Range-Energy Relations for N, Na, and Ar Ions (0.3-2.0 MeV) in Ar, N_2, O_2, and Air*

Robert A. Langley
Sandia Laboratories, Albuquerque, New Mexico 87115
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Ionization extrapolated ranges have been measured for N ions (0.3–2.0 MeV) in N_2 and air, for Na ions (0.3–2.0 MeV) in air, and for Ar ions (0.3–1.0 MeV) in Ar, N_2, O_2, and air. A magnetically analyzed beam of ions was brought through a differential-pumping system into a gas stopping cell. A single-grid ionization chamber, movable along the beam axis, was used to obtain the data necessary to plot a Bragg curve. The observed range-energy relations are compared with previous data where possible. The accuracy of the measurements is ± 3%.

I. INTRODUCTION

Energy-loss data and range-energy relations for heavy particles $Z > 2$ in gases have been sparingly studied in the past. Investigations using accelerators have been performed mainly by Soviet investigators.[1–4] Many laboratories have investigated the range-energy relations of fission fragments,[5–11] but these latter studies are generally confined to specific energies and usually do not cover an energy range. The theoretical study of heavy-ion ranges is very limited[12,13] although heavy-ion ranges are important in various fields of science, e.g., nuclear medicine, atmospheric explosions, and atomic and molecular interaction theory. For these reasons range-energy measurements were made for N, Na, and Ar ions in N_2, O_2, and air for ion energies between 0.3 and 2.0 MeV.

The ionization extrapolated range was determined by impinging a monoenergetic beam of ions on a gas target and obtaining a Bragg curve. The Bragg curve was determined by plotting the ionization produced by the beam in a collection chamber vs the distance of the collection chamber from the entrance aperture of the gas cell. By drawing the steepest tangent to this curve and extrapolating to zero ionization current, the ionization extrapolated range is obtained. Unless otherwise noted, "range" means the ionization extrapolated range in cm at 15 °C and 760 Torr. To obtain the range in units of mg/cm^2, multiply the range in centimeters by 0.0424 times the molecular weight of the target.

This paper describes experimental apparatus and procedures, presents collected data, and discusses and compares these results with previous measured and calculated results.

II. EXPERIMENTAL APPARATUS AND PROCEDURE

A schematic diagram of the experimental system is shown in Fig. 1. In this work, as in a previous work,[14] a focused magnetically analyzed ion beam was obtained from a Van de Graaff generator. The beam passed through a differential pumping section before entering the stopping chamber (gas cell). The system was energy calibrated by studying resonances of $F^{19}(p, \alpha)O^{16}$. The possible error in the calibration was 1%. A complete description of the physical aspects of the experiment is given in Ref. 14.

The energy degradation of the incident particles in the differential pumping chamber was not neg-

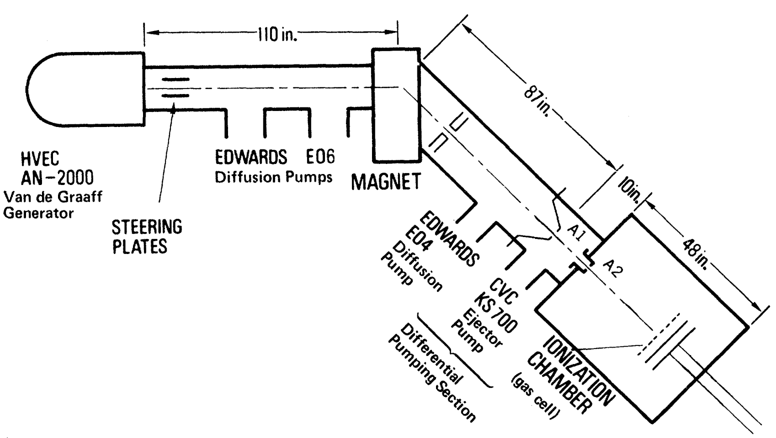

FIG. 1. Schematic diagram of the accelerator and experimental apparatus.

