amagat^{-1}, which is equivalent to $Z_{eff} = 34$.

The Ps quenching rate in pure NO gas is estimated to be roughly $0.15D$ nsec^{-1} amagat^{-1}. When N_2 is added to the NO gas, the quenching rate is changed to about $0.029D$ nsec^{-1} amagat^{-1}. This type of quenching is very similar to Ps in nitric acid. The formation of a metastable positronium compound PsNO* is proposed to account for the higher quenching rate for higher-energy *o*-Ps atoms The addition of N_2 increases the slowing-down rate. Therefore the quenching rate is reduced. However, the quenching rate for NO in NO + N_2 mixtures is not linearly dependent on the density of NO. At very low densities of NO the quenching rate increases to about $0.12D$ nsec^{-1} amagat^{-1}, which is the same as the rate in pure NO. It seems that in N_2 gas *o*-Ps atoms are not fully thermalized before their annihilation. A more detailed investigation in NO with various diluents may clarify this point. It is also surprising to find that the annihilation rates of both free positrons and Ps are much higher in nitrogen dioxide than those in NO.

ACKNOWLEDGMENT

The authors wish to thank Dow Chemical for their support in the gas-handling system used in this work.

*Work partially supported by U. S. AEC under Contract No. AT(11-1)-3373.

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PHYSICAL REV'EW A VOLUME 6, NUMBER 5 NOVEMBER 1972

Evolution of Explosively Unstable Systems

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The present study concerns the evolution in time of explosively unstable systems under the influence of time-dependent dissipative and nonlinear coupling effects. Generalized criteria for instability as well as new mode solutions are constructed by introducing a suitable transformation in time.

I. INTRODUCTION

So far, the studies on explosive instability[1–12] have been devoted to systems where the parameters of the problem, i.e., the coefficients of nonlinear coupling and of dissipation, are considered as constants with respect to time. Accordingly the analysis of the associated problems needs further detailed consideration, taking time dependence of the above parameters into account. An extended theory is particularly motivated by the interesting possibilities of comparison offered by recent experiments, e.g., in the field of plasma physics.[13,14]

In the approach to the problems of nonlinear interaction of waves forming a closed system, essentially two different methods have been used. These are the random-phase approach and the method of well-defined phase description, or coherent wave interaction. The situations which the two methods are intended to describe differ on essential points. External perturbations on the closed system are accounted for in the random-phase approach by allowing an uncertainty in frequency $\Delta\omega$, fulfilling the condition $\tau\Delta\omega \gg 1$, where τ is a characteristic time of the nonlinear interaction. The phase-average process enables one then to formulate the nonlinear problems in terms of numbers of "quanta."[2–5] The averaging over phases means, however, a loss of information. In the coherent wave description, where instead the condition for the related spread in frequency is $\tau\Delta\omega \ll 1$, the details of the phases of the interacting waves are retained and studied dynamically. Influence of external perturbations on the closed system of interacting waves is then disregarded. It depends on the physical situation as to which of the two descriptions will come closest to reality. For a thermonuclear plasma it may be that the random-phase description is justified, whereas in other situations, in particular those where one or more of the participating modes are laser modes, the coherent wave description may be more suitable. Many physical systems are in fact intermediate between the two limiting approaches, and thus correspond to a partially coherent description. It has been demonstrated[6,9] how the results are sensitive to phase effects, and also that phase-locking occurs during the process.[10]

In the present paper the coherent-phase description is used to investigate the evolution in time of an explosively unstable three-wave system, for which the parameters of linear dissipation and of nonlinear coupling are allowed to vary in time. The variations admitted have to be slow so as to be consistent with the method of approach. As a result of the investigation we obtain generalized criteria for instability. By means of functional transformations and a convenient transformation in time we also gain detailed information on the general solutions for the problem where the coefficients of dissipation and the nonlinear couplings vary with time in a mutually different manner.

We do not consider a simultaneous variation in time of the frequencies ω_i of the coupled waves. Such variations may naturally occur in certain physical situations and should then, in order to cause resonant interaction, fulfill the frequency matching conditions and be in accordance with the dispersion relations for the waves. Effectively there would be a separate variation in time of the coupling coefficients due to the varying frequency shifts. However, the dynamic relations will not be changed in form, provided the time-dependent frequencies fulfill the frequency-matching conditions, as can easily be checked. Since we allow for a general time variation of the coupling coefficients we have therefore, in fact, implicitly included in our formal treatment of the problem the possibility of slight variations in the frequencies. Besides the dissipative terms, in particular, seem to be especially sensitive to changes of the system parameters for practical situations that we have investigated.

