Measurement scheme for relative phase diffusion between two Bose-Einstein condensates

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We propose a method of measuring diffusion of the relative phase between two Bose-Einstein condensates occupying different nuclear or electronic spin hyperfine states coupled by a two-photon magnetic-dipole transition via an intermediate level. Due to the macroscopic quantum coherence the condensates can be decoupled from the electromagnetic fields. The rate of decoherence and the phase collapse may be determined from the occupation of the intermediate level or the absorption of radiation. [S1050-2947(99)50404-6]

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In this paper we propose a method of measuring the diffusion of the relative phase between two Bose-Einstein condensates (BECs) formed in different hyperfine states of the same atom. These two states are coupled by a two-photon magnetic-dipole transition via an intermediate atomic level. Due to the macroscopic quantum coherence of the BECs the two-photon transition between the two hyperfine states is suppressed by quantum interference effects. This effect is similar in origin to that occurring in electromagnetically induced transparency (EIT) [1,2] and lasing without inversion [3]. Atomic interactions give rise to phase diffusion and destroy the coherence, so that the two-photon transition is no longer completely suppressed. The rate of phase diffusion may be determined by monitoring the population in the intermediate level or the absorption rate.

Since the first realizations of dilute gas alkali-metal BECs [4] the experiments have been broadened to include two- and multiple-condensate systems [5–9]. Myatt *et al.* [5] produced two overlapping BECs of $|F=1,m=-1\rangle$ and $|F=2,m=2\rangle$ states of ^{87}Rb using sympathetic cooling. The stability of this pair is due to an unexpectedly small inelastic collision rate between these states [10]. Recently, another BEC pair of $|1,-1\rangle$ and $|2,1\rangle$ states of ^{87}Rb has been realized at JILA [7–9]. These two states have essentially identical magnetic moments and an adjustable spatial overlap allowing the creation of a fully interpenetrating binary mixture. An especially interesting property is that these two BECs can be coupled by a two-photon transition (one microwave and one radio-frequency photon).

In general, inelastic collisions will limit the possibilities of magnetically trapping BEC pairs. However, optical dipole traps, which use optical forces to trap atoms, have a major advantage over magnetic traps since they can stably trap atoms in arbitrary hyperfine states. An evaporatively cooled ^{23}Na gas has been succesfully confined in a dipole trap with a simultaneous observation of BECs in several different hyperfine states [6].

A fascinating property of BECs is that they exhibit a macroscopic quantum coherence that is absent in thermal atomic ensembles [11]. Since one needs a phase reference to observe a phase, binary mixtures of BECs are especially useful in the

studies of coherence properties. The atom-atom interactions in finite-sized BECs affect the matter wave coherence. The width of the number distribution in the ground state has a dispersive effect on the BEC self-interactions and the relative phase undergoes quantum collapses and revivals [12,13]. Additional sources of phase diffusion are spatial mode fluctuations [14] and finite temperature decoherence due to the interactions between condensate and noncondensate atoms [15,16].

In this paper we consider a system closely related to the recent experiments of overlapping BECs in different hyperfine levels coupled by a two-photon transition [7–9]. Analogous BEC pairs could possibly be produced also in dipole traps [6]. As a consequence of the macroscopic quantum coherence of BECs the two hyperfine levels coupled by a two-photon transition exhibit two-photon coherence. By adjusting the initial conditions of the BECs and the driving electromagnetic (em) fields, the atoms in the BECs can be decoupled from the em fields. However, as a consequence of the decoherence and the collapse of the relative phase the two-photon coherence is reduced, and the cw em fields start inducing atomic transitions. This provides an excellent scheme for measuring phase dynamics in the present experimental setups [7–9]. The two-photon coherence of two optically coupled BECs has been previously predicted to result in various dramatic properties of the scattered light [17,18]. Measurements of magnetic coherence-related phenomena in atomic BECs have been previously addressed, e.g., in spin-polarized hydrogen [19] and in the electronic spin resonance of alkali-metal gases [20].