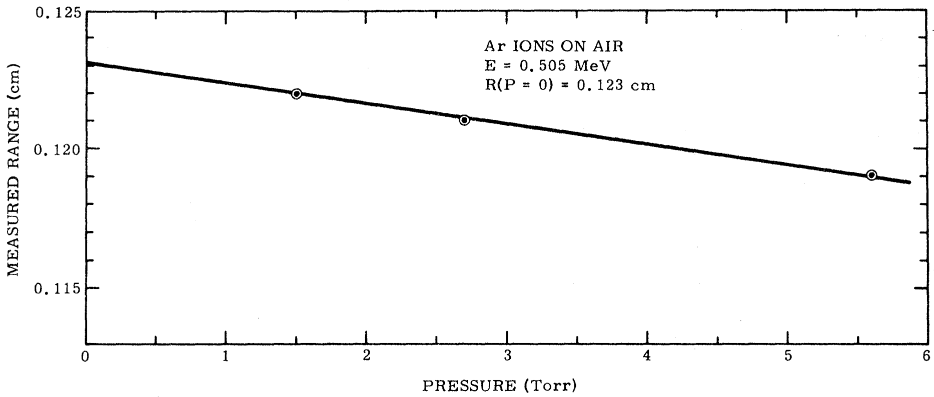

FIG. 2. Measured range vs stopping-cell pressure.

ligible in the present experiment as it was in the previous experiment.[14] This effect was demonstrated by measuring the "range" as a function of gas-cell pressure P. Figure 2 shows a typical set of these data. Data of this type were taken for all collision partners at all of the measured energies. The "correct" ionization extrapolated range was taken as the $P=0$ intercept of the best-fit line through the experimental points. This "correction" was usually within 2% of all measured ranges. It was found necessary to use this correction procedure here in contrast to the previous experiment (projectiles: H^+ and He^+) because the ratio of energy lost in the differential pumping region to energy lost in the gas cell was considerably higher for the heavy ions than for the light ions. This effect is due to the difference in the shapes of the stopping-power curves for the light ions and heavy ions in the energy range up to 2 MeV.

All gases were from commercial tanks. The stated purity was 99.995% for nitrogen, 99.5% for oxygen, and 99.99% for argon. Air was obtained from bottled dry air which had less than one part in 10^5 water vapor.

The data were taken by impinging the monoenergetic beam of ions on a gas absorber and measuring the ionization current produced by the beam in the ion chamber as the chamber was moved continuously into or out of the gas cell. The ionization extrapolated range was determined by plotting a Bragg curve and extrapolating the steepest tangent to this curve to zero current, with this intercept being defined as the ionization extrapolated range.[15]

Typical Bragg curves of the heavy and light ions in gases are shown in Fig. 3. The target gas for all three projectiles was air. The scales are arbitrary and different for each projectile. The origin is zero for both the abscissa and ordinate. Since the stopping cross sections for heavy ions reach their maximum at energies much higher than the maximum of this experiment, the stopping cross section is decreasing as the energy decreases, and the break in the Bragg curve near the end of the range is not as steep as it is for light projectiles. Under these conditions any relationship between the mean range and the extrapolated range

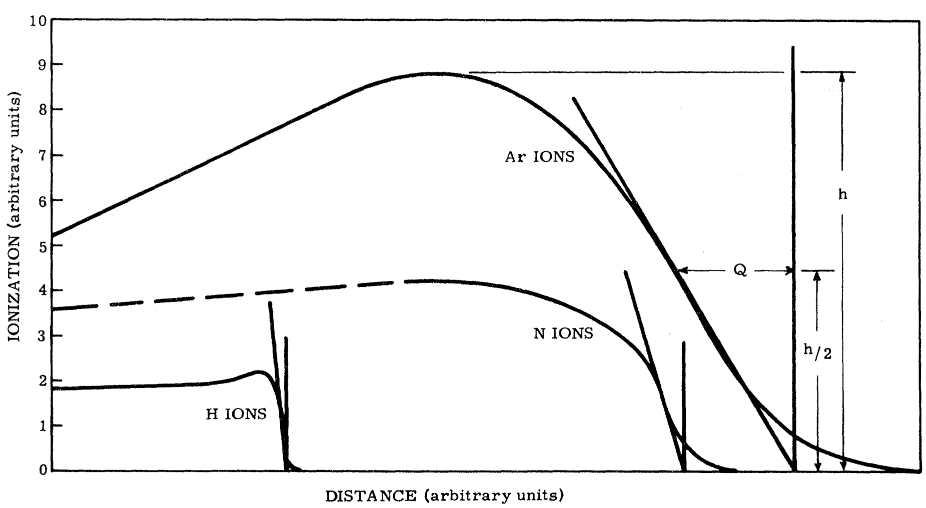

FIG. 3. Specific ionization curves of Ar^+, N^+, and H^+ in air (arbitrary units).

