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Improved Stark-Profile Calculations for the He II Lines at 256, 304, 1085, 1216, 1640, 3203, and 4686 Å*

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The frequency distributions of the He II lines at 256, 304, 1085, 1216, 1640, 3203, and 4686 Å, broadened by the local fields of both ions and electrons in a plasma, are calculated in the classical-path approximation. The calculations for the electron broadening are based on an impact theory which accounts for perturbations to both upper and lower levels of the radiating ion. The theory allows (approximately) for the level splitting caused by the fields of the (perturbing) ions, finite duration of the collisions, screening of the electron fields, and includes (approximately) the effects of inelastic collisions. The effect of perturbations by singly charged ions is calculated in the quasistatic linear Stark-effect approximation using Hooper's microfield distribution functions. Comparison to previous calculations shows fair agreement with some increase in linewidth. Comparison with experimental results is less conclusive. Fair agreement is found with some results while others reveal a discrepancy of as much as a factor of 2 in the line wings.

I. INTRODUCTION

In a previous paper[1] (hereafter referred to as KG) the results of calculations of the Stark broadening by ions and electrons of the first four Balmer lines were presented. Comparison with experimental results showed deviations of less than or about 10% between measured and calculated half-widths. In the present paper we present the results of similar calculations for the lines of ionized helium at 256, 304, 1085, 1216, 1640, 3203, and 4686 Å. As in KG, the calculations utilize the classical-path approximation. The calculations for the electron broadening are based on an impact theory which accounts for perturbations to both upper and lower levels of the radiating ion, and allows (approximately) for the level splitting caused by the fields of the (perturbing) ions, for finite duration of the collisions, and for screening of the electron fields.

The effects of perturbations by singly charged,[2] structureless ions is calculated in the quasistatic linear Stark-effect approximation, using Hooper's[3] microfield distribution. In KG it was assumed that inelastic collisions[4] between electrons and the radiating system (atom) could be neglected; in the present paper the effects of such inelastic collisions are (at least approximately) included in the line profile.

To first nonvanishing order in a perturbation expansion, the operator which describes the impact broadening by all the electrons is

$$\Phi_{ab}=2\pi vN\int d\rho\,\rho\Big[\Big(\frac{i}{\hbar}\Big)^2\Big(\int_{-\infty}^{\infty}dt\,e^{iH_at/\hbar}\,V_a(t)e^{-iH_at/\hbar}\int_{-\infty}^{t}dt'\,e^{iH_at'/\hbar}\,V_a(t')e^{-iH_at'/\hbar}$$
$$-\int_{-\infty}^{\infty}dt\,e^{iH_at/\hbar}\,V_a(t)e^{-iH_at/\hbar}\int_{\infty}^{\infty}dt\,e^{iH_bt/\hbar}\,V_b^*(t)e^{-iH_bt/\hbar}$$
$$+\int_{-\infty}^{\infty}dt\,e^{iH_bt/\hbar}\,V_b^*(t)e^{-iH_bt/\hbar}\int_{-\infty}^{t}dt'\,e^{iH_bt'/\hbar}\,V_b^*(t')e^{-iH_bt'/\hbar}\Big)\Big]\,. \quad (1)$$

[See, for instance, Eq. (2) of Ref. 5, hereafter referred to as GS.] If one replaces all the exponentials in Eq. (1) by unity, considers the case of dipole interaction, assumes elastic collisions, and assumes that the perturbing electrons follow straight-line paths, Eq. (3) of KG is obtained. If we retain the other assumptions but allow the perturbing electrons to follow hyperbolic paths (a necessary change if the radiator is an ion) and introduce the z (charge number) dependence of the matrix elements, we obtain Eq. (11) of GS. We write this equation in the slightly different form

$$\langle \Phi_{ab} \rangle = \left\langle \frac{-4\pi N}{3v} \left(\frac{\hbar}{mZ} \right)^2 [\mathcal{H}(v)] \times (\vec{R}_a \cdot \vec{R}_a - 2\vec{R}_a \cdot \vec{R}_b + \vec{R}_b \cdot \vec{R}_b) \right\rangle , \tag{2}$$

where $\mathcal{H}(v)$ is given by[6]

$$\mathcal{H}(v) = \tfrac{1}{2} \ln \left[1 + \left(\frac{mv^2 \rho_{\max}}{(Z-1)e^2} \right)^2 \right] , \qquad v \le v_c \tag{3}$$

$$= \tfrac{1}{2}\delta^2 \left[1 - \left(\frac{v_c}{v} \right)^2 \right] + \ln \left(\frac{v_c}{v} \right) + \tfrac{1}{2} \ln \left[1 + \left(\frac{mv^2 \rho_{\max}}{(Z-1)e^2} \right)^2 \right] , \qquad v > v_c . \tag{4}$$

