

Collisional relaxation in a fermionic gas

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(Received 1 March 1999)

We propose a method to study the degeneracy of a trapped atomic gas of fermions through the relaxation of the motion of a test particle. In the degenerate regime, and for an energy of the test particle well below the Fermi energy, we show that the Fermi-Dirac statistics is responsible for a strong decrease in the relaxation rate. This method can be used to directly measure the temperature of the fermionic gas. [S1050-2947(99)50506-4]

PACS number(s): 03.75.-b, 05.30.Fk, 32.80.Pj

New techniques for cooling atomic gases, based on either laser manipulation or evaporative cooling, have led to spectacular progress in the realization of degenerate atomic samples. The most striking example is the Bose-Einstein condensation of alkali-metal vapors [1–3] and, more recently, of an atomic hydrogen gas [4]. These techniques can also be applied to fermionic samples, in which spectacular phenomena, such as the Cooper pairing of atoms [5], or the inhibition of the spontaneous emission of an excited atom [6] have been predicted. In this Rapid Communication, we present a simple and powerful tool to study the degeneracy parameters of such a fermionic sample, which we consider ideal for the sake of simplicity.

Numerous possible ways to analyze an atomic Fermi gas in the degenerate regime have recently been studied from a theoretical point of view. Several authors focus on the interaction of the atomic cloud with light: modifications of the refraction index or of the absorption coefficient of the gas [7,8], reduction of the spontaneous-emission rate of an excited atom inside the atomic cloud, and angular dependence of the radiation pattern [9,10]. Spectacular effects are expected when the Fermi momentum p_F of the cloud is larger than the photon momentum. Unfortunately, in this regime, a quantitative analysis of the interaction of the gas with resonant light is difficult. Indeed, it corresponds to $n\lambda^3 > 1$, where n is the spatial density of the gas and λ is the wavelength of the light. Light propagation in the medium is then strongly affected by multiple scattering effects, which are difficult to handle.

Other proposed diagnostics of Fermi degeneracy of a trapped gas involve the study of its spatial distribution, either for a pure fermionic sample [11–14], or for a mixture with a Bose-Einstein condensate [15]. These methods are well adapted to study the region where the temperature T is around the Fermi temperature T_F , but become less sensitive to temperature in the strongly degenerate regime where $T \ll T_F$. Also, as pointed out in [10], one can take advantage of inelastic processes inside the gas to collect information about its degeneracy parameter.

In the present paper we investigate the dynamics of a probe particle (P) in the same trapping potential as the Fermi gas. We assume that we can prepare P on an arbitrary orbit of the trapping potential. An experimental procedure for such a preparation is outlined in the last part of this paper. Neglecting any inelastic process, the only way to change the trajectory of P is an interaction with the Fermi gas. We show

that the collisional dynamics of P gives access to the statistical properties of the Fermi gas, in particular its temperature T .

The trapping potential for the test particle $U(\mathbf{r})$ is assumed to be harmonic and isotropic, with a frequency $\omega/2\pi$. For simplicity we neglect the modification of the potential due to the mean-field interaction between P and the Fermi gas. We calculate the dynamics of P in the Fermi sea by means of the Boltzmann transport equation [16] in the local-density approximation:

$$\left. \frac{\partial w}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} w + \nabla_{\mathbf{r}} U \cdot \nabla_{\mathbf{p}} w \right|_{\text{coll}} = \frac{dw}{dt} \Big|_{\text{coll}}, \quad (1)$$

where $w(\mathbf{r}, \mathbf{p}, t)$ is the phase-space density of p at time t . We put $\mathbf{v} = \mathbf{p}/M$, where M is the mass of P . With a notation similar to [16] the collisional contribution to the quantum Boltzmann transport equation for P is

$$\left. \frac{dw}{dt} \right|_{\text{coll}}(\mathbf{r}, \mathbf{p}, t) = - \frac{\sigma}{4\pi h^3} \int d^3 p_f d^2 \Omega \times [w f (1 - f') - w' f' (1 - f)] |\mathbf{v} - \mathbf{v}_f|, \quad (2)$$