TABLE I. Ionization produced beyond the ionization extrapolated range as a percentage of the total ionization produced.

Collision partners	Energy (MeV)	%
$Ar^+ + Ar$	0.3	3.7
	0.5	2.0
	1.0	0.83
$Ar^+ + N_2$	0.5	2.3
$Ar^+ + O_2$	0.3	3.1
	0.5	2.4
	1.0	0.52

becomes tenuous. If one is interested in the ionization effect of the ions, then the extrapolated range is the more meaningful of the two ranges. As the projectile energy increases, ionization that is produced in the region beyond the extrapolated range must become a smaller fraction of the total ionization. To obtain quantitative values for this decrease, the area under the Bragg curve out to the extrapolated range and the total area under the curve were determined for argon projectiles. Presented in Table I are the results, which are given as the percentage of total ionization that is produced in the region beyond the extrapolated range.

A range straggling factor can be defined by extracting from the Bragg curve the value Q/R, where Q is the distance from the Bragg curve at half-maximum height to the extrapolated range R (see Fig. 3). The value Q/R is essentially ϵ of Ref. 16 except for the contribution to Q due to multiple scattering effects.[17] For the case of light ions, $Z \leq 2$, this contribution is small, but for heavier ions it is possible that this contribution can become a substantial portion of Q.

Since the slope of the Bragg curve near the end of the range for the heavy ions is less than that for the light ions, an additional error is introduced over the errors in the light-ion data because the zero-ionization intercept is less precisely defined. The beam current was kept below 1×10^{-7} A to reduce the error produced by the effect of local heating.[18] A complete discussion of the errors involved and their magnitude is given in Ref. 14. The estimated total possible error in any measured range was $\pm 3\%$.

III. EXPERIMENTAL RESULTS

Ionization extrapolated ranges of N, Na, and Ar ions in Ar, N_2, O_2, and air, normalized to 760 Torr and 15 °C, are shown in Tables II–IV. The actual measured ranges are presented along with interpolations from the measured ranges (so that values are given every 100 keV) using the "cubic-spline-fitting" method.[19] The energies at which the data were taken varied from 0.3 to 2.0 MeV except for Ar ions; an upper energy limitation of 1.0 MeV for Ar ions was imposed by the limiting field of the magnet.

Also shown in Tables II–IV is the range straggling factor Q/R. This is given in percentage. The values of Q/R decrease with increasing energy and increase with increasing mass of the projectile. For example, the values of $Q/R\times100$ for the target gas air at a projectile energy of about 1.0 MeV are 2.0, 6.4, 10.0, and 11.0 for H, N, Na, and Ar projectiles, respectively.

TABLE II. Ionization extrapolated ranges in cm[a] for N ions in gaseous targets at 15 °C and 760 Torr and range straggling factor in percentage.

Energy	N_2		Dry air	
(MeV)	Range (cm)	Q/R (%)	Range (cm)	Q/R (%)
		Experimental results		
0.384	0.163	9.0		
0.799	0.235	8.0		
1.000	0.270	7.0		
1.197	0.297	6.5		
1.415	0.329	6.3		
1.577	0.346	6.0		
1.830	0.389	5.3		
2.024	0.403	5.0		
0.451			0.169	9.0
0.711			0.221	8.0
1.005			0.273	6.8
1.194			0.300	6.4
1.398			0.329	5.8
1.659			0.358	5.3
1.874			0.384	5.0
2.055			0.403	4.7
		Interpolated results		
0.300	0.140		0.129	
0.400	0.167		0.157	
0.500	0.187		0.180	
0.600	0.204		0.200	
0.700	0.219		0.219	
0.800	0.235		0.238	
0.900	0.253		0.256	
1.000	0.270		0.272	
1.100	0.284		0.287	
1.200	0.297		0.301	
1.300	0.313		0.315	
1.400	0.327		0.329	
1.500	0.338		0.341	
1.600	0.349		0.352	
1.700	0.366		0.363	
1.800	0.384		0.375	
1.900	0.398		0.387	
2.000	0.403		0.398	

[a] N_2: 1.0 cm = 1.187 mg/cm^2; air: 1.0 cm = 1.221 mg/cm^2.

