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Three-particle entanglement from entangled photon pairs and a weak coherent state

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We show that a three-photon entangled state can be selected from the product state of an entangled photon pair and a weak coherent state. An experiment producing about 20 counts/sec of three-photon events can be extrapolated from existing results. [S1050-2947(99)50101-7]

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The realization of a quantum computer [1] is dependent on the ability to form arbitrary superpositions and entanglements of large numbers of initially unconnected single-particle states. In optics this can be done, in theory, using an ideal Kerr nonlinear medium. Such an element, with single-photon sensitivity, may soon be realized in high finesse cavity experiments [2]. However, in the absence of such a device many groups have studied two-particle entanglement effects using sources that create photons in pairs. Two popular sources are cascade emission from a three-level atomic system [3] and, more recently, the process of parametric down-conversion [4–6]. One could extend these approaches to look for four-level cascades or three-photon down-conversion processes but simple calculations based on realistic medium properties show that the rates of creation of entangled three-photon states will be extremely low (of order 10^{-10} sec^{-1} [7]). As a result there have been no experiments showing entanglement of larger numbers (>2) of particles to date. More recently, various theoretical arrangements for entangling initially separate photons have been proposed [8–11] but largely not pursued due to impracticality and/or experimental complexity. Recently we have built a pulsed source of single photons and have performed an experiment showing interference effects between separate photons [12–14]. The pulsed source of single photons is made from a parametric crystal pumped by a femtosecond laser. The crystal is arranged to produce pairs of down-converted photons of a few nanometers bandwidth and coincident to within a few hundred femtoseconds. Detection of one photon of the pair can be used to gate the detection of the second, producing a source rich in single-photon states [15,16]. The narrow bandwidth and short pulse length mean that our single-photon states are near time-bandwidth limited (or Heisenberg limited). It is this aspect that allows us to overlap separate single-photon pulses (in single spatial modes) and make them indistinguishable, giving high visibility interference effects [17,10,12]. Our initial work has been followed by a key experiment in which interference of separate photons has been used to illustrate the phenomenon of quantum teleportation [18].

Unfortunately the creation efficiency η_c of single photons from gated parametric sources remains low ($\eta_c \simeq 2.10^{-4}$ per pulse), and the effective detector efficiencies are low $\eta_d \sim 0.2$. For experiments where multiple-gated single-photon sources are used to create N-fold entangled states, the result is low N-fold coincidence rates (in the best case $\propto \eta_c^N \eta_d^N$). The multiple coincidence rates are usually well below 0.2 counts/sec for experiments involving two separate down-conversion crystals. The second major drawback has been the inevitable background coincidence rates arising from the non-negligible probabilities of two photons occupying the same mode.

In our experiments we have established that nonclassical interference and evidence of entanglement of separate particles occur when we mix a gated single-photon state with a weak coherent state at a beam splitter [12,13]. We have also proposed an extension of this apparatus to show entanglement effects between initially separate particles [14]. Here we show an apparatus that will select a three-photon polarization entangled state in a remarkably simple extension of our beam-splitter experiment. A slightly more elaborate apparatus is required to establish a momentum-phase entangled state. Common to all these experiments is the selection of entangled states from initial product states by measurement of threefold coincident photodetections. Based on optimizing the counting rates seen in past experiments [13], we expect to see threefold coincidence rates greater than 20 counts/sec. In these experiments we use a weak pulsed laser source as the coherent state. The weak laser pulses are still much brighter than the parametric pair pulses; thus the probability of seeing two down-converted pairs in one pulse, giving rise to a constant background of triply coincident detections, can be made negligible.