In Eqs. (2)–(4), a and b are the principal quantum numbers of the upper and lower states, $\vec{R}_a$ and $\vec{R}_b$ are the electron position operators for hydrogen (in atomic units), $\rho_{\max}$ is the maximum impact parameter cutoff and will be discussed further below, $\langle \cdots \rangle$ indicates an average over the velocity distribution, δ is a parameter which, for the time being, we set equal to one, and

$$v_c = \sqrt{\tfrac{3}{2}} \, [e^2 Z(Z-1)/\hbar (a^2 - b^2)] . \tag{5}$$

In KG it was argued that the Lewis and adiabatic cutoffs could be incorporated by letting

$$\rho_{\max} = \min\{v/\omega_c , \rho_D\} , \tag{6}$$

where ρ_D is the Debye length and ω_c is the larger of the Stark splitting caused by ion fields and the difference between the observed and ion-field-shifted frequencies. It was further argued that one can, to within a factor of 2, set ω_c equal to the mean quasistatic splitting of the levels in a given group. Thus we set

$$\omega_c = 5 \, a^2 \hbar N^{2/3} / Zm . \tag{7}$$

As in KG we define[7]

$$\rho'_{\min} = (\hbar/mv)(a^2 - b^2)/Z , \tag{8}$$

$$y_1 = \frac{mv_1^2}{2kT} = \frac{5a^2}{Z^2} (a^2 - b^2) \frac{E_H}{kT} a_0^2 N^{2/3} , \tag{9}$$

$$y_2 = \frac{mv_2^2}{2kT} = \frac{25}{8\pi} \frac{a^4}{Z^2} a_0 N^{1/3} , \tag{10}$$

where v_1 and v_2 are the velocities such that $\rho_{\max} = \rho'_{\min}$ and the "Lewis" cutoff is equal to the Debye length ($v_2/\omega_c = \rho_D$). We further introduce

$$y_3 = \frac{E_H}{kT} \left(\frac{5(Z-1) a^2 a_0 N^{2/3}}{Z} \right)^{2/3} , \tag{11}$$

$$y^* = e^2 (Z-1)/2kT\rho_D , \tag{12}$$

$$y_c = \frac{mv_c^2}{2kT} = \frac{3E_H}{2kT} \left(\frac{Z(Z-1)}{a^2 - b^2} \right)^2 . \tag{13}$$

In terms of these parameters the result of the velocity average is[8]

$$\left\langle \frac{\mathcal{H}(v)}{v} \right\rangle \equiv G(y_1, y_2, y_3, y^*, y_c, \delta)$$

$$= \left(\frac{2m}{\pi kT} \right)^{1/2} \left[e^{-y'_c} \left[\delta^2 + \ln \left(\frac{y_c}{y'_c} \right) \right] - E_1(y'_c)(1 + \delta^2 y_c) + \Theta(y_2 - y_1) \left(e^{-y_1} \ln \left[1 + \left(\frac{y_1}{y_3} \right)^3 \right] - e^{-y_2} \ln \left[1 + \left(\frac{y_2}{y_3} \right)^3 \right] \right.$$

$$\left. + e^{y_3} [E_1(y_1 + y_3) - E_1(y_2 + y_3)] + e^{-y_3 c} [E_1(y_1 - y_3 c) - E_1(y_2 - y_3 c)] + e^{-y_c \bar{c}} [E_1(y_1 - y_3 \bar{c}) - E_1(y_2 - y_3 \bar{c})] \right)$$

$$+ e^{-y'_2} \ln \left[1 + \left(\frac{y'_2}{y^*} \right)^2 \right] - e^{iy^*} [E_1(iy^*) - E_1(y'_2 + iy^*)] - e^{-iy^*} [E_1(-iy^*) - E_1(y'_2 - iy^*) + 2g(y^*)] \Big] , \tag{14}$$

where $y'_c = \max(y_1, y_c)$, $y'_2 = \max(y_1, y_2)$, Θ is the unit step function, $c = \frac{1}{2}(1 + i\sqrt{3})$, $\bar{c} = \frac{1}{2}(1 - i\sqrt{3})$, and

$$g(Z) \equiv -\mathrm{Ci}(Z)\cos(Z) - \mathrm{si}(Z)\sin(Z)$$

is discussed in Ref. 9. The source of the largest uncertainty in Eq. (2) is ω_c. This quantity enters G through y_1, y_2, and y_3, where the dependence is linear, quadratic, and $\frac{2}{3}$ power, respectively. Since these uncertainties are correlated, the total uncertainty can be reduced by replacing G by the following average:

$$\bar{G} = \tfrac{1}{2} [G(2y_1, 4y_2, 2^{2/3} y_3, y^*, y_c, \delta) + G(\tfrac{1}{2} y_1, \tfrac{1}{4} y_2, 2^{-2/3} y_3, y^*, y_c, \delta)] . \tag{15}$$

To assess the accuracy of the dipole approxima-

tion, we now calculate a quadrupole term. For the quadrupole interaction one has

$$\frac{1}{\hbar}\int_{-\infty}^{\infty} U_q(t)\,dt = \frac{-3e^2}{2\hbar}\,r_a^2\int\frac{\cos^2\theta-\frac{1}{3}}{|r(t)|^3}\,dt\ . \quad (16)$$

Since the perturbing electrons follow hyperbolic paths one can use

$$1/r = (1/P)(1-\epsilon\cos\theta)\ , \quad (17)$$

where

$$\epsilon = \{1+[m\rho v^2/(Z-1)e^2]^2\}^{1/2} \quad (18)$$

and

$$P = \rho(\epsilon^2-1)^{1/2}\ . \quad (19)$$

The integration can be carried out explicitly (reduced to a single quadrature); however, since it is only a correction to the dipole term, an approximation derived by Griem[10] is adequate. With this approximation to the quadrupole term added as a correction to the dipole term, the electron broadening operator becomes

$$\Phi_{ab} = \frac{-4\pi}{3}N\left(\frac{\hbar}{mZ}\right)^2\left(\frac{m}{2\pi kT}\right)^{1/2} \times\left[\bar{G}+\frac{8kT}{3E_H}+\left(\frac{2\pi}{\delta(a^2-b^2)}\right)^2\right] \times\{\vec{R}_a\cdot\vec{R}_a-2\vec{R}_a\cdot\vec{R}_b+\vec{R}_b\cdot\vec{R}_b\}\ . \quad (20)$$

We recognize the first term as the dipole contribution, the second as the quadrupole contribution in the case of straight-line paths (KG), and thus the last term as the correction to the quadrupole contribution due to hyperbolic paths. This latter can be quite large in the case of L_α and B_α. The parameter δ was introduced to guard against the violation of unitarity of the S matrix due to the quadrupole contribution. Thus, for L_α (B_α), δ^2 must be set equal to 2 (1.4). To remain consistent one must also increase $\rho_{\min}$ by δ in the dipole term.