where σ is the collisional cross section for interactions between P and fermions. Collisions occur in the low-energy range where s -wave scattering is dominant; hence the cross section is isotropic and independent of energy. The first part of the collisional integral corresponds to the decay of the phase-space density in \mathbf{r}, \mathbf{p} due to a collision between P and a fermion with momentum \mathbf{p}_f : $\mathbf{p} + \mathbf{p}_f \rightarrow \mathbf{p}' + \mathbf{p}'_f$. The final relative velocity points in the direction given by Ω . The second part of the integral describes the reverse process, and we put $\mathbf{v}_f = \mathbf{p}_f/m$, where m is the fermionic mass. We assume that a single probe particle is present, or that the gas of the probe particles is sufficiently dilute to be treated as an ideal Boltzmann gas. In the latter case we suppose that the number of test particles is sufficiently low that the relaxation does not significantly perturb the distribution of the Fermi gas. We use the abbreviations $w' = w(\mathbf{r}, \mathbf{p}', t)$, $f = f_T(\mathbf{r}, \mathbf{p}_f)$, and $f' = f_T(\mathbf{r}, \mathbf{p}'_f)$. The quantity $f_T(\mathbf{r}, \mathbf{p})$ represents the steady-state phase-space distribution of the Fermi gas in the local-density approximation:

$$f_T(\mathbf{r}, \mathbf{p}) = \frac{1}{1 + \exp\{[p^2/2m + U(r) - \mu]/(k_B T)\}}, \quad (3)$$

where μ is the chemical potential. The Pauli exclusion principle is represented in the collisional integral by the factors $1-f'$ and $1-f$, which give the occupation of the final state of the collision [17]. The local-density approximation is valid when $r_F p_F \gg \hbar$, where r_F and p_F are the sizes in position and momentum space of the Fermi gas. For a temperature much lower than the Fermi temperature T_F [where $k_B T_F = (6N)^{1/3} \hbar \omega$], this requires that $N \gg 1$. Indeed, one gets in this case $r_F = (48N)^{1/6} a_{\text{HO}}$ and $p_F = (48N)^{1/6} p_{\text{HO}}$, where $a_{\text{HO}} = [\hbar/(m\omega)]^{1/2}$ and $p_{\text{HO}} = (m\hbar\omega)^{1/2}$ are the spatial and momentum extensions of the ground-state harmonic oscillator.

We shall assume that the $w(\mathbf{r}, \mathbf{p})$ is initially a distribution centered in $\mathbf{r}_0, \mathbf{p}_0$, much narrower than the steady-state distribution of Eq. (2): $w_{\text{SS}} \propto \exp\{-[p^2/2M + U(r)]/(k_B T)\}$. As the collisional relaxation proceeds, the population of the narrow peak is transferred to a broad distribution proportional to w_{SS} . In the following we focus on the initial stage of this relaxation phenomenon, namely the decay of the narrow peak, which occurs at the rate $\Gamma_T(\mathbf{r}_0, \mathbf{p}_0)$ deduced from Eq. (2):

$$\Gamma_T(\mathbf{r}_0, \mathbf{p}_0) = \frac{\sigma}{4\pi\hbar^3} \int d^3 p_f d^2 \Omega f(1-f') |\mathbf{v}_0 - \mathbf{v}_f|. \quad (4)$$

This rate could also be derived from the Fermi golden rule, within the local-density approximation.

We now consider three different classes of trajectories for P and discuss how the damping of those trajectories is affected by the Fermi statistics of the cloud. Most of the calculations assume equal mass for the P and the fermionic atoms. Such a condition is approximately realized if P and the fermions are isotopes of the same element (i.e., ${}^7\text{Li}$ and ${}^6\text{Li}$, ${}^{39}\text{K}$ and ${}^{40}\text{K}$). For each trajectory class we calculate the multiple integral (4) for an arbitrary temperature numerically and we derive scaling laws for interesting limiting cases.