TABLE III. Ionization extrapolated range in cm[a] for Na ions in air at 15 °C and 760 Torr and range straggling factor in percentage.

Energy (MeV)	Dry air Range (cm)	Q/R (%)
	Experimental results	
0.397	0.179	13.0
0.600	0.242	12.0
0.783	0.286	11.0
0.949	0.326	10.0
1.149	0.361	9.5
1.385	0.394	8.7
1.563	0.419	8.1
1.773	0.446	7.0
	Interpolated results	
0.300	0.142	
0.400	0.180	
0.500	0.213	
0.600	0.242	
0.700	0.266	
0.800	0.290	
0.900	0.315	
1.000	0.336	
1.100	0.354	
1.200	0.368	
1.300	0.382	
1.400	0.396	
1.500	0.410	
1.600	0.424	
1.700	0.437	
1.800	0.449	

[a]Air: 1.0 cm = 1.221 mg/cm.

IV. DISCUSSION OF RESULTS

Figures 4–6 show the range-energy relations for N ions, Na ions, and Ar ions, respectively. The solid symbols on each graph show the present results and the open symbols show previous measurements.

Figure 4 shows the present results with the results of Teplova *et al.*,[1] Evans *et al.*,[4] and Blackett and Lees.[6] The ranges as measured by Teplova *et al.* are about 10% longer than the present measured ranges. This difference is within the experimental error of 11% quoted by Teplova *et al.* Also shown are the results of Blackett and Lees, which agree quite well with the present data considering that their results were obtained by studying the tracks of recoil atoms in a Wilson cloud chamber. The data for Na ions are presented in Fig. 5. Also shown are the results of Teplova *et al.* Although referred to by them as the "maximum range," the quantity measured by Teplova *et al.* is the ionization extrapolated range as defined by Holloway and Livingston.[15] Teplova *et al.* also measured the mean range and found the "maximum range" to be 3–5% longer. In Fig. 6 are shown the present data for argon ions incident on Ar, N_2, O_2, and air. Also included are the results of Evans *et al.* ($E < 200$ keV), Teplova *et al.*, and Blackett and Lees.

When the projectile atom mass is greater than the target atom mass, the possibility exists that in

TABLE IV. Ionization extrapolated ranges in cm[a] for Ar ions in gaseous targets at 15 °C and 760 Torr and range straggling factor in percentage.

Energy (MeV)	N_2 Range (cm)	N_2 Q/R (%)	O_2 Range (cm)	O_2 Q/R (%)	Dry air Range (cm)	Dry air Q/R (%)	Ar Range (cm)	Ar Q/R (%)
				Experimental results				
0.358	0.094	19						
0.514	0.133	16						
1.034	0.217	11						
0.321			0.083	19				
0.510			0.126	16				
1.030			0.211	11				
0.326					0.085	19		
0.505					0.123	16		
1.029					0.213	11		
0.325							0.100	22
0.509							0.144	16
1.025							0.231	5
				Interpolated results				
0.300	0.079		0.078		0.079		0.093	
0.400	0.105		0.102		0.101		0.119	
0.500	0.130		0.124		0.122		0.142	
0.600	0.153		0.145		0.141		0.163	
0.700	0.173		0.163		0.160		0.182	
0.800	0.191		0.180		0.177		0.199	
0.900	0.205		0.195		0.193		0.214	
1.000	0.215		0.208		0.209		0.228	

[a]N_2 = 1.0 cm = 1.187 mg/cm^2; O_2: 1.0 cm = 1.357 mg/cm^2; air: 1.0 cm = 1.221 mg/cm^2; Ar: 1.0 cm = 1.696 mg/cm^2.