If one relaxes the assumption of elastic collisions [does not replace the exponentials in Eq. (1) by unity or zero], the evaluation of the broadening operator becomes quite tedious. Since we expect that inelastic collisions do not contribute a major part to the broadening, any reasonable estimate of their broadening strength will suffice. In this spirit we use a semiempirical Gaunt factor for their broadening strength.[11] With the inelastic collisions thus accounted for, an explicit representation of the electron broadening operator is

$$\langle\alpha'|\beta'|\Phi_{ab}|\alpha''\rangle\beta''\rangle = \delta_{\beta'\beta''}\Big[G^*\sum_{\sigma\alpha}\langle\alpha'|R_{a\sigma}|\alpha\rangle\langle\alpha|R_{a\sigma}|\alpha''\rangle + \Big(\sum_{\substack{\sigma\gamma\\ n_\gamma\neq n_\alpha}}\frac{\pi}{\sqrt{3}}\langle\bar{g}(\Delta E_{n_\alpha,n_\gamma})\rangle\langle\alpha'|R_{a\sigma}|\gamma\rangle\langle\gamma|R_{a\sigma}|\alpha''\rangle\Big)\Big] + \delta_{\alpha'\alpha''}\Big[G^*\sum_{\sigma\beta}\langle\beta'|R_{b\sigma}|\beta\rangle\langle\beta|R_{b\sigma}|\beta''\rangle + \Big(\sum_{\substack{\sigma\gamma\\ n_\gamma\neq n_\beta}}\frac{\pi}{\sqrt{3}}\langle\bar{g}(\Delta E_{n_\beta,n_\gamma})\rangle\langle\beta'|R_{b\sigma}|\gamma\rangle\langle\gamma|R_{b\sigma}|\beta''\rangle\Big)\Big] - 2G^*\sum_\sigma\langle\alpha'|R_{a\sigma}|\alpha''\rangle\langle\beta'|R_{b\sigma}|\beta''\rangle\ , \quad (21)$$

where G^* is the coefficient of the matrix elements in Eq. (20), $\langle\bar{g}(\Delta E)\rangle$ is the thermal average of the semiempirical Gaunt factors,[12] and $\Delta E_{n,n'}$ is the difference in energy levels between states of ionized helium with principal quantum number n and n'. The matrix elements for the inelastic collision terms can be written as

$$\sum_{\substack{\nu\\ \nu\neq n}} A_n^\nu,\ m,\ j,\ k\ ,$$

where

$$A_n^\nu,\ m,\ n,\ k \equiv \sum_{\sigma,i,l}\langle n,\ m,\ j|R_\sigma|\nu,\ i,\ l\rangle \times\langle\nu,\ i,\ l|R_\sigma|n,\ m,\ k\rangle\ .$$

Since the A_n^ν near the diagonal decrease rapidly as $|n-\nu|$ increases (see Appendix III of Ref. 13), the sum can be approximated by the term with $\nu=n+1$. In fact, for the elements near the diagonal the term with $\nu=n+1$ is at least 60% of the sum over ν. (The Gaunt factors, which multiply the matrix elements, also decrease as $|n-\nu|$ increases; however, they are temperature dependent so that no single error bound can be given). The relative error incurred by using only the $\nu=n+1$ term for the elements further off the diagonal can be quite large, however, since these off-diagonal terms are much smaller than the near diagonal ones, the actual error (the error in the electron contribution to the line profile) is negligible. Thus, since the entire inelastic contribution is small, all terms except those with $n_\gamma=n_\alpha(n_\beta)+1$ were dropped in the final calculations.

The probable error in the ϕ matrix elements may be estimated from

$$\Delta G^2 = \tfrac{1}{2}\Big\{\tfrac{1}{4}[G(2y_1,\ 4y_2,\ 2^{2/3}y_3,\ y^*,\ y_c,\ \delta) - G(\tfrac{1}{2}y_1,\ \tfrac{1}{4}y_2,\ 2^{-2/3}y_3,\ y^*,\ y_c,\ \delta)]^2 + \tfrac{1}{4}\Big[\frac{8kT}{3E_H}+\Big(\frac{2\pi}{\delta(a^2-b^2)}\Big)^2\Big]^2 + \tfrac{1}{4}[\langle\bar{g}(\Delta E_{n_a,n_{a+1}})\rangle]^2 M_a + \tfrac{1}{4}[\langle\bar{g}(\Delta E_{n_b,n_{b+1}})\rangle]^2 M_b + \tfrac{1}{4}y_1^2\Big\}\ . \quad (22)$$

TABLE I. Fractional $(1/n)$ widths A_n of the reduced Stark profiles $S(\alpha)$ from the present calculation, G^* is the coefficient of the matrix elements in Eq. (21), C_a and C_b are the thermal averages of the Gaunt factors for upper and lower states (multiplied by $2\pi/\sqrt{3}$, and E is the error estimate (%).