The first situation consists of P at rest in the trap center. For a Boltzmann gas with the same number of atoms, we would expect $\Gamma_T^{(\text{Bolt.})}(0,0) = n\sigma v_{\text{th}}$, where n is the spatial density at the center of the trap for this gas and v_{th} is the most probable speed ($v_{\text{th}} = [8k_B T/(\pi m)]^{1/2}$). To put in evidence the effects of the Fermi statistics, we plot in Fig. 1 the rate $\Gamma_T(0,0)$ normalized by $\Gamma_T^{(\text{Bolt.})}(0,0)$. At high temperature ($T > T_F$) the deviations due to Fermi statistics are negligible. On the other hand, for $T < T_F$, these deviations are spectacular and we find that $\Gamma_T/\Gamma_T^{(\text{Bolt.})} \propto T^3$. This power-law dependence originates from two different phenomena both related to Fermi statistics. (i) For the Boltzmann gas, $n \propto T^{-3/2}$ and $v \propto \sqrt{T}$, while in the fermionic gas the spatial density and the most probable speed remain constant for vanishing T . This accounts for a factor $\propto T$. (ii) For $T \ll T_F$, P has a finite collision probability only with fermions within an energy interval $\Delta E \sim k_B T$ (see Fig. 2) at the surface of the Fermi sphere [18]. The solid angle $\Delta\Omega$ available for the allowed fermionic final states is also proportional to T . Therefore the collisional rate is reduced by an additional factor $\Delta E \Delta\Omega \propto T^2$.

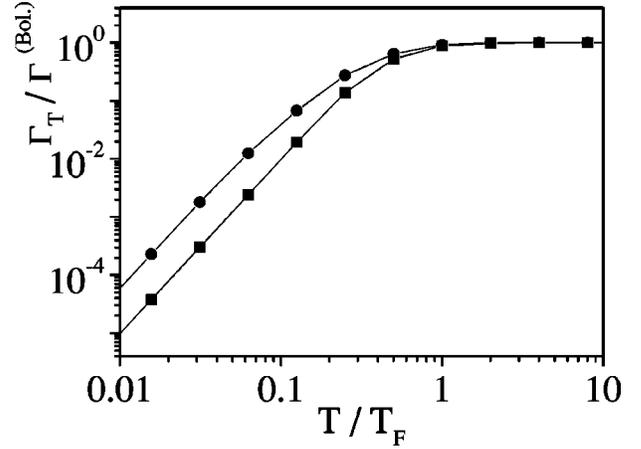


FIG. 1. Collisional rate $\Gamma_T(\mathbf{r}=0, \mathbf{p}=0)$ of a probe particle (P) of mass M at rest in the bottom of a trap containing a fermionic gas of temperature T (atomic mass m). The temperature is plotted in units of the Fermi temperature T_F , and Γ_T in units of the rate $\Gamma_T^{\text{Bolt.}}$ for a Boltzmann gas at the same temperature and with the same number of atoms. Squares, $M = m$; circles, $M = (133/6)m$ (case of a ${}^6\text{Li}$ gas probed by ${}^{133}\text{Cs}$).

This situation is well suited for determining the temperature of the Fermi gas in the degenerate regime. Its absolute calibration depends on the exact mass ratio between P and the fermions. Consequently, in Fig. 1 we also plot $\Gamma_T(0,0)$ for the specific case of a cesium atom (${}^{133}\text{Cs}$) as P for a lithium gas (${}^6\text{Li}$).

