \quad (1)$$

where $n(\vec{p})$ is the helium momentum distribution, $\vec{Q}$ is the momentum transfer, and M is the neutron mass.

The result of the scattering experiment would thus be a peak centered at $\hbar^2Q^2/2M$ whose shape reflects the momentum distribution of the helium atoms. If Bose–Einstein condensation takes place below the λ point, one would expect to observe a broad distribution representing the momentum of the normal helium atoms plus a sharp peak representing that fraction of atoms occupying the zero-momentum state. The peak representing the condensate atoms will be broadened by final-state interactions; however, it is expected that it can be resolved from the broad distribution of the normal helium atoms.[2–6]

Previous experiments searching for Bose–Einstein condensation in ^{4}He have been performed by Cowley and Woods[7,8] and by Harling.[9,10] These experiments had either too broad a resolution or insufficient statistical accuracy to observe the two-component scattering distribution indicative of a condensate. A condensate fraction was estimated in each case; however, this estimate could not be made reliably since it was based only on the narrowing of the measured neutron distribution below the λ point.

In order to confirm directly the existence of a condensate, an experiment was undertaken on a triple-axis spectrometer at the high-flux isotope reactor. The experiment was performed with a fixed incoming neutron energy E_0 of 182.47 ± 0.07 meV and a fixed scattering angle of $135.00° \pm 0.02°$, which gives a momentum transfer of 14.33 ± 0.01 Å^{-1}. The fixed-scattering-angle experiment was employed to obtain the largest momentum transfer possible and to facilitate stacking extra shielding around the spectrometer. The extra shielding was particularly important since the signal was only about 1 count per minute in the peak. The resolution energy width for the constant angle scan used in the experiment was about 2.1 meV.

The four-dimensional resolution ellipsoid for the triple-axis spectrometer was calculated using methods similar to those suggested by Cooper and Nathans.[11] The dimensions of the ellipsoid were then checked by measurements with single crystals around the elastic scattering position for several different energies. In addition the resolution of the monochromator and its associated collimators was checked by measuring the energy width of the incoherent scattering from vanadium using a high-resolution analyzing system. The analyzer resolution was then checked by doing similar scans using high-resolution incident beams. All resolution measurements were in good agreement with the calculated resolution function. The value of the resolution width quoted above was then calculated by passing the four-dimensional resolution ellipsoid in a series of steps through an infinitely thin ^{4}He dispersion surface along the trajectory in energy-momentum space corresponding to the constant scattering angle scan and by obtaining the intersected area for each step. The resolution width quoted is the full width at half-maximum of the distribution so obtained. With the calculations as a guide, each resolution element of the spectrometer was optimized to obtain the maximum neutron intensity.

The measurements were alternated between helium at 4.2 and 1.2 °K. The 4.2 °K results serve as a check on the experiment since no condensate should be observed above the λ point. Because the signal was very small, long counting times were needed and the total counting time amounted to about 5 mon. The measured scattering law for the two temperatures is shown in Fig. 1. All data have been normalized to one run, which represents about 20-min counting time per point. Of course many runs were performed, especially in the area near the peak top, where evidence for a condensate is expected. The data at 1.2 °K do exhibit a two-component distribution indicative of a Bose–Einstein condensation. The effect, however, is quite subtle and to demonstrate it more clearly the absolute value of the slope of the measured data is plotted in Fig. 2 starting from the point on the peak sides where the slope is the largest. The change in slope on both sides of the 1.2 °K peak representative of a two-part distribution is clearly observable.

$S(\vec{Q}, \omega)$ was not corrected for the resolution width since it is not necessary for the observation of the condensate; also the resolution does not materially affect the assignment of a condensate fraction since the main distribution and condensate peak are broadened equally. However, $S(\vec{Q}, \omega)$ has been corrected for the change in volume of the resolution ellipsoid and the change in analyzer and counter efficiency when the outgoing energy E' is changed in the experiment. This is necessary since as E' gets smaller the analyzer resolution gets narrower. Techniques for performing these corrections are discussed by Tucciarone, Lau, Corliss, Delapalme, and Hastings.[12] In our case the correction is small, because when the outgoing energy decreases and gives better resolution, the efficiencies of the analyzer crystal and of the neutron detector increase, and the two effects tend to counterbalance each other. The correction essentially consists of multiplying each point in turn by a slowly varying function, and it is somewhat similar to correcting for a small sloping background.