N_e	A_2	A_4	A_8	G^*	C_a	C_b	E
			$NA=1$ $NB=2$				
			$\lambda=304$ Å				
			$T=5\,000$ K				
10^{14}	4.60-7	8.21-7	1.31-6	14.68	0.73	0.73	7.8
10^{15}	8.36-7	1.48-6	2.35-6	12.40	0.73	0.73	9.2
			$T=10\,000$ K				
10^{14}	3.75-7	6.63-7	1.03-6	16.84	0.73	0.73	6.9
10^{15}	6.96-7	1.27-6	1.93-6	14.54	0.73	0.73	8.0
10^{16}	1.26-6	2.27-6	3.58-6	12.27	0.73	0.73	9.5
10^{17}	2.23-6	3.98-6	7.09-6	10.04	0.73	0.73	11.6
10^{18}	3.86-6	7.62-6	1.54-5	7.99	0.73	0.73	14.6
10^{19}	7.11-6	1.47-5	2.34-5	6.46	0.73	0.73	18.1
			$T=20\,000$ K				
10^{14}	3.03-7	5.29-7	8.30-7	19.08	0.73	0.73	6.3
10^{15}	5.67-7	1.01-6	1.61-6	16.78	0.73	0.73	7.2
10^{16}	1.06-6	1.88-6	2.88-6	14.49	0.73	0.73	8.3
10^{17}	1.91-6	3.47-6	5.76-6	12.21	0.73	0.73	9.9
10^{18}	3.41-6	6.64-6	1.29-5	10.00	0.73	0.73	12.1
10^{19}	6.15-6	1.30-5	2.26-5	7.97	0.73	0.73	15.2
			$T=40\,000$ K				
10^{15}	4.58-7	8.16-7	1.30-6	19.19	0.73	0.73	6.7
10^{16}	8.69-7	1.53-6	2.43-6	16.89	0.73	0.73	7.6
10^{17}	1.62-6	2.84-6	4.56-6	14.59	0.73	0.73	8.8
10^{18}	2.92-6	5.53-6	1.02-5	12.32	0.73	0.73	10.5
10^{19}	5.40-6	1.10-5	2.00-5	10.10	0.73	0.73	12.8
			$T=80\,000$ K				
10^{15}	3.70-7	6.54-7	1.01-6	21.91	0.73	0.81	6.7
10^{16}	7.12-7	1.29-6	1.96-6	19.61	0.73	0.81	7.4
10^{17}	1.35-6	2.43-6	3.77-6	17.30	0.73	0.81	8.4
10^{18}	2.53-6	4.59-6	7.72-6	15.00	0.73	0.81	9.7
10^{19}	4.72-6	9.67-6	1.83-5	12.71	0.73	0.81	11.5
			$T=160\,000$ K				
10^{16}	5.88-7	1.07-6	1.67-6	22.92	0.73	1.03	7.9
10^{17}	1.15-6	2.01-6	3.25-6	20.61	0.73	1.03	8.7
10^{18}	2.19-6	3.89-6	6.65-6	18.30	0.73	1.03	9.8
10^{19}	4.18-6	8.19-6	1.62-5	15.97	0.73	1.03	11.3
			$NA=1$ $NB=3$				
			$\lambda=256$ Å				
			$T=5\,000$ K				
10^{14}	6.45-5	9.99-5	1.46-4	10.90	0.73	0.73	2.8
10^{15}	6.32-5	9.85-5	1.45-4	8.62	0.73	0.73	3.6
			$T=10\,000$ K				
10^{14}	6.63-5	1.03-4	1.50-4	13.03	0.73	0.73	2.5
10^{15}	6.61-5	1.03-4	1.51-4	10.73	0.73	0.73	3.0
10^{16}	6.55-5	1.03-4	1.52-4	8.45	0.73	0.73	3.8
10^{17}	6.33-5	1.01-4	1.52-4	6.21	0.73	0.73	5.2
10^{18}	6.14-5	1.00-4	1.52-4	4.14	0.73	0.73	7.8
10^{19}	5.36-5	9.13-5	1.42-4	2.60	0.73	0.73	12.6

The first term is the uncertainty in the (elastic) dipole contribution, the second is the uncertainty in the quadrupole term (estimated to be $\frac{1}{2}$ of the quadrupole term), the third and fourth come from the uncertainties in the Gaunt factors (again estimated to be $\frac{1}{2}$ of the Gaunt factors). The quantities M_a and M_b are the ratio of inelastic to elastic dipole matrix elements and are set to one (worst case) in the calculation of error estimates presented in Table I. The last term in the error estimate is from the

TABLE I. (*Continued*)