II. BASIC EQUATIONS

In the well-defined phase description the basic equations for three interacting modes are[9]

$$\frac{\partial a_0}{\partial t} - i\omega_0 a_0 = c_{12}^* a_1 a_2 ,$$
$$\frac{\partial a_1}{\partial t} - i\omega_1 a_1 = c_{02} a_0 a_2^* , \qquad (1)$$
$$\frac{\partial a_2}{\partial t} - i\omega_2 a_2 = c_{10} a_0 a_1^* ,$$

where the amplitudes of the normal modes are denoted by a_j and the asterisk refers to the complex conjugate.

The wave vectors $\vec{k}_j$ are assumed to satisfy the relation

$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2 ,$$

and the frequencies fulfill the condition

$$\mathrm{Re}(\omega_0) = \mathrm{Re}(\omega_1) + \mathrm{Re}(\omega_2) .$$

We may consider here the imaginary part of the frequency, $\mathrm{Im}(\omega_j)$, as well as the coupling coefficients c_{ij}, to be dependent on time. The coefficients c_{ij} are generally complex quantities in the presence of linear damping or growth $[\mathrm{Im}(\omega_j) \equiv \nu_j(t) \neq 0]$.

It is convenient then to introduce the following notations:

$$c_{ij}(t) \equiv |c_{ij}(t)|\, e^{i\theta_{ij}(t)}$$

and

$$g_{ij}(t) \equiv |c_{ij}(t)| / |c_{ij}(0)| .$$

Putting

$$a_j(t) = A_j(t)\, e^{i[\mathrm{Re}(\omega_j)]t} , \quad A_j(t) \equiv |A_j(t)|\, e^{i\phi_j(t)}$$

and introducing furthermore the real quantities

$$u_0 = [|c_{01}(0)|\, |c_{02}(0)|]^{1/2} |A_0(t)| ,$$
$$u_1 = [|c_{01}(0)|\, |c_{12}(0)|]^{1/2} |A_1(t)| ,$$
$$u_2 = [|c_{02}(0)|\, |c_{12}(0)|]^{1/2} |A_2(t)| ,$$

and

$$\Phi = \phi_0(t) - \phi_1(t) - \phi_2(t) ,$$

we obtain from Eq. (1)

$$\frac{\partial u_0}{\partial t} + \nu_0(t) u_0 = g_{12}(t) u_1 u_2 \cos[\Phi + \theta_{12}(t)] ,$$

$$\frac{\partial u_1}{\partial t} + \nu_1(t) u_1 = g_{02}(t) u_0 u_2 \cos[\Phi + \theta_{02}(t)] , \qquad (2)$$

$$\frac{\partial u_2}{\partial t} + \nu_2(t) u_2 = g_{01}(t) u_0 u_1 \cos[\Phi + \theta_{01}(t)] ,$$

and

$$\frac{\partial \Phi}{\partial t} = -g_{12}(t) \frac{u_1 u_2}{u_0} \sin[\Phi + \theta_{12}(t)] - g_{02}(t) \frac{u_0 u_2}{u_1} \sin[\Phi + \theta_{02}(t)] - g_{01}(t) \frac{u_0 u_1}{u_2} \sin[\Phi + \theta_{01}(t)] . \qquad (3)$$

III. EXPLOSIVE INSTABILITY

As in the case where the coefficients of dissipation and mode coupling do not vary in time[6–12] we would expect the system of Eqs. (2) and (3) to have unbounded solutions for certain conditions of the coefficients when these are also dependent on time. These would then correspond to the "explosive" instability and diverge, in the well-defined phase description, as $u_j \sim (t_\infty - t)^{-1}$.