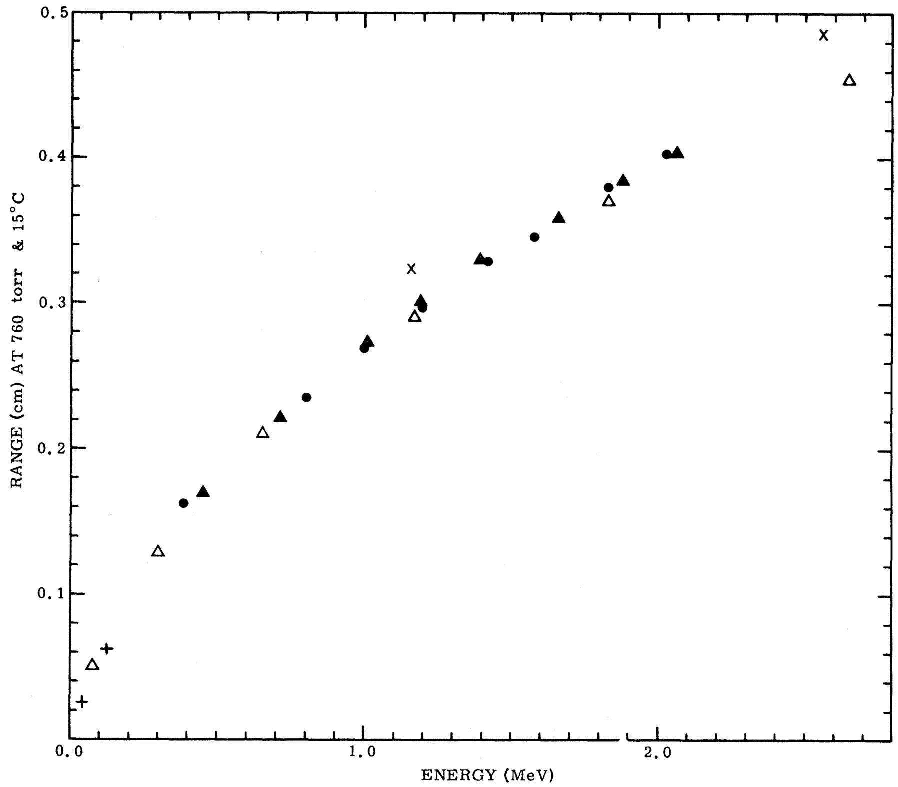

FIG. 4. Range of N ions in air and N_2. ▲, N ions on air (present results); ●, N ions on N_2 (present results); ×, N ions on air (Teplova *et al.*, 1958); +, N ions on N_2 (Evans *et al.*); △, N ions on air (Blackett and Lees).

a nuclear collision a velocity is imparted to a target atom which is greater than the incoming projectile atom velocity. In this case the measured range will result from the range of the projectile atoms plus an additional contribution from the struck target atoms. The size of the contribution depends on the range of the target atoms in the target gas, the number of these target atoms which undergo a hard collision, and the energy at which these collisions become dominant (i.e., the energy at which the nuclear stopping cross section σ_N becomes greater than the electronic stopping cross section σ_e).

The total stopping cross section consists of the electronic stopping cross section and the nuclear stopping cross section, i.e.,

$$\sigma = \sigma_e + \sigma_N \ . \qquad (1)$$

In the energy range of this experiment,

$$\sigma_e \cong KV \ , \qquad (2)$$

where V is the lab velocity of the projectile and K is a constant for each set of collision partners but increases as the projectile mass increases for a particular target. This relationship is approximately true as long as the projectile velocity is less than the velocity at which σ_e passes through its maximum. This implies that at a particular velocity the lighter the projectile the longer the range. As the projectile velocity decreases, the contribution from the nuclear stopping becomes increasingly important[20]; it has been estimated