We consider a three-level system with the energies of levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ denoted by ω_1, ω_2, and ω_3. The weak microwave or rf cw em fields $\mathbf{B}_A$ and $\mathbf{B}_B$ induce magnetic-dipole transitions between the levels $1\leftrightarrow2$ and $2\leftrightarrow3$, respectively. For simplicity, we assume that the radiative lifetimes are long, so that the level widths can be ignored. The EM interaction introduces the following terms into the Hamiltonian density:

$$\mathcal{H}_{em}=-\boldsymbol{\mu}_{12}\cdot\mathbf{B}_A\psi_1^\dagger\psi_2-\boldsymbol{\mu}_{23}\cdot\mathbf{B}_B\psi_2^\dagger\psi_3+\text{H.c.}, \qquad (1)$$

where $\psi_i(\mathbf{r})$ is the field operator for the hyperfine state $|i\rangle$ and $\boldsymbol{\mu}_{ij}$ denotes the magnetic moment for the transition $i\leftrightarrow j$.

We consider a situation where two macroscopic, perfectly overlapping BECs occupy levels $|1\rangle$ and $|3\rangle$ and level $|2\rangle$ is initially empty. In Refs. [7–9] the double BEC system in levels $|1\rangle$ and $|3\rangle$ is prepared from the single BEC in state

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$|1\rangle$ by a two-photon transition. If the occupation of level $|2\rangle$ is small, the collisional interactions are mainly between atoms in levels $|1\rangle$ and $|3\rangle$. We write the interaction Hamiltonian density in terms of the field operators $\psi_1(\mathbf{r})$ and $\psi_3(\mathbf{r})$,

$$\mathcal{H}_{int} = \sum_{i=1,3} \frac{u_i}{2} \psi_i^\dagger \psi_i^\dagger \psi_i \psi_i + u_{13} \psi_1^\dagger \psi_3^\dagger \psi_3 \psi_1, \qquad (2)$$

where $u_i = 4\pi\hbar^2 a_i/m$. We could directly obtain the equations of motion for the condensate mean fields from Eq. (2), but the interactions are significantly simplified in the case of approximately equal scattering lengths resulting in $u_1 \simeq u_3 \simeq u_{13} \equiv u$. In ^{87}Rb [7,8], the scattering lengths satisfy $a_1 : a_{13} : a_3 :: 1.03 : 1 : 0.97$, where a_1 (a_3) denotes the intraspecies scattering length for state $|1,-1\rangle$ ($|2,1\rangle$), and a_{13} is the interspecies scattering length. We approximate the field operators $\psi_i(\mathbf{r}) \simeq \phi_i(\mathbf{r}) b_i$ for levels $|1\rangle$ and $|3\rangle$ in terms of the BEC annihilation operator b_i and the corresponding spatial wave function $\phi_i(\mathbf{r})$. It is also assumed that $\phi_1 = \phi_3 \equiv \phi$. Our aim is to write the equations of motion for the expectation values $\sigma_{ij} \equiv \langle b_i^\dagger b_j \rangle / N$, where the total initial number of BEC particles is denoted by N. The effect of noncondensate atoms is treated by a phenomenological damping in the equation of motion for the coherence between the two BECs. By approximating the scattering lengths to be equal and by assuming $\langle b_2^\dagger b_2 \rangle \ll N$ we may then approximate the interaction Hamiltonian [Eq. (2)] by

$$H_{int} \simeq \frac{\hbar\kappa}{2}(\hat{N}^2 - \hat{N} - 2\hat{N} b_2^\dagger b_2), \qquad (3)$$