In order to formulate, in the general case where $\nu_j(t) \neq 0$ and $g_{ij}(t) \neq 1$, a necessary condition for an explosive instability to occur, we write

$$u_j = f_j(t)/(t_\infty - t) , \qquad (4)$$

where $f_j(t)$ is assumed to be a positive and slowly varying function of time as t approaches the explosion time at t_∞. We then have from Eq. (2) three equations of the type

$$f_j^2(t_\infty) = g_{jk}^{-1}(t_\infty) g_{jl}^{-1}(t_\infty) \times \{\cos[\Phi(t_\infty) + \theta_{jk}(t_\infty)] \cos[\Phi(t) + \theta_{jl}(t_\infty)]\}^{-1} , \qquad (5)$$

with the constraint that all

$$\cos[\Phi(t_\infty) + \theta_{ij}(t_\infty)] > 0 . \qquad (5a)$$

Since furthermore all g_{ij} are positive, we conclude from Eqs. (5) and (5a) that a *necessary condition* for the presence of an explosive instability is that the phase angles $\theta_{ij}(t)$ of the coupling coefficients $c_{ij}(t)$ all lie in the same half-plane at the time of explosion. Since we may assume that the time variation of the angles $\theta_{ij}(t)$, and then also of their mutual differences, is known beforehand, this condition could in each case be retraced to a certain necessary initial condition on the angles $\theta_{ij}(0)$. When θ_{ij} are constants these conditions are obviously identical.[9] From Eqs. (3)–(5) we obtain

$$\tan[\Phi(t_\infty) + \theta_{12}(t_\infty)] + \tan[\Phi(t_\infty) + \theta_{02}(t_\infty)] + \tan[\Phi(t_\infty) + \theta_{01}(t_\infty)] = 0, \qquad (6)$$

which, as we notice, is independent of $g_{ij}(t)$ and which together with (5a) determines uniquely the asymptotic value of the phase $\Phi(t_\infty)$.

Let us investigate, furthermore, the time t_∞ for the special case $\mathrm{Im}(\omega_j) \equiv \nu_j(t) = \nu(t)$, where all $\nu_j(t)$ are equal but time dependent and $g_{ij}(t) = g(t)$, i.e., all $g_{ij}(t)$ are equal, but all θ_{ij} are constants in time. We then introduce the transformation

$$U_j = u_j \exp[\textstyle\int_0^t \nu(t') dt'] , \qquad (7)$$

$$\tau = \textstyle\int_0^t \exp\{-\int_0^{t'} [\nu(t'') + \eta(t'')] dt''\} dt' , \qquad (8)$$

where

$$\eta(t) = -\frac{d}{dt}[\ln g(t)] . \qquad (8a)$$

The forms (8) and (8a) enable us to treat the two time-dependent effects, represented by the coefficients $\nu(t)$ and $\eta(t)$ on the same basis, which is an essential fact for the discussion of the problem. From Eqs. (2) and (3) we then obtain the following simple set of equations:

$$\frac{\partial U_0}{\partial \tau} = U_1 U_2 \cos(\Phi + \theta_{12}) ,$$

$$\frac{\partial U_1}{\partial \tau} = U_0 U_2 \cos(\Phi + \theta_{02}) , \qquad (9)$$

$$\frac{\partial U_2}{\partial \tau} = U_0 U_1 \cos(\Phi + \theta_{01}) ,$$

$$\frac{\partial \Phi}{\partial \tau} = -\frac{U_1 U_2}{U_0} \sin(\Phi + \theta_{12}) - \frac{U_0 U_2}{U_1} \sin(\Phi + \theta_{02}) - \frac{U_0 U_1}{U_2} \sin(\Phi + \theta_{01}) . \qquad (10)$$

We nctice that Eqs. (9) and (10) are identical to Eqs. (2) and (3) with the terms $\nu_j(t) u_j$ omitted and the factors $g_{ij}(t)$ put equal to 1. The new mode solutions u_j accounting for time-dependent coefficients are then obtained by substituting for the time τ in the arguments of the solutions to Eqs. (9) and (10) the expression (8) and furthermore making use of relation (7).