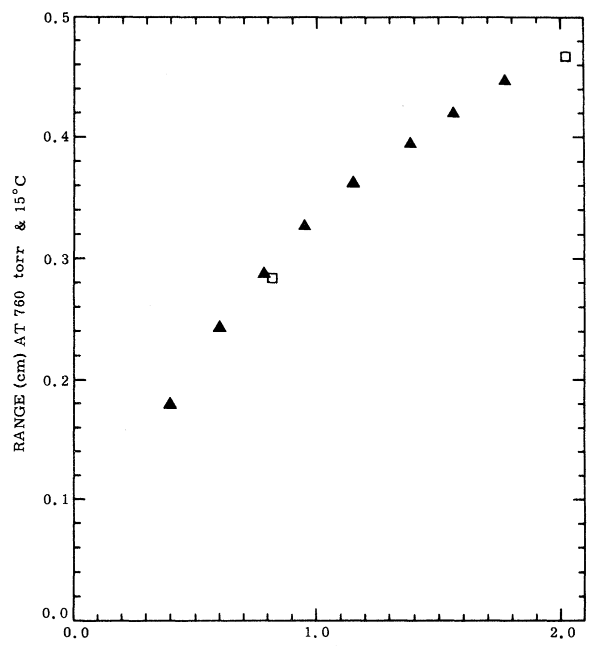

FIG. 5. Range of Na ions in air. ▲, present data; □, Teplova *et al.*, 1959.

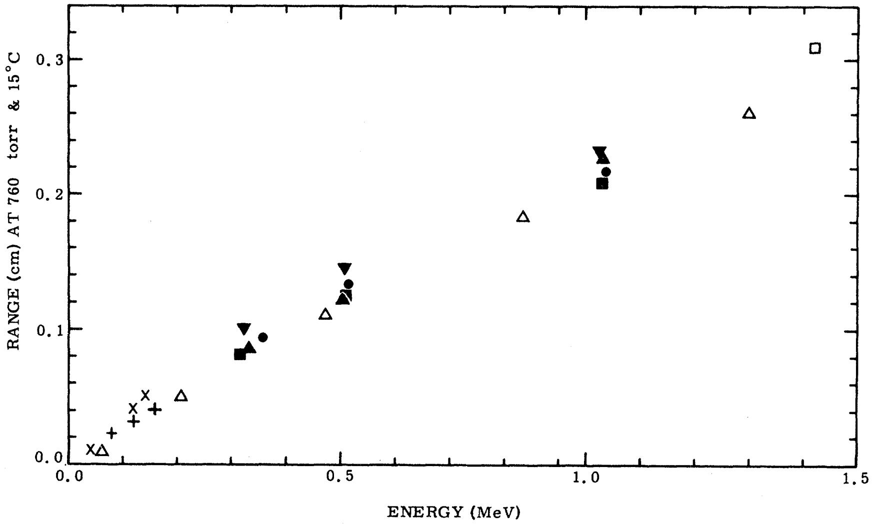

FIG. 6. Range of Ar ions in air, N_2, O_2, and Ar. ▲, Ar ions on air; ●, Ar ions on N_2; ■, Ar ions on O_2; ▼, Ar ions on Ar; +, Ar ions on N_2 (Evans *et al.*); ×, Ar ions on Ar (Evans *et al.*); □, Ar ions on air (Teplova *et al.*, 1959); Δ, Ar ions on air (Blackett and Lees).

that σ_N could be the same order of magnitude as σ_e at energies of 200–300 keV for argon projectiles in nitrogen.[21] Thus, the present range data may contain a contribution from the struck target atoms. Consider, for example, argon projectiles in a nitrogen target. The ranges for argon and nitrogen in air are given in Tables IV and II, respectively. For this set of collision partners the maximum velocity which could be imparted to the nitrogen atom is 1.7 times the argon ion velocity. These results imply that toward the end of the "argon-ion range" a substantial portion of the ionization may be produced by the struck nitrogen ions. This experiment does not distinguish between the separate contributions made to the range by the argon and nitrogen ions.

It is stated in the article by Teplova *et al.* that "the range of ions with velocities $\leq 6\times10^8$ cm/sec should be approximately proportional to the velocity" and that "the experimental points deviate from