Consider the arrangement shown in Fig. 1. A femtosecond pulsed laser is doubled in a nonlinear crystal. The doubled beam illuminates a type-II phase-matched parametric down-conversion crystal arranged to emit polarization entangled photon pairs into the two modes shown [6]. The idler mode polarization is rotated through 90° using a half-wave plate and any crystal birefringence is compensated for using birefringent plates. A weak beam of undoubled laser light is linearly polarized at 45° using a half-wave plate. At the line labeled A in the figure the resulting state is represented by the product of weak coherent states in vertical and horizontal polarizations and a weak (thermal) source of two-photon polarization entangled states:

$$|\Psi\rangle = \frac{1}{2}|\alpha\rangle_{hc}|\alpha\rangle_{vc}[\sqrt{1-2g^2}|0\rangle_s|0\rangle_i + g(|1\rangle_{hs}|1\rangle_{hi} + |1\rangle_{vs}|1\rangle_{vi}) + O(g^2)]. \quad (1)$$

$|\rangle_{hc}$ ($|\rangle_{vc}$) represents the horizontally (vertically) polarized laser mode, which is populated by α representing the coher-

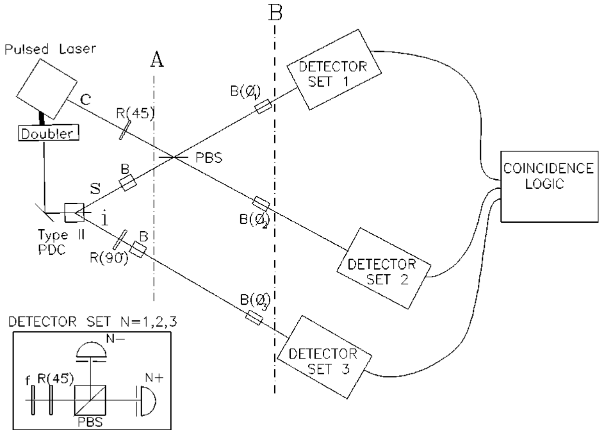

FIG. 1. Proposed polarization GHZ experiment described in the text. Key: PDC—parametric down-conversion crystal; c,s,i—laser mode and signal and idler down-converted modes, respectively; $R(45°)$, $R(90°)$—polarization rotation elements; B—birefringent compensators; $B(\phi_N)$ $(N=1,2,3)$—birefringent elements inducing a phase shift of ϕ between vertical and horizontal polarizations; PBS—polarizing beam splitters; $N+,N-$—photon-counting detectors viewing the $+$ or $-$ output mode from the polarizing beam splitters; f—narrow band filter.

ent state. The parametric down-conversion signal and idler modes $|\rangle_s$ and $|\rangle_i$ are populated mostly by vacuum plus a small amplitude $g\sqrt{2}\ll 1$ of the entangled state consisting of *either* one horizontal polarization signal photon with one horizontal polarization idler photon $|1\rangle_{hs}|1\rangle_{hi}$ *or* one vertical polarization signal photon with one vertical polarization idler photon $|1\rangle_{vs}|1\rangle_{vi}$. The signal mode is then mixed with the laser mode in a polarizing beam splitter as shown so that the horizontal (vertical) component of the coherent state is mixed with the vertically (horizontally) polarized signal photons. The three resulting beams now propagate to remote detector sets labeled 1,2,3 as shown. Each detector set is preceded by a variable thickness birefringent plate, which introduces a relative phase shift between horizontal and vertical polarizations (ϕ_1,ϕ_2,ϕ_3). The horizontal and vertical polarization modes are then combined by rotating their polarization by 45° (using a half-wave plate), and viewing through a polarizing beam-splitter cube. We now limit ourselves to those situations where we see triply coincident photodetections from detector sets 1, 2, and 3. We also consider only a weak coherent state $\alpha\ll 1$ such that

$$|\alpha\rangle_h\simeq\sqrt{1-\alpha^2}|0\rangle+\alpha|1\rangle_h+O(\alpha^2), \tag{2}$$

and similarly with h replaced by v. In this situation, when we look at the line labeled B we find that the terms that can contribute to the triple coincidence rate are