where $\hat{N} = \Sigma_i b_i^\dagger b_i$ is assumed to be a constant of the motion and $\hbar\kappa = u \int d^3 r |\phi|^4$. We see that for perfectly overlapping BECs with approximately equal scattering lengths the effect of BEC self-interactions is strongly suppressed. Finally, the detunings are defined by $\delta_{21} \equiv \omega_A - \omega_{21}$ and $\delta_{32} \equiv \omega_B - \omega_{32}$, where $\omega_A(\omega_B)$ is the frequency of the field $\mathbf{B}_A$ ($\mathbf{B}_B$) and the transition frequency between the hyperfine levels $1 \leftrightarrow 2$ ($2 \leftrightarrow 3$) is ω_{21} (ω_{32}). In the rotating-wave approximation we then obtain the following equations of motion for the expectation values σ_{ij}:

$$\dot{\sigma}_{11} = \Omega_A \, \text{Im}(\sigma_{21}), \qquad (4a)$$

$$\dot{\sigma}_{22} = -\Omega_A \, \text{Im}(\sigma_{21}) + \Omega_B \, \text{Im}(\sigma_{32}), \qquad (4b)$$

$$\dot{\sigma}_{33} = -\Omega_B \, \text{Im}(\sigma_{32}), \qquad (4c)$$

$$\dot{\sigma}_{21} = -i(\delta_{21} + N\kappa)\sigma_{21} + \frac{i\Omega_A}{2}(\sigma_{22} - \sigma_{11}) - \frac{i\Omega_B}{2}\sigma_{31}, \qquad (4d)$$

$$\dot{\sigma}_{32} = -i(\delta_{32} - N\kappa)\sigma_{32} + \frac{i\Omega_B}{2}(\sigma_{33} - \sigma_{22}) + \frac{i\Omega_A}{2}\sigma_{31}, \qquad (4e)$$

$$\dot{\sigma}_{31} = -i(\delta_{32} + \delta_{21} - i\gamma)\sigma_{31} + \frac{i\Omega_A}{2}\sigma_{32} - \frac{i\Omega_B}{2}\sigma_{21}, \qquad (4f)$$

where the Rabi frequencies are given by $\Omega_A \equiv 2\int d^3 r \phi_2^* \phi_1 \boldsymbol{\mu}_{21} \cdot \mathbf{B}_A/\hbar$, $\Omega_B \equiv 2\int d^3 r \phi_2^* \phi_3 \boldsymbol{\mu}_{23} \cdot \mathbf{B}_B/\hbar$, and Im denotes the imaginary part. For simplicity, in Eq. (4) we have set Ω_A and Ω_B to be real. Here the equations correspond to the cascade or ladder three-level system with $\omega_1 < \omega_2 < \omega_3$. In the case of a Λ (V) three-level scheme the sign of δ_{32} (δ_{21}) should be changed. The dominant mean-field contribution of the BEC self-interactions is to shift the resonance conditions in Eqs. (4d) and (4e).

Level $|2\rangle$ is assumed to be initially empty and two BECs occupy levels $|1\rangle$ and $|3\rangle$. The off-diagonal element σ_{31} describes the macroscopic coherence between the two BECs. As explained earlier this collapses and decoheres due to the atom-atom interactions. We have included the effect of the decay of the matter wave coherence in Eq. (4) in terms of a phenomenological damping parameter γ in the equation of motion for σ_{31}. This damping parameter includes contributions from both the quantum effects of BEC self-interactions and collisions between condensate and noncondensate atoms. We assume that these dominate other damping mechanisms as long as the population in level $|2\rangle$ remains small.

For an initial condition for Eq. (4) we set $\sigma_{11} = \sigma_{33} = \sigma_{31} = 1/2$, indicating a well-established coherence between the two BECs with a vanishing relative phase. This corresponds, e.g., to a situation where a BEC is first prepared in level $|1\rangle$ and half of the BEC atoms are then coherently transferred to level $|3\rangle$, so that the atoms remain entangled. Initially there are no atoms in the intermediate level $\sigma_{22} = 0$. The em fields are two-photon resonant ($\delta_{21} = -\delta_{32}$) and $\Omega_A = -\Omega_B \equiv \Omega$. It is easy to see from Eq. (4) that for $\gamma = 0$, this corresponds to a steady-state situation. Both BECs are decoupled from the em fields and the absorption described by $\text{Im}(\sigma_{21})$ and $\text{Im}(\sigma_{32})$ vanishes [21]. However, for nonzero γ, $\text{Im}(\sigma_{21})$ and $\text{Im}(\sigma_{32})$ also become nonzero, and atoms start accumulating in level $|2\rangle$ due to the absorption of em radiation. The phase damping parameter γ may be determined by measuring the oscillations of the em fields or the population in level $|2\rangle$.