N_e	A_2	A_4	A_8	G^*	C_a	C_b	E
			$NA=1$ $NB=3$				
			$\lambda=256$ Å				
			$T=20\,000$ K				
10^{14}	6.73-5	1.04-4	1.53-4	15.18	0.73	0.73	2.3
10^{15}	6.79-5	1.05-4	1.54-4	12.87	0.73	0.73	2.7
10^{16}	6.81-5	1.07-4	1.57-4	10.56	0.73	0.73	3.3
10^{17}	6.89-5	1.09-4	1.62-4	8.26	0.73	0.73	4.3
10^{18}	7.03-5	1.15-4	1.69-4	6.01	0.73	0.73	5.9
10^{19}	6.82-5	1.13-4	1.69-4	3.93	0.73	0.73	9.1
			$T=40\,000$ K				
10^{15}	6.90-5	1.07-4	1.56-4	15.07	0.73	0.91	2.9
10^{16}	6.98-5	1.09-4	1.59-4	12.75	0.73	0.91	3.4
10^{17}	7.13-5	1.13-4	1.66-4	10.42	0.73	0.91	4.2
10^{18}	7.45-5	1.24-4	1.76-4	8.09	0.73	0.91	5.4
10^{19}	8.02-5	1.36-4	1.87-4	5.79	0.73	0.91	7.8
			$T=80\,000$ K				
10^{15}	6.96-5	1.08-4	1.57-4	17.47	0.73	1.17	3.4
10^{16}	7.08-5	1.10-4	1.61-4	15.15	0.73	1.17	4.0
10^{17}	7.27-5	1.16-4	1.67-4	12.80	0.73	1.17	4.7
10^{18}	7.67-5	1.29-4	1.78-4	10.43	0.73	1.17	5.8
10^{19}	8.77-5	1.46-4	1.94-4	8.04	0.73	1.17	7.7
			$T=160\,000$ K				
10^{16}	7.14-5	1.11-4	1.61-4	18.11	0.73	1.53	5.1
10^{17}	7.36-5	1.17-4	1.68-4	15.76	0.73	1.53	5.9
10^{18}	8.02-5	1.31-4	1.79-4	13.37	0.73	1.53	7.0
10^{19}	9.18-5	1.50-4	1.97-4	10.93	0.73	1.53	8.6
			$NA=2$ $NB=3$				
			$\lambda=1640$ Å				
			$T=5\,000$ K				
10^{14}	7.72-5	1.64-4	5.05-4	11.87	0.73	0.73	4.1
10^{15}	1.56-4	4.09-4	9.36-4	9.59	0.73	0.73	5.1
			$T=10\,000$ K				
10^{14}	6.21-5	1.24-4	2.99-4	14.02	0.73	0.73	3.6
10^{15}	1.25-4	3.01-4	8.29-4	11.72	0.73	0.73	4.3
10^{16}	2.55-4	6.53-4	1.29-3	9.44	0.73	0.73	5.4
10^{17}	4.37-4	9.50-4	1.67-3	7.20	0.73	0.73	7.0
10^{18}	6.27-4	1.24-3	1.96-3	5.13	0.73	0.73	9.9
			$T=20\,000$ K				
10^{14}	4.91-5	9.32-5	1.80-4	16.25	0.73	0.73	3.3
10^{15}	9.81-5	2.24-4	6.78-4	13.95	0.73	0.73	3.9
10^{16}	2.10-4	5.60-4	1.19-3	11.64	0.73	0.73	4.7
10^{17}	3.94-4	9.15-4	1.66-3	9.34	0.73	0.73	5.8
10^{18}	6.20-4	1.27-3	2.03-3	7.09	0.73	0.73	7.7
			$T=40\,000$ K				
10^{15}	7.94-5	1.65-4	5.19-4	16.29	0.73	0.91	3.9
10^{16}	1.69-4	4.53-4	1.06-3	13.98	0.73	0.91	4.5
10^{17}	3.52-4	8.55-4	1.61-3	11.65	0.73	0.91	5.5
10^{18}	5.99-4	1.27-3	2.11-3	9.31	0.73	0.91	6.8
10^{19}	8.75-4	1.70-3	2.67-3	7.02	0.73	0.91	9.2

neglect of electrons with velocity less than v_1 (see KG).

II. RESULTS AND COMPARISON TO PREVIOUS CALCULATIONS AND EXPERIMENTS

The results of these calculations are summarized in Table I. Reference 13 contains the complete profiles, both as curves (Appendix I) and tables (Appendix II).

In Table I, A_n are the (half) fractional $1/n$ widths and are tabulated for $n=2$, 4, and 8. The coefficient G^* is the coefficient of the matrix elements in Eq. (21). C_a and C_b are the thermal averages of the Gaunt factors multiplied by $2\pi/\sqrt{3}$, and E is the

TABLE I. (*Continued*)