$$\frac{1}{2}\alpha g[\exp\, i(\phi_1+\phi_2+\phi_3)|1\rangle_{vc1}|1\rangle_{vs2}|1\rangle_{vi3} + |1\rangle_{hc1}|1\rangle_{hs2}|1\rangle_{hi3}]+O(\alpha^2 g,g^2), \tag{3}$$

where subscripts now describe the polarization h,v, source c,s,i, and destination of the mode 1,2,3. We then apply suitable filtering and time gating [17,10,12] to make the single photons, originating from the laser, indistinguishable from signal photons emitted by the crystal, which allows us to erase all source subscripts in the above, leaving

$$\frac{1}{2}\alpha' g'[\exp\, i(\phi_1+\phi_2+\phi_3)|1\rangle_{v1}|1\rangle_{v2}|1\rangle_{v3} + |1\rangle_{h1}|1\rangle_{h2}|1\rangle_{h3}]+O(\alpha^2 g,g^2), \tag{4}$$

which is the well-known maximally entangled GHZ state introduced by Greenberger, Horne, and Zeilinger [19]. There is some loss introduced by the filtering [17]; hence we relabel with $\alpha'<\alpha$, $g'<g$. To demonstrate interference effects with this state, we have to mix the horizontal and vertical modes at each detector set. The action of the 45° rotation followed by a polarizing beam splitter is to transform

$$|1\rangle_{vj}\rightarrow\frac{1}{\sqrt{2}}[i|1\rangle_{-j}+|1\rangle_{+j}],$$
$$|1\rangle_{hj}\rightarrow\frac{1}{\sqrt{2}}[-i|1\rangle_{-j}+|1\rangle_{+j}], \tag{5}$$

where $j=1,2,3$, $|\rangle_{+j}$ represents the transmitted modes, and $|\rangle_{-j}$ represents the orthogonally polarized reflected modes. It is then easy to show that the probability $P_{123}(\phi_1,\phi_2,\phi_3)$ of detecting a triple coincidence in detector sets 1, 2, and 3 is given by

$$P_{123}(\phi_1,\phi_2,\phi_3)=\eta_d^3/16\{\alpha'^2 g'^2[1+(-1)^m \times\cos 2(\phi_1+\phi_2+\phi_3)]+O(g^4) +O(\alpha^4 g^2)\}, \tag{6}$$

where we write $m=k_1+k_2+k_3$, and denote $k_{1,2,3}=+1$ for detections in the $+$ detectors and $k_{1,2,3}=-1$ for detections in the $-$ detectors. We also include detector efficiencies η_d and note that the total coincidence rate when we add all P_{123} triple coincidences is $\eta_d^3 g'^2\alpha'^2/2$, half that predicted at line A. In effect we have postselected the GHZ state from the product state represented in Eq. (1).

The higher-order terms of order $g^4,\alpha^4 g^2$ could cause a reduction in the visibility of any interference effect. However, it will be negligible if we arrange $g^2\ll\alpha^2\ll 1$. This naturally occurs in the experiment [13,12] as the weak laser pulses contain on the order of $\alpha^2\sim 5\times 10^{-2}$ photons per pulse. This is much stronger than the parametric source with $\eta_c\equiv g^2\simeq 5\times 10^{-4}$ pairs per pulse per single spatial mode. Given these values we expect a triple coincidence probability of 2×10^{-7} per pulse. With a pulse repetition rate of 100 Mhz this gives a triple coincidence rate of 20 counts/sec, an order of magnitude brighter than two-crystal sources [18].