Equations (4) may be integrated numerically, but it is also illuminating to look at analytic estimates. We consider a situation where the em fields are resonant, $\delta_{21} + N\kappa = \delta_{32} - N\kappa = 0$, and the damping γ is much smaller than the Rabi frequency Ω. We look for an exponential solution to Eq. (4) to leading order in the small parameter γ/Ω. By also determining the coefficients to first order in γ/Ω we obtain $\sigma_{31}(t)$, describing the matter wave coherence between the BECs as

$$\sigma_{31}(t) \simeq \frac{1}{2} e^{-3\gamma t/4} - \frac{\gamma}{8\sqrt{2}\Omega} e^{-\gamma t/8} \sin(\sqrt{2}\Omega t). \qquad (5)$$

The absorption of the field $\mathbf{B}_A$ is proportional to

$$\text{Im}[\sigma_{21}(t)] \simeq -\frac{\gamma}{8\Omega} e^{-3\gamma t/4} + \frac{\gamma}{8\Omega} e^{-\gamma t/8} \cos(\sqrt{2}\Omega t). \qquad (6)$$

The absorption results in oscillating em fields with the amplitude of the oscillations given by $\gamma/(8\Omega)e^{-\gamma t/8}$. The real parts satisfy $\text{Re}[\sigma_{21}(t)] = \text{Re}[\sigma_{32}(t)] = 0$. Finally, the occupation in level $|2\rangle$ is
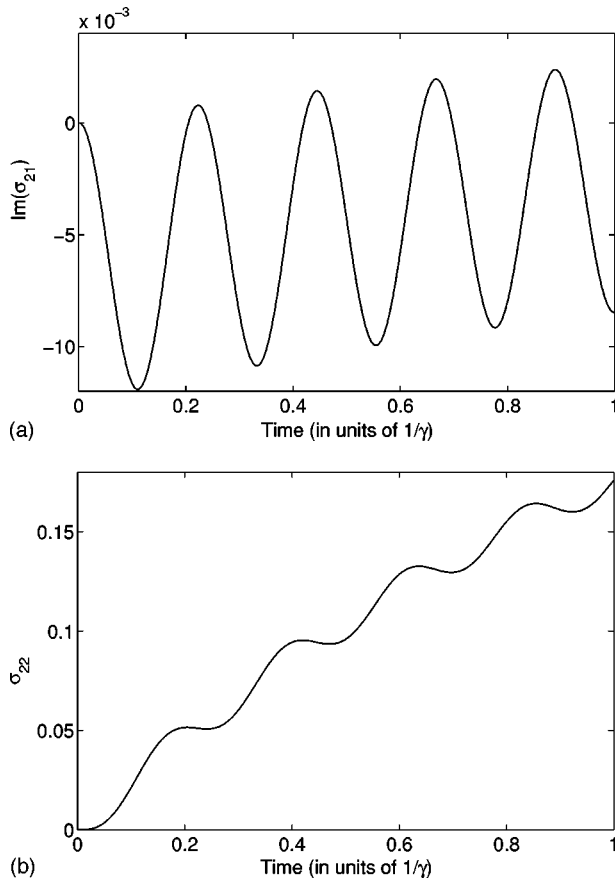
FIG. 1. Particular example of the response of the medium in the case of $\Omega = 20\gamma$. (a) $\text{Im}[\sigma_{21}(t)]$ proportional to the absorption of the em radiation from the field driving the transition between levels $|1\rangle$ and $|2\rangle$. (b) $\sigma_{22}(t)$ proportional to the population of the intermediate level. The em fields are assumed to be resonant.