One must be careful that multiple reflections in the analyzing crystal do not distort the neutron data. The effect of multiple reflections was assayed over the energy range of interest by observing the reflected intensity of a white neutron beam incident upon the analyzer crystal. The analyzer was positioned about its scattering vector so that multiple reflections did not influence the

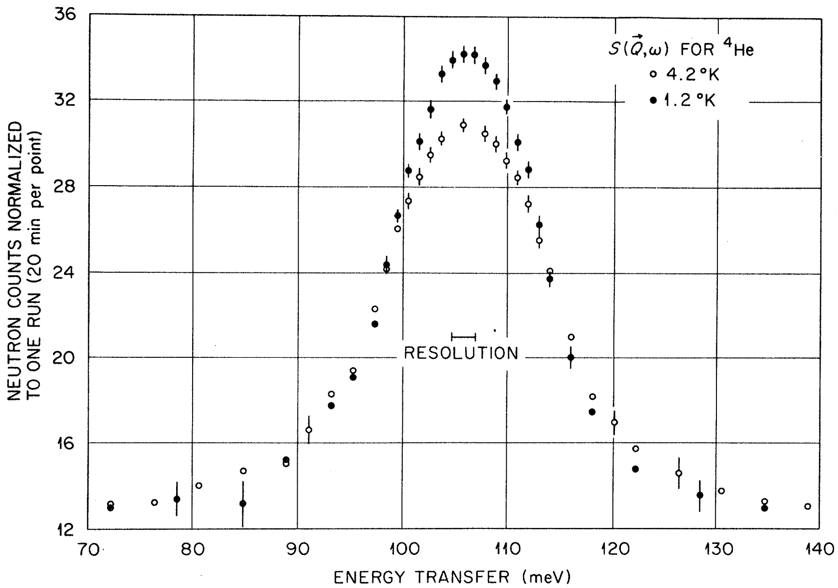

FIG. 1. $S(\vec{Q},\omega)$ for ^{4}He at 4.2 and 1.2 °K.

measured results and no correction was applied for them. A detailed account of the data analysis and the result of extraction of momentum distributions from the data will be reported elsewhere. It is interesting to note, however, that the width of the condensate peak is considerably larger than our resolution and that broadening from final-state effects appears to be larger than is expected.[2-6]

The 4.2 °K data have been corrected for the change in density of liquid He between 4.2 and 1.2 °K since the decreased density at 4.2 °K means fewer scattering centers would be available at this temperature. The areas of the curves then appear to be equal, although the areas are somewhat uncertain because of uncertainties in where to place the background. If the background is taken as 12.9, the areas are equal to within 1%.

Since the change in slope of the 1.2 °K curve indicates the onset of the condensate distribution, it is easy to get a rough idea of the condensate fraction by drawing a curve through the slope change that completes the distribution of the normal helium atoms and by taking the area above this curve to be the condensate. This type of estimate gives

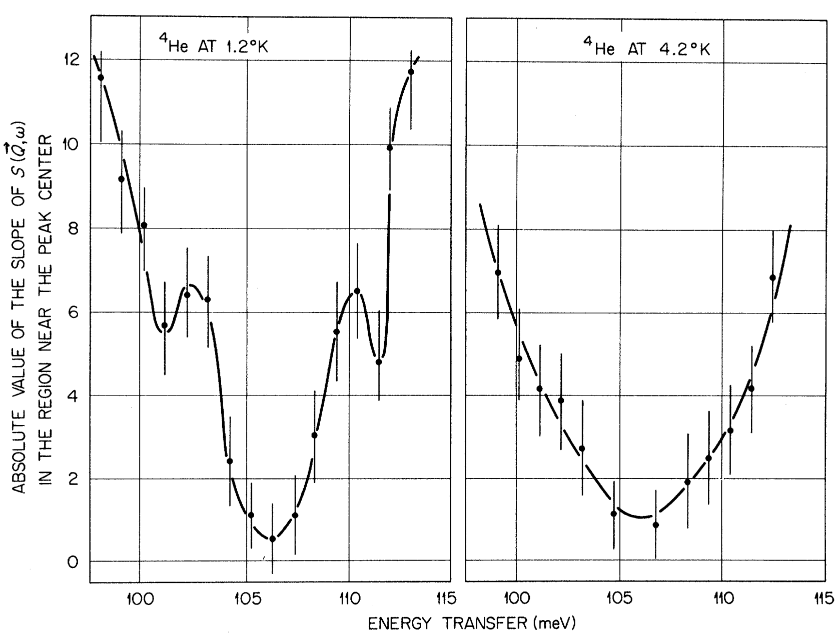

FIG. 2. Absolute value of the slope of $S(\vec{Q},\omega)$ taken directly from the measured data plotted from the point on sides of the distribution where the slope is near its greatest value. The lines are merely smooth curves drawn through the points and not the result of any analytical fit to the data.

TABLE I. Parameters from Eq. (2) obtained by a least-squares fitting of the measured data. Parameters giving peak heights are in counts per run and parameters representing peak widths or positions are in mV. Width parameters are left undivided by $2(\ln 2)^{1/2}$ so that they can be directly compared with the measured full width at half-maximum of $S(\vec{Q}, \omega)$.