We can also produce a momentum entangled version of this state on using the apparatus shown in Fig. 2. Now the crystal is chosen for type-I nondegenerate phase matching. In this situation we can arrange each down-converted beam to consist of two modes with a small difference in frequency [20] and, as a result, slightly different propagation direction. These modes are separated by pickoff mirrors into r (reflected) and t (transmitted) modes. The frequencies f_r and f_t

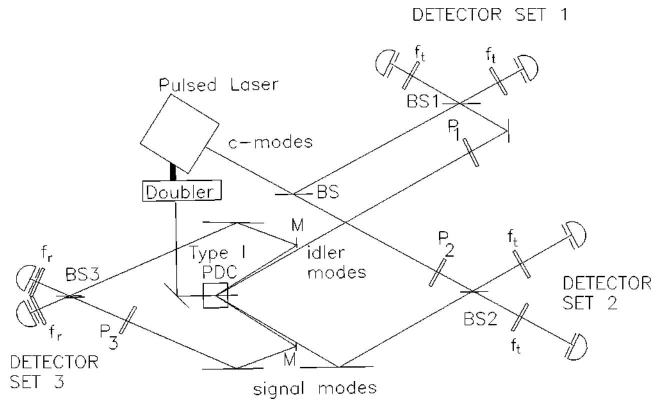

FIG. 2. Proposed momentum-phase GHZ experiment described in the text. Key: PDC—parametric down-conversion crystal; M—r wavelength pickoff mirrors; BS—nonpolarizing beam splitter; BS1,BS2,BS3—remote mode recombining beam splitters; $P_{1,2,3}$—variable; $\phi_{1,2,3}$ radians—phase shifters; f_t, f_r—narrow band filters tuned to the t or r wavelength.

are chosen to satisfy energy conservation between the two down-converted beams and the pump. As a result the presence of an idler photon in an r mode is directly correlated with a signal photon in a t mode, and *vice versa* (for example, see Ref. [5]). The weak coherent beam is split at a standard beam splitter into two c modes, which recombine with the transmitted signal and idler modes at two separate beam splitters labeled 1 and 2 (we select only f_t from the weak coherent state using narrow-band filters). The reflected signal and idler modes recombine at a third beam splitter BS3. Using the same approximations as in Eqs. (1)–(3) the state before the remote beam splitters is

$$|\Psi\rangle = \frac{1}{2}[(\exp^{i\phi_1}|1\rangle_{ct1} + |1\rangle_{ct2}) \times(\exp^{i(\phi_2+\phi_3)}|1\rangle_{st2}|1\rangle_{ir3} + |1\rangle_{sr3}|1\rangle_{it1})], \quad (7)$$

where subscripts again uniquely label the source (c,s,i), frequency (f_t, f_r), and destination (BS1,BS2,BS3). When we limit ourselves to threefold coincidences beyond the beam splitters we select only

$$|\Psi\rangle = \frac{1}{2}[\exp^{i(\phi_1+\phi_2+\phi_3)}|1\rangle_{ct1}|1\rangle_{st2}|1\rangle_{ir3} + |1\rangle_{it1}|1\rangle_{ct2}|1\rangle_{sr3}]. \quad (8)$$

This is again analogous to the GHZ state [19]. An analysis of the coincidence rates gives a similar result to that shown in Eq. (6).

The nonlocality inherent in these states is highlighted when we specialize to the situations of maximum correlation with $\phi_1+\phi_2+\phi_3=n\pi$. The knowledge of the outcome of the measurement for two of the particles uniquely determines the outcome of the third measurement independent of the individual values of ϕ_1, ϕ_2, ϕ_3. If we were to assume that the photons have phase or polarization fixed at the point of creation (''locally realistic'') the above statement would only be valid for certain selected phases. Of course, in the second arrangement we can uniquely determine the phase of the laser by independent measurements on the unattenuated beam. The measurements made after beam splitters 1 and 2 can thus be thought of as binary measurements of the phase of the signal and idler modes (relative to the laser). This then collapses the relative phase of the reflected (r modes) onto one of two values, which is measured after beam splitter 3.