$$\sigma_{22}(t) \simeq \frac{1}{3}(1 - e^{-3\gamma t/4}) - \frac{\gamma}{4\sqrt{2}\Omega} e^{-\gamma t/8} \sin(\sqrt{2}\Omega t). \quad (7)$$

Due to the decoherence of the BECs the em fields absorb radiation and atoms start occupying level $|2\rangle$. By measuring the number of atoms in state $|2\rangle$ at time t, after switching on the driving em fields, one could determine the damping rate of the matter wave coherence γ. Alternatively, the damping rate could be observed from the amplitude of the oscillating em signal.

In Fig. 1 we have plotted one example of the signal corresponding to a particular value $\Omega = 20\gamma$. We plot $\text{Im}[\sigma_{21}(t)]$ and $\sigma_{22}(t)$ obtained by numerically integrating Eq. (4) for $\delta_{21} + N\kappa = \delta_{32} - N\kappa = 0$, and for the initial conditions $\sigma_{11}(0) = \sigma_{33}(0) = \sigma_{31}(0) = 1/2$ and $\sigma_{21}(0) = \sigma_{32}(0) = 0$. The oscillating signal (a) and the accumulating population in the intermediate level (b) are clearly observed. The graphs are also well represented by the approximate analytic results [Eqs. (6) and (7)].

If the fields are off-resonant from the intermediate level $|2\rangle$, the absorption of the radiation and the occupation of state $|2\rangle$ are reduced, but $\text{Re}(\sigma_{21})$ and $\text{Re}(\sigma_{32})$ are, in this case, nonzero.

In EIT the interference between the off-diagonal density matrix elements leads to an initially opaque medium being

rendered almost transparent [1,2]. The transition amplitudes driven by two oppositely phased em fields destructively interfere. The effect of the medium on the em fields is canceled. The crucial quantities for the interference are the coherences σ_{21} and σ_{32} describing the transition dipole matrix elements excited by the driving em fields. In EIT the coherence between levels $|1\rangle$ and $|3\rangle$ is present only as a consequence of the em coupling via level $|2\rangle$. For BECs in hyperfine states $|1\rangle$ and $|3\rangle$ the coherence σ_{31} is present from the start without being created by the driving fields due to the macroscopic matter wave coherence. As a result, the em fields couple σ_{31} to the oscillating dipoles. The presence of the macroscopic quantum coherence of BECs can then completely inhibit the em fields from establishing any coherence between levels $|1\rangle$ and $|2\rangle$ or $|2\rangle$ and $|3\rangle$. On the other hand, the rate at which coherences σ_{21} and σ_{32} are induced describes the decoherence rate of BECs.

Although so far only a two-photon coupling between a BEC pair has been experimentally implemented, one may wonder whether the coherence of BECs coupled by a single-photon transition could also suppress absorption in such a way that the phase diffusion rate could be measured. In the absence of matter wave coherence between levels $|g\rangle$ and $|g'\rangle$, a driving em field will start inducing oscillations of the atom numbers if the population is initially unbalanced or if the em field is nonresonant. The macroscopic quantum coherence of BECs in levels $|g\rangle$ and $|g'\rangle$ results in a nonvanishing initial value of $\sigma_{gg'}$. For an unbalanced population and with appropriately selected initial values of $\sigma_{gg'}$ and the detuning, the absorption may be completely canceled according to the Bloch equations. Instead of being due to the quantum interference of transition amplitudes, the cancellation of the absorption in this case results from the suppression of the phase rotation of the dipole matrix element. As the coherence is reduced due to the phase diffusion, the absorption becomes nonzero and the population balanced. However, the validity of the present formalism is questionable in the case of significant population imbalance, because the BECs may then occupy different spatial regions.