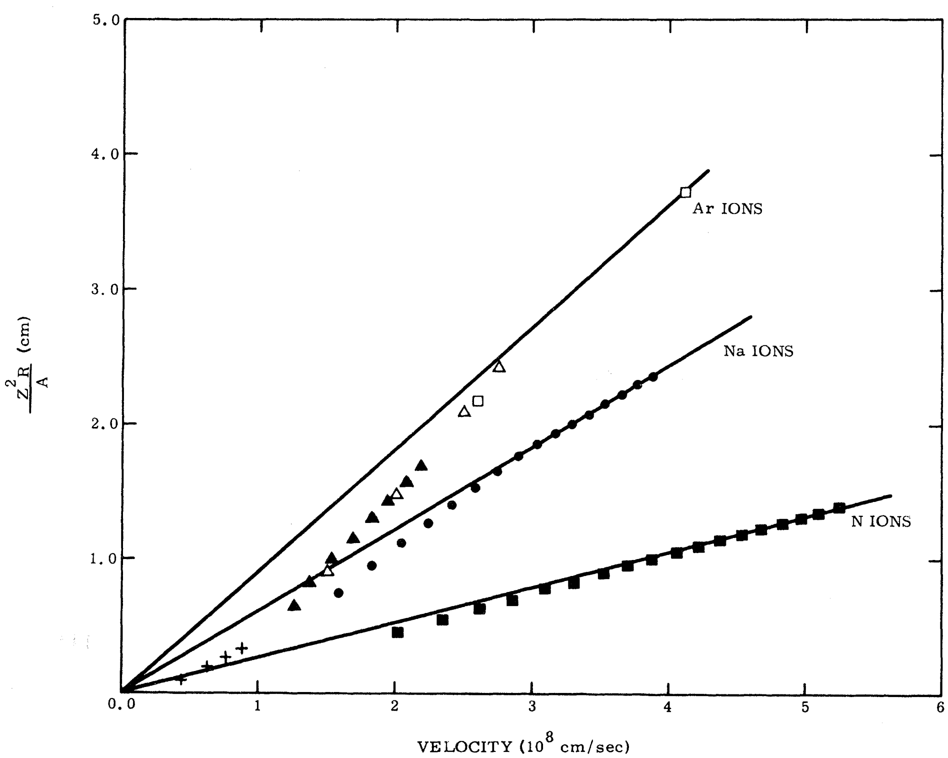

FIG. 7. Comparison of range vs projectile velocity for Ar, Na, and N ions. The range is expressed in units of Z^2/A in order to facilitate display of the data on a single graph and comparison with the data of Ref. 1, where Z is the atomic number of the projectile and A is its atomic mass. ▲, Ar^+ in air (present results); ●, Na^+ in air (present results); ■, N^+ in air (present results); +, Ar^+ in N_2 (Evans *et al.*); Δ, Ar^+ in air (Blackett and Lees); □, Ar^+ in air (Teplova *et al.*, 1959).

straight lines drawn through the origin by less than 5%." This statement does not appear to be strictly true. Figure 7 shows the range in units of $(Z^2/A)R$ as a function of the velocity of the projectile where Z is the atomic number of the projectile and A is its atomic weight. The results for N ions show a break from the straight line through the origin at about 3.5×10^8 cm/sec and for Na ions at about 2.7×10^8 cm/sec. For Ar ions the present results do not go high enough in velocity to ascertain the exact situation, but they nevertheless suggest that a break might occur at 2.7×10^8 cm/sec. Since the range is zero at zero velocity, the slope of each curve must decrease as $V=0$ is approached, as is demonstrated by observing the low-energy curve of Evans *et al.* for Ar ions in N_2. It appears that the straight-line-through-the-origin approximation of Teplova *et al.* is not correct below a velocity of about 3×10^8 cm/sec; at higher velocities it appears to be valid, but it is not expected to be correct at velocities higher than the maximum of the stopping-power curve.[1]

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Multiphoton Ionization of Hydrogen Atoms in the Semiclassical Treatment of an Intense Radiation Field

Surya P. Tewari*

Department of Physics and Astrophysics, University of Delhi, Delhi, India

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The transition probability for the multiphoton ionization of the ground-state hydrogen atom by an intense electromagnetic field has been calculated following a nonperturbative method developed by Reiss. One distinctive feature of the calculation is a minor reduction in the power-law dependence of the interaction parameter in the transition probability.

I. INTRODUCTION

Recently Bebb and Gold[1] calculated the ionization probability of the hydrogen atom by simultaneous absorption of several photons using the perturbation technique. Although the perturbation technique is useful in ordinary quantum electrodynamics, its application to the interaction of an intense radiation field with matter suffers from difficulties. For instance, at high intensities of the electromagnetic