We now consider a second type of trajectory consisting of an oscillation of P with an energy E , and an angular momentum L which we set equal to zero (see inset of Fig. 3). To calculate the rate at which P is ejected from this trajectory by collisions, we suppose that the relaxation is slow with respect to the period of an oscillation (collisionless regime). Since $\Gamma_T(\mathbf{r}, \mathbf{p})$ is not constant over the trajectory, we define an average collision rate:

$$\gamma_T(E, L=0) = \frac{\omega}{2\pi} \oint \Gamma_T(r(t), p(t)) dt. \quad (5)$$

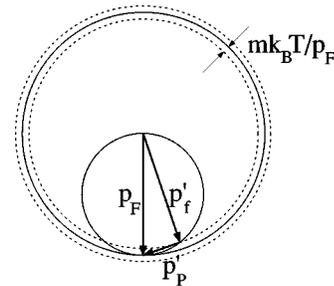


FIG. 2. Collision between a fermion and P at rest. The fermion has an initial momentum equal to the Fermi momentum p_F . p'_P and p'_f are the momenta of P and the fermion after collision. The circumference with diameter p_F passing through the center of the Fermi sphere gives the possible final states in the case of equal mass. The shell of thickness $mk_B T/p_F$ represents the final states available to the Fermi particle.

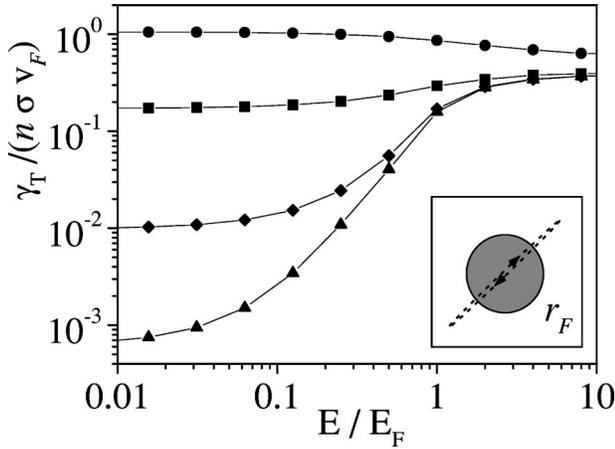


FIG. 3. Damping rate $\gamma_T(E, L=0)$ for a linear oscillation of P as a function of its excitation energy E . E is expressed in units of the Fermi energy and γ_T is normalized by the rate $n\sigma v_F$, where n is the fermionic density at the bottom of the trap and v_F the Fermi velocity. Circles, $T=T_F$; squares, $T=T_F/4$; diamonds, $T=T_F/16$; triangles, $T=T_F/64$. The inset shows a trajectory of P through the fermionic cloud.

Depending on the energy of the oscillation and the temperature of the fermionic gas, we find different regimes (see Fig. 3). As expected, for $T > T_F$ the relaxation rate is the same as the one predicted for a Boltzmann gas. It does not depend much on the energy of the excitation, even for $E > k_B T$ (as long as we assume a constant s -wave elastic cross section). Indeed, when E grows, the relative velocity between P and the cloud increases as $E^{1/2}$, while the fraction of the time P spends within the cloud decreases as $E^{-1/2}$, leading to a constant average rate. For the degenerate case $T < T_F$, three energy domains have to be considered for P . (i) For $E < k_B T < E_F$, we recover the rate $\Gamma_T(0,0)$ displayed in Fig. 1. (ii) For $k_B T < E < E_F$, the rate varies as E^2 . This can be easily understood at zero temperature with $E \ll E_F$. In this case only collisions with fermionic particles with energy close to E_F contribute (first factor E), and the solid angle available for the allowed final states brings an extra factor E . (iii) For $k_B T < E_F < E$, almost all final states for the fermions after the collision lie above the Fermi surface so that the inhibition due to statistics is no longer effective. One recovers in this case a rate independent of E , as for a Boltzmann gas.

The last type of trajectory considered in this paper consists of a circular orbit of the test particle in the trapping potential ($L = E/\omega$). The corresponding decay rate is plotted in Fig. 4 as a function of the mechanical energy of P . For a weakly degenerate gas we recover the same result as for a Boltzmann gas. When one increases the energy of P , the damping rate is constant up to $k_B T$ and then decreases, as P is orbiting outside the cloud. In the degenerate case the damping rate presents a resonant behavior around $E = E_F$. For $E \ll E_F$ this rate is decreased because of Pauli's exclusion principle, while for $E \gg E_F$ it is small since P is outside the Fermi cloud.