Parameters	1.2 °K	4.2 °K
A_0	12.90	12.90
A_1	12.47	11.97
A_2	106.22	106.22
A_3	$20.76/2(\ln 2)^{1/2}$	$23.28/2(\ln 2)^{1/2}$
A_4	7.48	5.69
A_5	106.22	106.22
A_6	$12.82/2(\ln 2)^{1/2}$	$13.48/2(\ln 2)^{1/2}$
A_7	1.60	0.0
A_8	106.22	...
A_9	$4.92/2(\ln 2)^{1/2}$	...

a condensate fraction between 2 and 3%. To obtain a more reliable estimate, a Gaussian analysis of the data similar to that used by Harling[9,10] and by Puff and Tenn[13] was undertaken. This proved to be completely unsatisfactory as the measured distributions are decidedly non-Gaussian. $S(\vec{Q}, \omega)$ is much too steep on the sides and has too large a wing contribution to be well fitted by any Gaussian distribution. However, the distribution could be fitted satisfactorily by adding a higher-order term to the Gaussian distribution and thus three more parameters. A Gaussian distribution appears to represent satisfactorily the condensate contribution. It was thus assumed that the measured distribution could be fitted by the function

$$I(E)=A_0+A_1\, e^{(E-A_2)^2/A_3^2}+A_4\, e^{(E-A_5)^4/A_6^4}+A_7\, e^{(E-A_8)^2/A_9^2}, \quad (2)$$

where the first two terms fit the main momentum distribution and the last describes the condensate. The parameters were found by a least-squares analysis of the data and are given in Table I. A good fit to the data at 4.2 °K was found with parameter A_7 equal to zero; however, the 1.2 °K data required a nonzero A_7 parameter, as shown in the table. The condensate fraction was determined to be $(2.4 \pm 1)\%$. Preliminary results of a more sophisticated analysis of our data by Gersch *et al.* indicate a similar value for the condensate fraction.[14] These results are considerably smaller than the theoretical estimates of the condensate fraction, which range from 6% to 25%.[15-18,3]

Another possible way of analyzing the data would be to say that all the ^{4}He atoms represented by the area above the 4.2 °K distribution and below the 1.2 °K distribution belong in the condensate and that the structure in the peak of the curve reflects structure in the condensate itself. This would give a condensate fraction of about 10%. However, this seems extremely unlikely since there appears to be no *a priori* reason to expect structure in the scattering from the atoms in the condensate. Furthermore, the wing contributions to the peak at 4.2 °K, which probably stem from the roton excitations, are much smaller at 1.2 °K. This area in the wings would be expected to merge into the main body of the peak for 1.2 °K and not necessarily contribute to the condensate. Also, some narrowing of the momentum distribution for the atoms not in the condensate would be expected as the temperature is decreased from 4.2 to 1.2 °K. Taking all considerations into account, we find that the small value for the condensate fraction gives the best interpretation of the experimental data.

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[1]F. London, Nature 141, 643 (1938).

[2]P. C. Hohenberg and P. M. Platzman, Phys. Rev. 152, 198 (1966).

[3]W. C. Kerr, K. N. Pathak, and K. S. Singwi, Phys. Rev. A 2, 2416 (1970).

[4]W. C. Kerr, K. N. Pathak, and K. S. Singwi, Phys. Rev. A 4, 2413 (1971).

[5]H. A. Gersch and Phil N. Smith, Phys. Rev. A 4, 281 (1971).

[6]H. A. Gersch, L. J. Rodriguez, and Phil N. Smith, Phys. Rev. A 5, 1547 (1972).

[7]R. A. Cowley and A. D. B. Woods, Phys. Rev. Letters 21, 787 (1968).

[8]R. A. Cowley and A. D. B. Woods, Can. J. Phys. 49, 177 (1971).

[9]O. K. Harling, Phys. Rev. Letters 24, 1046 (1970).

[10]O. K. Harling, Phys. Rev. A 3, 1073 (1971).

[11]M. J. Cooper and R. Nathans, Acta Cryst. 23, 357 (1967).

[12]A. Tucciarone, H. Y. Lau, L. M. Corliss, A. Delapalme, and J. M. Hastings, Phys. Rev. B 4, 3206 (1971).

[13]R. D. Puff and J. S. Tenn, Phys. Rev. A 1, 125 (1970).

[14]H. A. Gersch, Phil N. Smith, and L. J. Rodriguez (private communication). The results of this analysis will be published by these authors at a later date.

[15]O. Penrose and L. Onsager, Phys. Rev. 104, 576 (1956).

[16]W. L. McMillan, Phys. Rev. 138, A442 (1965).

[17]D. Schiff and L. Verlet, Phys. Rev. 160, 208 (1967).

[18]W. E. Parry and C. R. Rathbone, Proc. Phys. Soc. (London) 91, 273 (1967).