N_e	A_2	A_4	A_8	G^*	C_a	C_b	E
			$NA=2$ $NB=3$				
			$\lambda=1640$ Å				
			$T=80\,000$ K				
10^{16}	1.34-4	3.44-4	9.01-4	16.55	0.81	1.17	4.9
10^{17}	2.92-4	7.54-4	1.48-3	14.21	0.81	1.17	5.7
10^{18}	5.56-4	1.22-3	2.07-3	11.84	0.81	1.17	6.9
10^{19}	8.68-4	1.72-3	2.71-3	9.44	0.81	1.17	8.7
			$T=160\,000$ K				
10^{16}	1.09-4	2.47-4	7.56-4	19.67	1.03	1.53	5.9
10^{17}	2.45-4	6.61-4	1.37-3	17.33	1.03	1.53	6.7
10^{18}	4.98-4	1.15-3	1.98-3	14.94	1.03	1.53	7.8
10^{19}	8.39-4	1.69-3	2.70-3	12.49	1.03	1.53	9.4
			$NA=2$ $NB=4$				
			$\lambda=1216$ Å				
			$T=5\,000$ K				
10^{14}	2.18-3	3.19-3	4.37-3	10.51	0.73	0.73	2.6
10^{15}	2.17-3	3.22-3	4.45-3	8.21	0.73	0.73	3.3
			$T=10\,000$ K				
10^{14}	2.28-3	3.32-3	4.49-3	12.55	0.73	0.75	2.3
10^{15}	2.29-3	3.38-3	4.60-3	10.23	0.73	0.75	2.8
10^{16}	2.34-3	3.51-3	4.80-3	7.91	0.73	0.75	3.6
10^{17}	2.38-3	3.63-3	5.02-3	5.62	0.73	0.75	5.2
10^{18}	2.24-3	3.49-3	4.91-3	3.48	0.73	0.75	8.9
			$T=20\,000$ K				
10^{14}	2.33-3	3.36-3	4.52-3	14.55	0.73	0.94	2.3
10^{15}	2.35-3	3.44-3	4.64-3	12.22	0.73	0.94	2.7
10^{16}	2.43-3	3.60-3	4.87-3	9.87	0.73	0.94	3.4
10^{17}	2.57-3	3.84-3	5.40-3	7.50	0.73	0.94	4.6
10^{18}	2.66-3	4.14-3	5.79-3	5.15	0.73	0.94	7.2
10^{19}	2.29-3	3.60-3	5.15-3	2.96	0.73	0.94	14.0
			$T=40\,000$ K				
10^{15}	2.38-3	3.47-3	4.65-3	14.26	0.73	1.20	3.0
10^{16}	2.47-3	3.63-3	4.88-3	11.89	0.73	1.20	3.6
10^{17}	2.65-3	3.91-3	5.46-3	9.49	0.73	1.20	4.6
10^{18}	2.88-3	4.42-3	6.23-3	7.03	0.73	1.20	6.6
10^{19}	2.92-3	4.56-3	6.58-3	4.60	0.73	1.20	11.3
			$T=80\,000$ K				
10^{16}	2.49-3	3.64-3	4.86-3	14.16	0.81	1.58	4.3
10^{17}	2.68-3	3.92-3	5.43-3	11.73	0.81	1.58	5.2
10^{18}	2.98-3	4.52-3	6.34-3	9.23	0.81	1.58	6.9
10^{19}	3.32-3	5.13-3	7.37-3	6.68	0.81	1.58	10.3
			$T=160\,000$ K				
10^{17}	2.71-3	3.93-3	5.41-3	14.59	1.03	2.11	6.5
10^{18}	3.07-3	4.59-3	6.43-3	12.06	1.03	2.11	8.1
10^{19}	3.58-3	5.52-3	7.83-3	9.45	1.03	2.11	10.7
			$NA=2$ $NB=5$				
			$\lambda=1085$ Å				
			$T=5\,000$ K				
10^{14}	4.94-4	3.27-3	5.24-3	10.04	0.73	0.73	2.6
10^{15}	1.74-3	4.20-3	5.96-3	7.71	0.73	0.73	3.5
			$T=10\,000$ K				
10^{14}	3.67-4	2.89-3	5.04-3	11.87	0.73	0.91	2.5
10^{15}	1.30-3	4.11-3	5.96-3	9.51	0.73	0.91	3.2
10^{16}	2.47-3	4.89-3	7.01-3	7.12	0.73	0.91	4.5
10^{17}	2.96-3	5.29-3	7.53-3	4.74	0.73	0.91	7.5

TABLE I. (*Continued*)

N_e	A_2	A_4	A_8	G^*	C_a	C_b	E
			$NA=2$ $NB=5$				
			$\lambda=1058$ Å				
			$T=20\,000$ K				
10^{14}	2.69-4	2.27-3	4.72-3	13.67	0.73	1.17	2.6
10^{15}	6.98-4	3.92-3	5.81-3	11.29	0.73	1.17	3.2
10^{16}	2.28-3	4.85-3	6.96-3	8.87	0.73	1.17	4.3
10^{17}	3.12-3	5.60-3	7.86-3	6.39	0.73	1.17	6.6
10^{18}	3.23-3	5.60-3	7.92-3	3.92	0.73	1.17	12.5
			$T=40\,000$ K				
10^{15}	4.95-4	3.55-3	5.57-3	13.17	0.73	1.53	3.6
10^{16}	1.93-3	4.68-3	6.79-3	10.72	0.73	1.53	4.5
10^{17}	3.09-3	5.66-3	7.88-3	8.20	0.73	1.53	6.3
10^{18}	3.72-3	6.43-3	9.14-3	5.62	0.73	1.53	10.5
			$T=80\,000$				
10^{16}	1.43-3	4.46-3	6.57-3	12.87	0.81	2.03	5.2
10^{17}	2.94-3	4.59-3	7.75-3	10.32	0.81	2.03	6.8
10^{18}	3.89-3	6.72-3	9.49-3	7.68	0.81	2.03	9.8
			$T=160\,000$ K				
10^{17}	2.76-3	5.49-3	7.62-3	13.10	1.03	2.70	7.9
10^{18}	3.97-3	6.85-3	9.62-3	10.43	1.03	2.70	10.3
			$NA=3$ $NB=4$				
			$\lambda=4686$ Å				
			$T=5\,000$ K				
10^{14}	2.84-3	8.18-3	1.61-2	11.08	0.73	0.73	3.1
10^{15}	4.92-3	1.16-2	1.99-2	8.78	0.73	0.73	3.9
			$T=10\,000$ K				
10^{14}	2.48-3	7.54-3	1.51-2	13.22	0.73	0.75	2.7
10^{15}	4.46-3	1.10-2	1.96-2	10.90	0.73	0.75	3.3
10^{16}	6.78-3	1.46-2	2.45-2	8.58	0.73	0.75	4.2
10^{17}	9.80-3	1.93-2	3.00-2	6.29	0.73	0.75	5.8
10^{18}	1.24-2	2.33-2	3.67-2	4.16	0.73	0.75	9.0
			$T=20\,000$ K				
10^{15}	4.11-3	1.05-2	1.93-2	13.07	0.73	0.94	3.2
10^{16}	6.39-3	1.43-2	2.43-2	10.72	0.73	0.94	3.9
10^{17}	9.21-3	1.87-2	2.94-2	8.35	0.73	0.94	5.1
10^{18}	1.22-2	2.33-2	3.67-2	6.01	0.73	0.94	7.4
			$T=40\,000$ K				
10^{15}	3.21-3	9.02-3	1.77-2	15.31	0.91	1.20	3.5
10^{16}	5.81-3	1.36-2	2.36-2	12.95	0.91	1.20	4.1
10^{17}	8.90-3	1.85-2	2.93-2	10.54	0.91	1.20	5.1
10^{18}	1.26-2	2.42-2	3.78-2	8.09	0.91	1.20	7.0
10^{19}	1.64-2	2.98-2	4.75-2	5.69	0.91	1.20	10.5
			$T=80\,000$ K				
10^{16}	5.18-3	1.26-2	2.25-2	15.39	1.17	1.58	4.8
10^{17}	8.37-3	1.79-2	2.87-2	12.96	1.17	1.58	5.7
10^{18}	1.24-2	2.42-2	3.78-2	10.46	1.17	1.58	7.3
10^{19}	1.74-2	3.27-2	5.02-2	7.93	1.17	1.58	10.1
			$T=160\,000$ K				
10^{17}	7.99-3	1.75-2	2.84-2	15.96	1.53	2.11	6.9
10^{18}	1.24-2	2.44-2	3.81-2	13.43	1.53	2.11	8.3

