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Determining the electron forward-scattering amplitude using electron interferometry

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We describe a method for measuring the forward-scattering amplitude for electron collisions with atoms or molecules. Our scheme uses a gas cell in one arm of an electron interferometer and measures the resulting attenuation and phase shift of the electron matter wave. The complex index of refraction of the gas is determined along with the forward-scattering amplitude. Calculations of the scattering of electrons by atoms are performed using a self-energy potential obtained by treating the atom as an inhomogeneous electron gas. The results indicate that the proposed experiments are feasible. [S1050-2947(99)50902-5]

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I. INTRODUCTION

The quantum-mechanical theory of scattering of particles predicts the amplitude and phase, whereas most scattering experiments yield only the modulus of the scattered wave function. One possibility of measuring the phase of the scattered wave is to use interferometric techniques, as has been successfully applied in neutron [1,2] and atom/molecular optics [3,4].

Information about the scattering length for neutron collisions with a medium has been obtained from the analysis of measurements of the complex index of refraction [1] using neutron interferometers. Recently, atom interferometry [5] has been successfully applied to measuring the index of refraction for atomic sodium [3,4] and molecular sodium dimer [6,7] matter waves passing through a noble-gas medium. With the application of multiple-scattering theory [8–10] these experiments provide the capability for directly measuring the forward-scattering amplitude of an atomic collision. In the present work, we extend these ideas to the use of electron interferometry for measurements of the forward-scattering amplitude for electron collisions with atoms and molecules.

Electron interferometry and electron holography [11,12,2] are well established techniques used in fundamental measurements as well as solid-state physics experiments. The first electron interferometers were built using microfabricated gratings in 1953 for electrons [13] and a biprism for electrons in 1955 [14]. The latter proved to be very versatile, and electron interference experiments were carried out with electrons ranging in energy from a few tens of eV to 1 MeV [11,12]. In some of these experiments the two interfering electron beams were separated by over 300 μm and a physical barrier was inserted between the beams [15], achieving parameters similar to or even more favorable than those achieved in the original atom forward-scattering experiments [3,4,7]. Therefore it should be possible to measure the forward-scattering amplitude for electron collisions with atoms and molecules using electron interferometers.

For the proposed electron interferometry measurements of the forward-scattering amplitude $f(k,0)$, the electron wave is split into two parts and allowed to propagate along physically separate paths before being recombined for the interference measurement (see Fig. 1). If the separation of the two paths of the electron is large a physical barrier can be inserted and one path can interact with an atomic or molecular gas, whereas the other path propagates in a vacuum. The final interference fringes then allow a measurement of the phase shift $\Delta\varphi$ introduced by the electron-atom collisions. The phase shift is proportional to the real part of the forward-scattering amplitude

$$\Delta\varphi=\frac{2\pi N}{k}\,\mathrm{Re}\,f(k,0), \tag{1}$$

where N is the number density of the medium and k is the momentum of the electron. The attenuation of the *amplitude* of the transmitted wave can also be measured. It is proportional to the imaginary part of the forward-scattering amplitude $f(k,0)$ and is related to the total cross section through the optical theorem

$$\sigma_{tot}=\frac{4\pi}{k}\,\mathrm{Im}\,f(k,0). \tag{2}$$

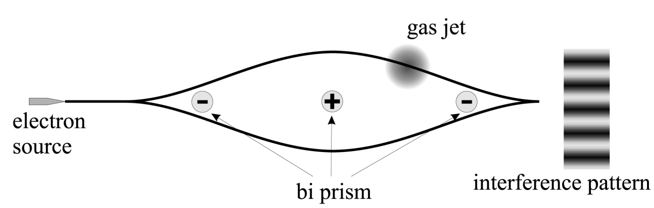

FIG. 1. Schematics, not to scale, of a biprism electron interferometer [14,15] adapted to measure the forward-scattering amplitude. Charged wires acts as biprisms. The negatively charged wires split/recombine the beam and the positively charged wire redirects the two beams back together. The refractive index of a gas is measured by inserting a gas cell or a gas jet in one arm of the interferometer.

By analogy to light optics, the index of refraction of the effective medium is given in terms of the forward-scattering amplitude by

$$n(k)=1+\frac{2\pi N}{k^2}f(k,0). \tag{3}$$

By measuring both the phase shift $\Delta\varphi$ and the amplitude of the interference pattern one can obtain a direct measurement of the forward-scattering amplitude for electrons colliding with atoms or molecules. As in the case of atom interferometry [3], the experiment would be sensitive to the ratio of the real to imaginary part of the forward-scattering amplitude,

$$R=\frac{\mathrm{Re}\, f(k,0)}{\mathrm{Im}\, f(k,0)}, \tag{4}$$

but not sensitive to the number density of the medium.