By means of Eq. (8) we may then directly relate the time t_∞ to the corresponding time $\hat{t}_\infty$ for the simplified problem where there is no linear damping or growth present and where the coupling constants are initially the same as in the actual problem, but independent of time. We thus have from Eq. (8)

$$\hat{t}_\infty = \int_0^{t_\infty} \exp[-\textstyle\int_0^{t'} [\nu(t'') + \eta(t'')] dt''] dt' \qquad (11)$$

which defines the new t_∞ in terms of $\hat{t}_\infty$. From relation (11) we conclude that if the integral

$$I(t') = \int_0^{t'} [\nu(t'') + \eta(t'')] \, dt'' \qquad (11a)$$

reaches a certain level for t' values in the interval of integration, then, assuming a given time $\hat{t}_\infty$, we have in the critical limit

$$\hat{t}_\infty = \beta_c^{-1} = \int_0^\infty \exp\{-\textstyle\int_0^{t'} [\nu_c(t'') + \eta_c(t'')] dt''\} dt' . \qquad (11b)$$

For larger values of the functions ν and η such that the corresponding value of β yields $\beta\hat{t}_\infty > 1$ we will not be able to find a real value of t_∞ as a solution to Eq. (11). The explosive instability is then nonexistent for the given value of $\hat{t}_\infty$, as determined by the initial values of the U_j's $[u_j(0)=U_j(0)]$, which we here assume fixed and subject to a definite upper limit.

In order to discuss the case where all $\nu_j(t)\neq 0$ and all $g_{ij}(t)\neq 1$ are mutually different it is convenient to introduce the function

$$v_j(t) = u_j(t)\exp[-\int_0^t \eta_j(t')\,dt'] , \tag{12}$$

where, e.g.,

$$\eta_0(t) = -\frac{d}{dt}\ln|g_{01}(t)\,g_{02}(t)|^{1/2} \tag{12a}$$

and the other η_j's are obtained by permuting indices. We then have from Eqs. (2) and (3)

$$\frac{dv_0}{dt} + [\nu_0(t)+\eta_0(t)]v_0 = v_1v_2\cos[\Phi+\theta_{12}(t)] ,$$

$$\frac{dv_1}{dt} + [\nu_1(t)+\eta_1(t)]v_1 = v_0v_2\cos[\Phi+\theta_{02}(t)] , \tag{13}$$

$$\frac{dv_2}{dt} + [\nu_2(t)+\eta_2(t)]v_2 = v_0v_1\cos[\Phi+\theta_{01}(t)] ,$$

and

$$\frac{d\Phi}{dt} = -\frac{v_1v_2}{v_0}\sin[\Phi+\theta_{12}(t)] - \frac{v_0v_2}{v_1}\sin[\Phi+\theta_{02}(t)] - \frac{v_0v_1}{v_2}\sin[\Phi+\theta_{01}(t)] . \tag{14}$$

If we specialize to $\theta_{ij}(t)=0$, Eqs. (13) and (14) yield

$$\frac{dv_0^2}{dt} + 2[\nu_0(t)+\eta_0(t)]v_0^2 = \frac{dv_1^2}{dt} + 2[\nu_1(t)+\eta_1(t)]v_1^2 = \frac{dv_2^2}{dt} + 2[\nu_2(t)+\eta_2(t)]v_2^2 = 2v_0v_1v_2\cos\Phi \tag{15}$$

and

$$v_0v_1v_2\sin\Phi = \Gamma\exp\{-\int_0^t \sum[\nu_j(t')+\eta_j(t')]dt'\} , \tag{16}$$

where

$$\Gamma = v_0(0)v_1(0)v_2(0)\sin\Phi(0) .$$

The relations (15) and (16), which may be regarded as generalizations of the expressions for the constants of motion[6] which exist in the case where $\nu_j\equiv 0$, $g_{ij}\equiv 1$, and $\theta_{ij}\equiv 0$, are useful for the problem of constructing mode solutions for the case where $\nu_{ij}(t)\neq 0$ and $g_{ij}(t)\neq 1$. In fact, we may conclude from Eqs. (15) and (16) that, in the initial phases of the evolution in time of the system, i.e., when the amplitudes are small, the modes will experience their individual dissipations $\nu_j(t)$ whereas closer to the time of explosion t_∞ the modes will become dissipated in a collective way and their common dissipation will tend towards a mean value of the dissipations of the single modes. This transition is a result of the nonlinear character of the interaction and in fact a phenomenon that is, it seems,[11] complicated to describe in detail through the transition domain in time. The main features in the region are, however, correctly described by means of the transformation

$$v_j = U_j(\tau)\exp[-\int_0^t h_j(t')\,dt'] , \tag{17}$$

$$\tau = \int_0^t \exp[-\int_0^{t'}\sigma(t'')\,dt'']dt' , \tag{18}$$

where

$$h_j(t) = \sigma(t) - [\sigma(t) - \nu_j(t) - \eta_j(t)]\frac{d}{dt}\left[\left(1-\frac{t}{t_\infty}\right)^\alpha t\right] \tag{17a}$$