We now address the preparation of P with arbitrary initial position and momentum. The simplest idea is to exploit the difference in the ground-state hyperfine splitting between the

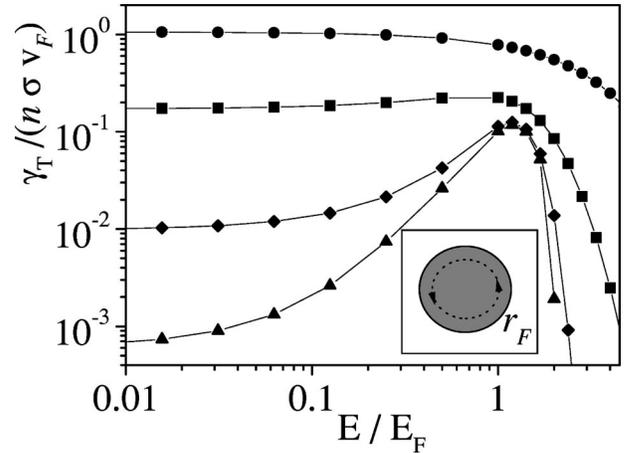


FIG. 4. Damping rate $\gamma_T(E, L=E/\omega)$ for a circular orbit of the TP as a function of its excitation energy E . The normalizations and symbols are the same as in Fig. 3.

various alkali-metal atoms, or between two isotopes of the same species. Consider, for example, the specific case of lithium atoms. If one starts with a mixture of ${}^6\text{Li}$ and ${}^7\text{Li}$, one can perform radio-frequency evaporation around a frequency $\nu_{\text{hf}} \sim 804$ MHz, corresponding to the ${}^7\text{Li}$ hyperfine splitting. One can prepare in this way an ultracold sample of ${}^7\text{Li}$, playing the role of P , without eliminating any fermionic atom ${}^6\text{Li}$, whose hyperfine splitting corresponds to 228 MHz [19]. The state obtained in this way corresponds to the first situation considered in this paper. One can then prepare P on an arbitrary trajectory by means of successive optical stimulated Raman transitions [20]. Due to the isotopic shift of the Li resonance line (10 GHz), these transitions can be made isotopically selective. In a realistic case, we can consider 10^8 fermions in an isotropic magnetic trap with $\omega/2\pi = 100$ Hz ($T_F = 4$ μK , $r_F = 0.17$ mm). The density of the fermionic gas is 5×10^{12} cm^{-3} , giving a mean-field energy created by the Fermi cloud on P equal to 10 nK [21,22], which is negligible with respect to E_F , as assumed in this paper.

To summarize, we have shown in this paper that the collisional relaxation of a probe particle embedded in a Fermi atomic cloud gives direct access to the quantum degeneracy of this gas. In the degenerate regime, P can be regarded as an excitation ‘‘frozen’’ by Pauli's exclusion principle. One can determine both the temperature of the Fermionic cloud from the value of the relaxation rate Γ for $E \sim 0$, and the value of the Fermi energy exploiting the resonant behavior of Γ for $E \sim E_F$. For simplicity we have considered here a noninteracting Fermi gas, but it is clear that this method can be extended to study the effects of interactions superimposed onto the fermionic excitation spectrum. In particular, we plan to address the consequences of Cooper pairing of the fermions [5,23–25] in a subsequent paper.

I am very grateful to Jean Dalibard for stimulating discussions and for a careful reading of the manuscript. I acknowledge useful discussions with Micha Baranov, Yvan Castin, Marc-Oliver Mewes, Christophe Salomon, and Florian Schreck. This work was partially supported by the CNRS, Collège de France, DRET, DRED, and EC (TMR Network No. ERB FMRX-CT96-0002).

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