In recent experiments Hall et al. [9] studied phase diffusion in a binary BEC mixture of ^{87}Rb. A BEC was first prepared in level $|1, -1\rangle$ and a part of the condensate was then transferred to level $|2, 1\rangle$. The relative phase between the two separated halves was determined by interfering the atoms at a later time. The phase diffusion rate was estimated by varying the evolution time of the two BECs before the interference measurement. In every interference measurement the BECs were destructively imaged and the repetitions of independent runs produced information about the uncertainty of the phase. Only weak phase diffusion was observed.

In addition to the environmentally induced decoherence due to the interactions between condensate and thermal noncondensate atoms [15,16], the quantum collapse due to the BEC self-interactions has an important effect on the phase diffusion [12,13]. This rate dramatically depends on the relative strengths of the three scattering lengths a_1, a_3, and a_{13} in Eq. (2). Under conditions where the scattering lengths are equal, the two BECs are perfectly overlapping, and only levels $|1\rangle$ and $|3\rangle$ are occupied; the interaction Hamiltonian in Eq. (3) depends only on the constant total atom number. This suppresses the phase collapse. If the mean-field contribution

of Eq. (2) vanishes, Eqs. (4) may also be defined (without the terms $N\kappa$) in terms of the expectation values for the total field operators $\sigma_{ij} \equiv \int d^3 r \langle \psi_i^\dagger(\mathbf{r}) \psi_j(\mathbf{r}) \rangle / N$.

A two-photon transition between a BEC pair of $|1,-1\rangle$ and $|2,1\rangle$ states of ^{87}Rb has been implemented via level $|2,0\rangle$ [7–9]. This intermediate state for the two-photon coupling is untrappable. The atoms in $|2,0\rangle$ can escape the trap. This would correspond in our scheme to an additional damping Γ for level $|2\rangle$. If the population in level $|2\rangle$ is small, this damping would be approximately independent of the number of atoms in $|2\rangle$. In that case we could add the following additional term to Eq. (4b): $\dot{\sigma}_{22} = \cdots - \Gamma \sigma_{22}$. Then the phase diffusion could possibly be measured by monitoring the number of atoms escaped through level $|2,0\rangle$ or by counting the atoms remained in levels $|1,-1\rangle$ and $|2,1\rangle$. Optical dipole traps [6] can stably trap atoms in arbitrary hyperfine levels. Suitable BEC pairs, without losses of atoms, could possibly be produced in dipole traps to implement the proposed scheme for the measurement of the phase diffusion.

Unlike the previous measurements of phase diffusion [9], our model may, in principle, allow continuous and nondestructive monitoring of the phase dynamics. However, in practice it may be difficult to realize the two-photon resonance condition over the duration of the experiment, e.g.,

due to the technical noise in magnetic fields. In that case γ may have to be determined by an ensemble of measurements. Another experimental challenge may be the assumption of the spatial overlap between the two BECs. In the case of magnetically trapped ^{87}Rb the BEC pair of $|1,-1\rangle$ and $|2,1\rangle$ states may easily separate [9]. Nevertheless, it is easy to see from the Thomas-Fermi solutions for the binary BECs that, at least in the Thomas-Fermi limit, the overlap of the ground states can be perfect, even if the two BECs ''see'' two different trap frequencies. It is also important to emphasize that in our measurement scheme, in the case of resonant em fields, the initially equal occupation of the two BECs remains equal also at later times, so that the relative spatial volume of the condensate wave functions does not change.

In conclusion, we have proposed a method of measuring the ''phase memory'' of a BEC pair. This method relies on the quantum interference of transition amplitudes and is similar in origin to that occurring in EIT and lasing without inversion.

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[20] W. Zhang and G.-Q. Liu, Phys. Rev. B **56**, 14 645 (1997).

[21] To decouple the BECs from the em fields in the case of a nonvanishing relative phase between the BECs, $\Delta\varphi \equiv \arg(\langle a_3^\dagger a_1 \rangle)$, we need to choose $\Delta\phi = \pi + \Delta\varphi$, where $\Delta\phi \equiv \arg(\Omega_B^* \Omega_A)$ is the phase difference between the Rabi frequencies.