error estimate $E = 100^* \Delta G/G$.

When one compares the A_n of Table I to similar quantities computed from the profiles of Griem and Shen we note first that, whereas the ratio $R_n = A_n/A_n(\mathrm{GS})$ for a given temperature and density is a smooth function of n for 3203, it is quite eratic for 4686. This indicates that the shapes of the profiles with unshifted central component have been changed while those without remain fairly similar. In addition a plot of A_n vs electron density (for

TABLE I. (*Continued*)

N_e	A_2	A_4	A_8	G^*	C_a	C_b	E
			$NA=3$ $NB=5$				
			$\lambda=3203$ Å				
			$T=10\,000$ K				
10^{14}	2.24-2	3.30-2	4.47-2	12.19	0.73	0.91	2.5
10^{15}	2.34-2	3.48-2	4.73-2	9.84	0.73	0.91	3.1
10^{16}	2.48-2	3.75-2	5.28-2	7.45	0.73	0.91	4.3
			$T=20\,000$K				
10^{14}	2.27-2	3.32-2	4.47-2	14.09	0.73	1.17	2.6
10^{15}	2.37-2	3.49-2	4.72-2	11.71	0.73	1.17	3.2
10^{16}	2.55-2	3.79-2	5.29-2	9.29	0.73	1.17	4.2
10^{17}	2.78-2	4.29-2	5.98-2	6.81	0.73	1.17	6.2
10^{18}	2.81-2	4.44-2	6.39-2	4.36	0.73	1.17	11.1
			$T=40\,000$ K				
10^{15}	2.38-2	3.48-2	4.68-2	13.66	0.91	1.53	3.6
10^{16}	2.57-2	3.77-2	5.21-2	11.21	0.91	1.53	4.5
10^{17}	2.84-2	4.34-2	5.99-2	8.69	0.91	1.53	6.3
10^{18}	3.12-2	4.83-2	7.01-2	6.12	0.91	1.53	9.9
			$T=80\,000$ K				
10^{16}	2.56-2	3.73-2	5.00-2	13.41	1.17	2.03	5.3
10^{17}	2.85-2	4.31-2	5.90-2	10.86	1.17	2.03	6.8
10^{18}	3.26-2	4.97-2	7.19-2	8.23	1.17	2.03	9.6
			$T=160\,000$ K				
10^{17}	2.85-2	4.27-2	5.81-2	13.66	1.53	2.70	7.9
10^{18}	3.34-2	5.10-2	7.25-2	11.00	1.53	2.70	10.2

fixed n) indicates that the present calculations agree (but with some scatter) with GS at $n_e=10^{15}$ cm^{-3}, but yield wider profiles for $N_e>10^{15}$ cm^{-3}. The deviation in fractional widths is about 10% per decade increase in N_e. These results are not unexpected. It is known that lines with unshifted central components are more sensitive to the details of the computation. The 4686-Å profiles of Griem and Shen probably suffer from some loss of accuracy in the numerical integration, in fact, it is a credit to their tenacity that they were able to perform the calculations at all on the computers available to them more than 10 years ago. The general increase in linewidth that we see in the present calculations is a result of inclusion of the quadrupole term and inelastic collisions as well as the inclusion of some off-diagonal matrix elements neglected in GS, namely those involving changes in magnetic quantum numbers.

Comparisons to experiments are not as conclusive. There is considerable scatter in the experimental results. Jones, Greig, Oda, and Griem[14] measured profiles of He II 304, 1640, 4686, and 10125 Å which are in agreement with the present calculations.[15,16] They do indicate however, that the inclusion of inelastic collisions by means of adding on terms proportional to the Gaunt factors tends to overestimate the effect. (see Appendix VI of Ref. 13 for a short discussion of the effects on the line profiles of the inclusion of inelastic collisions). Bogen[17] and Berg[18] also reported He II 4686 Å profiles that agree with the present calculations. Jenkins[19] measured the 3203-Å line profile, again in agreement with the present calculation. On the other hand Hessberg and Böttticher[20] measured profiles of He II 1640, 1215, and 1085 Å which deviate by as much as a factor of 2 beyond the half-intensity point. Similar results are reported by Eberhagen and Wunderlich[21] for the lines 4686, 3203, 1215, and 1085 Å. It is unlikely that these differences can be attributed to the different regions of temperature and electron density at which the various experiments were performed, nor can one account for these differences by allowing for some (or all) of the ions being doubly charged (see Appendix V of Ref. 13).