and

$$\sigma(t) \equiv \tfrac{1}{3}\sum[\nu_j(t)+\eta_j(t)] . \tag{18a}$$

The solution expressed by Eqs. (17) and (18), where $U_j(\tau)$ satisfies Eqs. (9) and (10), is correct in the limit of small times and has the desired asymptotic behavior for t approaching t_∞. The quantity α in Eq. (17a) determines where the point of inflection of the exponent in Eq. (17) occurs, i.e., it is a measure of where the transition takes place between individual and collective dissipation. It can be determined through detailed comparison with the results of computer experiments based on Eqs. (2) and (3).

For constant or slowly varying values of the quantities $\nu_j(t)$ and $\eta_j(t)$ we find from Eqs. (17) and (17a) that the time value for the inflection of the exponent of Eq. (17) may be expressed as

$$t_{\text{inf}}/t_\infty \cong 2(1+\alpha)^{-1} .$$

Preliminary computer experiments[12] indicate that α takes a value around 2, in a particular example near 1.5.

Equation (18) expresses the connection between the explosive times $t=t_\infty$ and $\tau=\hat{t}_\infty$, as related to the systems described by Eqs. (12)–(18) and (9) and (10), respectively. It thus accounts for the shift of the time of explosion caused by the influence of mutually different time-dependent dissipation and nonlinear mode coupling coefficients. From Eq. (18) we also find, as in Eq. (11b), the critical limit where the explosive instability is removed to infinitely large time values when the upper limit of integration in Eq. (18) is put equal to infinity, $t=\infty$, and for marginally critical choice of the function $\sigma(t)$ in Eqs. (18) and (18a).

IV. CONCLUDING REMARKS

We emphasize from Eqs. (18) that for each value of the time t we have a differential ratio $d\tau/dt$ that clearly demonstrates the effects of the time-dependent dissipation and mode coupling coefficients on the instantaneous time scale for the argument of the functions $U_j(\tau)$. From Eqs. (18) and (18a) we notice that the effects of a growing damping or a decrease in the nonlinear coupling coefficients are generally, as expected, to move the time value corresponding to a certain mode amplitude towards a higher value, and eventually to infinity for critical values of the time-dependent coefficients. The effect of, e.g., an increased damping in a limited time domain of the evolution of an explosive instability is therefore to flatten the curve in this region and to postpone the explosion. This is an interesting observation with respect to saturation effects originating, e.g., from higher-order nonlinear contributions.

It should be mentioned in conclusion, that it is a straightforward procedure to extend the methods that we have exploited here to a discussion of the evolution of explosively unstable systems consisting of more than three waves, i.e., to an n-wave system.

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PHYSICAL REVIEW A VOLUME 6, NUMBER 5 NOVEMBER 1972

Plane Waves of Constant Amplitude in Nonlinear Dielectrics

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Two rather unusual effects which are associated with the propagation of plane waves of constant amplitude in nondispersive isotropic nonlinear electromagnetic media are presented. One is the distortion-free transmission of information through a nonlinear medium, by "polarization modulation" of a plane wave of constant amplitude. The other is the reflection and refraction of a monochromatic wave, obliquely incident on the plane interface between a linear medium and a nonlinear one, so that the transmitted wave, in the nonlinear medium, is also monochromatic. These effects are described by exact solutions of the nonlinear system of equations and they should be of interest, in particular, in laser physics.

I. INTRODUCTION

In a previous paper,[1] it was shown that a *circularly polarized* monochromatic plane electromagnetic wave can propagate in a nondispersive isotropic nonlinear medium just as it does in a linear medium. The problem of the reflection and transmission of such waves at normal incidence on the plane interface between a linear medium and a nonlinear one was solved by elementary means, requiring only the solution of an algebraic equation for the amplitude of the transmitted wave (in the nonlinear medium). This transmitted wave is also circularly polarized and it is monochromatic, which is rather surprising.

The existence of a more general wave of *constant amplitude*, of which the circularly polarized wave is a special case, was pointed out in a footnote in Ref. 1. The primary purpose of this present paper is to draw attention to the unusual properties of waves of constant amplitude and especially to point to their possible use for distortion-free transmission of information through a nonlinear medium. There is also the hope that it might lead