All these experiments depend on postselection and we lose 50% of all possible triple coincidences due to pairs of photons traveling to one or another of the detector sets. Similarly, we have to rely on the random overlap of the laser and down-converted photons when both are emitted in only a fraction of the pulses. Both of these losses of efficiency could allow local realistic interpretations to be applied unless we can prove that fair sampling occurs. In a future experiment one might want to replace the laser with an efficient single-mode single-photon source where these problems would be reduced. We note that a GHZ experiment is in the process of being performed [21]. This experiment is an extension of the Innsbruck teleportation scheme [18] and suffers a three-photon coincidence rate of less than 10^{-2} sec^{-1}, three orders of magnitude below that predicted here.

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[1] P. W. Shor, in *Proceedings of the 35th Annual Symposium on the Foundations of Computer Science*, edited by S. Goldwasser (IEEE Computer Society Press, Los Alamitos, CA, 1994), p. 124.
[2] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).
[3] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982); W. Perrie, A. J. Duncan, H. J. Beyer, and H. Kleinpoppen, *ibid.* **54**, 1790 (1985); **54**, 2647(E) (1985).
[4] Z. Y. Ou and L. Mandel, Phys. Rev. Lett. **61**, 50 (1988); Y. H. Shih and C. O. Alley, *ibid.* **61**, 2921 (1988).
[5] J. G. Rarity and P. R. Tapster, Phys. Rev. Lett. **64**, 2495 (1990).
[6] P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, Phys. Rev. Lett. **75**, 4337 (1995).
[7] This is the result of a simplistic calculation based on an extension of the theory of parametric down-conversion, using optimistic values of nonlinearity χ^3.
[8] B. Yurke and D. Stoler, Phys. Rev. Lett. **68**, 1251 (1992).
[9] M. Zukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).
[10] J. G. Rarity, Ann. (N.Y.) Acad. Sci. **755**, 624 (1995).
[11] A. Zeilinger, M. A. Horne, H. Weinfurtur, and M. Zukowski, Phys. Rev. Lett. **78**, 3031 (1997).
[12] J. G. Rarity and P. R. Tapster, in *Quantum Interferometry*, edited by F. De Martini, G. Denardo, and Y. Shih (VCH, Weinheim, 1996), pp. 211–222.
[13] J. G. Rarity, P. R. Tapster, and R. Loudon, e-print quant-ph/9702032.

[14] J. G. Rarity and P. R. Tapster, Philos. Trans. R. Soc. London, Ser. A **355**, 2267 (1997).

[15] C. K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986).

[16] J. G. Rarity, P. R. Tapster, and E. Jakeman, Opt. Commun. **62**, 201 (1987).

[17] A. Heisenberg-limited single-photon wave packet has a duration set by the Fourier transform of its bandwidth. In practice the pump laser has a bandwidth $\Delta\omega_p$ and pulse duration $\Delta\tau_p \simeq 1/\Delta_p$. We use a detector filter with width $\Delta\omega_d$, giving an effective coherence length for the down-converted beams and the weak state $\Delta\tau_d \simeq 1/\Delta\omega_d$. To obtain high visibility interference we need coherence within the pulse, which we obtain when we limit the detector filter to give $\Delta\tau_d > \Delta\tau_p$. In other words, the detection of a photon has a fundamental time uncertainty set by the filter width, and is longer than the initial pump pulse length. Detections from the separate sources can then be made fully indistinguishable by spatial filtering into a single spatial mode. The filtering reduces the effective efficiency of the source by a factor of order $R \simeq \Delta\omega_d/\Delta\omega_p$, but with Gaussian filters the interference visibility falls like $(1 + \Delta\omega_d^2/2\Delta\omega_p^2)^{-1/2}$, which gives high visibility ($\simeq 0.9$) for $R \simeq 0.5$ (see Ref. [12]).

[18] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).

[19] D. Greenberger, M. Horne, and A. Zeilinger, Am. J. Phys. **58**, 1 (1990).

[20] J. G. Rarity and P. R. Tapster, Phys. Rev. A **41**, 5139 (1990).

[21] J.-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **80**, 3891 (1998).