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[1] P. Kepple and Hans R. Griem, Phys. Rev. 173, 317 (1968).

[2]At the time the calculations were performed microfield distributions produced by multiply ionized ions were not available. At first thought it might seem necessary to utilize a more detailed quantum-mechanical treatment which includes symmetrized wave functions when considering the case of perturbations by other He II ions, however the following argument shows that the present theory is adequate: At the beginning of the interaction the perturbing ion is in the ground state and the radiating ion in an excited state. Thus initially they are distinguishable; further the perturbation time ($\sim \rho_{\min}/v$) is much smaller than the time required to transfer the excitation energy, thus they remain distinguishable for the times of interest here, and symmetrized wave functions should *not* be used for optically thin emission line shapes (see also Refs. 13 and 14).

[3]C. F. Hooper, Jr., Phys. Rev. 165, 215 (1968); 169, 193 (1968).

[4]By inelastic collision we mean here one in which there is a change in the principal quantum number of the radiating ion.

[5]Hans R. Griem and K. Y. Shen, Phys. Rev. 122, 1490 (1961).

[6]The matrix elements of the operator ϕ_{as} contain terms of the form

$$\langle\alpha'|\vec{R}\cdot\vec{R}|\alpha''\rangle \equiv \sum_{\gamma} \langle\alpha'|\vec{R}|\gamma\rangle\cdot\langle\gamma|\vec{R}|\alpha''\rangle$$
$$= \sum_{\substack{\gamma \\ n_\gamma = n_{\alpha'} = n_{\alpha''}}} \langle\alpha'|\vec{R}|\gamma\rangle\cdot\langle\gamma|\vec{R}|\alpha''\rangle + \sum_{\substack{\gamma \\ n_\gamma \neq n_\alpha = n_{\alpha''}}} \langle\alpha'|\vec{R}|\gamma\rangle\cdot\langle\gamma|\vec{R}|\alpha''\rangle .$$

The elastic-collision approximation consists of neglecting the second term, and it is only when this is done that it is legitimate to neglect the exponentials in Eq. (1) and that Eqs. (3) and (4) are obtained. The elastic collision approximation is justified *a priori* by examination of the magnitude of the relevant matrix elements (see Appendix III of Ref. 13) and *a posteriori* by the agreement between measured profiles and those calculated on the basis of the approximation.

[7]The quantity $\rho'_{\min}$ in this paper is functionally the same as $\rho_{\min}$ in KG, however, the latter is the minimum impact parameter cutoff (necessary in order to avoid a divergence), whereas the former will be used to define a velocity below which the electrons should be treated in the quasistatic approximation. See GS for a discussion of the actual minimum impact parameter cutoff for the case of hyperbolic paths.

[8]Equation (14), which is derived in Ref. 13, Appendix IV, can be simplified for some (but not all) of the region of y values of interest. However, since the calculations were performed on a large digital computer, the approximations are not particularly helpful.

[9]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, Natl. Bur. Std. Appl. Math. Ser. 55 (U. S. GPO, Washington, D. C., 1964), p. 232.

[10]Hans R. Griem (private communication). Note that in this form the quadrupole matrix elements are approximated by dipole matrix elements.

[11]This approach was suggested by H. Van Regemorter [Astrophys. J. 136, 906 (1968)] and applied by Hans R. Griem [Phys. Rev. 165, 258 (1968)] to the case of isolated ion lines and by R. A. Hill, J. B. Gerardo, and Paul C. Kepple [Phys. Rev. A 3, 855 (1971)] to the cases H_β, H_γ, and H_δ. When the results of the hydrogen line profiles were compared to experiment, it was found that the Gaunt factors probably overestimate the effect of inelastic collisions. However, the inelastic collisions are more important in the case of ions, and thus it is felt that their inclusion (through the Gaunt-factor approximation) in the present calculation will result in a smaller error than their complete neglect would.

[12]We use the numerical values found in C. W. Allen, *Astrophysical Quantities*, 2nd ed. (Athelone, London, 1964), p. 43.

[13]P. Kepple, University of Maryland Technical Report No. 72-018, 1971 (unpublished).

[14]L. A. Jones, J. R. Greig, T. Oda, and Hans R. Griem, Phys. Rev. A 4, 833 (1971).

[15]The results of a single calculation (without inelastic collisions) of the profile of the line at 10 125 Å is included in Ref. 13.

[16]The resonance line is an exception here. However, since it is optically thick, its profile is strongly influenced by the shape of the absorption coefficient. For example, for the latter, two ions initially in the ground state must be considered indistinguishable. In contrast to the case of emission (see Ref. 2) symmetrization is thus necessary here, leading to a substantial reduction in the broadening caused by singly charged helium ions.

[17]P. Bogen, Z. Naturforsch. 25a, 1151 (1970).

[18]H. F. Berg, Z. Physik 191, 503 (1966).

[19]J. E. Jenkins, Ph. D. thesis (Imperial College, London, 1970) (unpublished).

[20]H. Hessberg and W. Bötticher, Z. Naturforsch. 22a, 316 (1967).

[21]A. Eberhagen and R. Wunderlich, Z. Physik 232, 1 (1970).