II. CALCULATIONS

In order to assess the feasibility of our proposed scheme, we calculated electron-atom scattering amplitudes by treating the atom as an inhomogeneous electron gas [16]. The use of electron-gas models in the theoretical description of low-energy electron diffraction and extended x-ray absorption fine structure has been well documented [17–19]. For the choice of scattering potential, we follow the derivation given by Lee and Beni [19]. When the electron density of the atom is slowly varying on the scale of the local de Broglie wavelength of the incident electron, the usual Dyson equation for the electron propagator reduces to a Schrödinger equation with a complex self-energy. The plasmon pole approximation [20] may be used to replace the elementary excitations of the electron gas by a single pole. The self-energy acts as an effective potential that accounts for the exchange and correlation effects caused by the electrons in the atom, with the imaginary part corresponding to emission and absorption of plasmons. Inelastic processes that arise from the plasmon pole approximation correspond physically to excitations of atomic bound states, although a precise connection is not possible within the inhomogeneous electron gas model of the atom. The accuracy of the approximation in describing the electron-atom scattering that gives rise to extended x-ray absorption fine structure, however, is known to be quite good [19,21,22] for electron energies above 50 eV, and we expect the approximation to be at least as accurate for estimating the ratio R at these energies. The self-energy potential is given by [19,20]

$$\Sigma(\vec{p},w)=-\int\frac{d\vec{q}}{(2\pi)^3}\frac{4\pi}{q^2}\left\{\frac{w_p^2}{2w_1(q)[w_1(q)+\tilde{w}]}+\frac{f(\vec{p}+\vec{q})}{\epsilon(q,\tilde{w})}-\frac{i\pi w_p^2}{2w_1(q)}I(\vec{p},\vec{q})\right\}, \tag{5}$$

with

$$I(\vec{p},\vec{q})=f(\vec{p}+\vec{q})\,\delta(\tilde{w}-w_1(q))+[1-f(\vec{p}+\vec{q})]\,\delta(\tilde{w}+w_1(q)), \tag{6}$$

$$\tilde{w}=\frac{1}{2}(\vec{p}+\vec{q})^2-w, \tag{7}$$

and

$$\epsilon(q,\tilde{w})^{-1}=1+\frac{w_p^2}{\tilde{w}^2-w_1^2(q)}, \tag{8}$$

where $f(\vec{q})$ is the Fermi distribution function and w_p is the plasma frequency. The quantity $w_1(q)$ must reduce to the plasma frequency for small q and to the free-particle energy $\frac{1}{2}q^2$ for large q. The choice of plasmon dispersion relation is given by [20]

$$w_1^2(q)=w_p^2+\epsilon_F^2\left[\frac{4}{3}\left(\frac{q}{k_F}\right)^2+\left(\frac{q}{k_F}\right)^4\right], \tag{9}$$

where k_F is the Fermi momentum and ϵ_F is the Fermi energy. The radial dependence of the self-energy potential comes from the plasma frequency and Fermi momentum

$$w_p(r)=[4\pi\rho_e(r)]^{1/2}, \tag{10}$$

$$k_F(r)=[3\pi^2\rho_e(r)]^{1/3}, \tag{11}$$

where $\rho_e(r)$ is the electron-charge-density function. The Thomas-Fermi description of the atom is used to obtain the local energy

$$w(r)=\tfrac{1}{2}p^2(r)=\tfrac{1}{2}k^2+\epsilon_F(r), \tag{12}$$

where k is the free-electron momentum. The full scattering potential is determined by adding the self-energy to the electrostatic potential

$$V_0(\vec{r})=-\frac{Z}{r}+\int\frac{\rho_e(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d\vec{r}\,', \tag{13}$$

where Z is the nuclear charge of the atom and $\rho_e(\vec{r})$ is the electron-charge-density function. The density function is obtained by solving the relativistic Kohn-Sham equation [23]

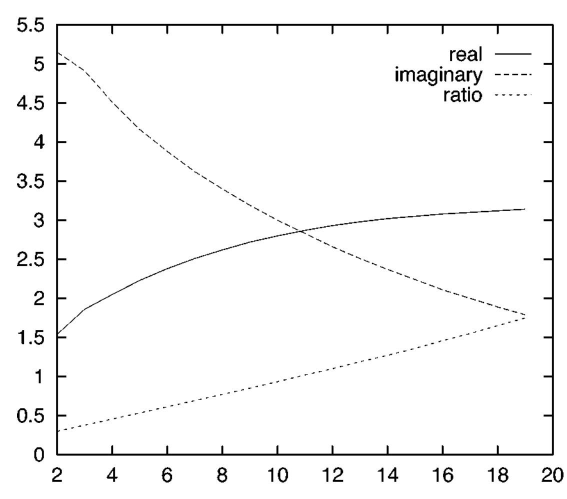

FIG. 2. Real, imaginary, and the ratio of real to imaginary parts of the forward-elastic-scattering amplitude as a function of k for electron collisions with neon. The scattering amplitudes are given in atomic units.

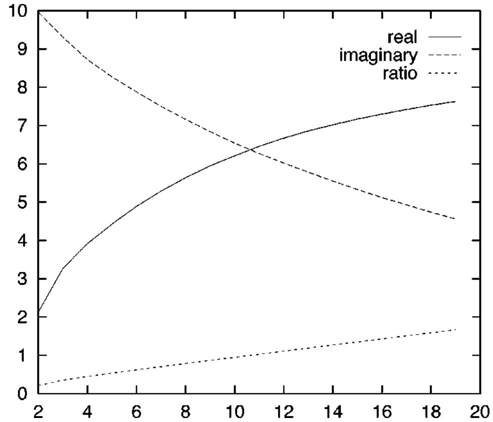

FIG. 3. Real, imaginary, and the ratio of real to imaginary parts of the forward-elastic-scattering amplitude as a function of k for electron collisions with argon. The scattering amplitudes are given in atomic units.

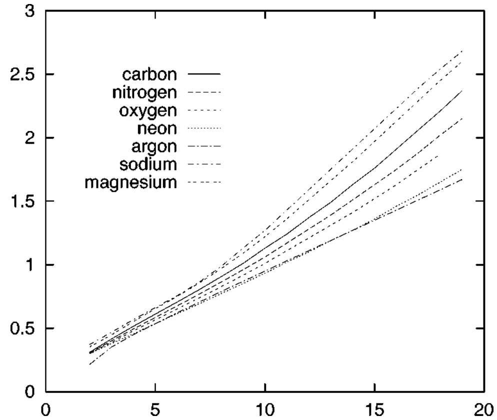

FIG. 4. Ratio R for several atomic gases.

using the local-density approximation. The Schrödinger equation for the central potential $V(r)=V_0(r)+\Sigma(\vec{p},w)$ is solved numerically to obtain the scattering amplitude

$$f(k,\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)[\exp(2i\delta_l)-1]P_l(\cos\theta), \tag{14}$$

where δ_l is the lth-order phase shift.

III. DISCUSSION

Figure 2 shows the real and imaginary parts, and the ratio of the real to imaginary part of the forward-elastic-scattering amplitude as a function of k for electron collisions with neon. The real part is slowly varying for $k>2$ a.u. The imaginary part decreases rapidly with k and crosses the real part at about $k=11$ a.u. The ratio appears to be linear with a slope of about 0.08. Figure 3 shows the same plots for electron collisions with argon. The ratio again increases linearly with k with a slope of about 0.09. Figure 4 shows the ratio for several other atoms. The linearity of the ratio of the real to the imaginary part of the forward-scattering amplitude is a general feature that emerges from our calculations for all the atoms. The values of the ratios of the real to imaginary parts of the scattering amplitude are not small and tend to increase with the overall size of the atom.

In general, experiments will be easiest if the ratio R is of order unity. Then both the attenuation and the phase shift can be measured in the same experiment. If $R\ll 1$ the phase-shift measurement is more difficult because of the large attenuation of the transmitted electron beam. Nevertheless, since the interference experiments are sensitive to the *amplitude* of the transmitted wave, successful experiments can still be done for high attenuation, as shown in the atom interferometer experiments [3] in which the transmission was as low as 10^{-4}, corresponding to a transmitted amplitude of 10^{-2}. For $R\gg 1$ the phase shifts are easy to measure, but the attenuation is more difficult, especially since a reduction of the interference contrast can also stem from the final coherence length of the electron beam. As can be seen in Fig. 4, the calculations presented here point in most of the cases to the favorable range of $R\sim 1$. Significant attenuation would only occur at low energies where the cross sections are large, but as the energy of the electron increases the attenuation becomes less severe. Our calculations show that for energies between 50 eV and 5 keV, the ratio of the real to the imaginary part of the forward-scattering amplitude generally increases with energy, indicating that electron interferometry experiments should be feasible with high-energy electrons. Similarly it should be possible to obtain information about the forward-scattering amplitudes for atoms in bulk material from experiments by inserting a very thin foil in one arm of the interferometer.

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