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RAPID COMMUNICATIONS

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Deterministic Bell states and measurement of the motional state of two trapped ions

E. Solano,[1,2] R. L. de Matos Filho,[1] and N. Zagury[1]

[1]*Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21945-970 Rio de Janeiro, RJ, Brazil*

[2]*Sección Física, Departamento de Ciencias, Pontificia Universidad Católica del Perú, Apartado 1761, Lima, Peru*

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We present a method for the deterministic generation of all the electronic Bell states of two trapped ions. It involves the combination of a purely dispersive with a resonant laser excitation of vibronic transitions of the ions. In contrast to other methods presented up to now, our proposal does not require differential laser addressing of the individual ions and may be easily implemented with present available techniques. It is further shown that this excitation scheme is highly adequate for the complete determination of the motional state of the ions. [S1050-2947(99)50604-5]

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The deterministic preparation of two-particle entangled states has become a subject of increasing interest. The quantum correlations between the particles in such states may give rise to striking phenomena, as the exclusion of a local realistic description of nature [1]. Very recently it has been shown that, beside their interest in studying the fundamentals of quantum physics, the deterministic generation of two-particle entangled states is of extreme importance to experimentalists for the implementation of quantum teleportation [2], quantum cryptography [3], and quantum computation [4]. Controlled entanglement between two massive particles has been achieved both in the case of atoms crossing a high Q cavity [5] and in the case of ions in a trap [6]. In the preparation and manipulation of entangled states of two separated particles it is very important to have a system that allows a well controllable coherent interaction between the particles and an effective protection of the entangled states against perturbations of the environment. The system composed by ions in a linear trap meets these requirements very well [7]. In such systems the interplay of the Coulomb interaction between the ions and their coherent interaction with laser fields can be explored to create entangled states of the ions.

Up to now most proposals presented to generate deterministic entangled states of trapped ions rely on the necessity of addressing individual ions by laser light [8–10]. As far as we know present experimental conditions are such that the equilibrium distance between the trapped and cooled ions is shorter than the beam waist of the applied laser fields [7]. Therefore, presently, laser beams cannot be focused exclusively on individual ions. Recently, the consequences of this experimental constraint have been partially circumvented in Ref. [6], by introducing a differential displacement of the individual ions from the trap center. Due to the ion micromotion, the coupling constants between the individual ions and the laser field become different. This allowed the authors of Ref. [6] to prepare electronic entangled states of two ions, which are good approximations of two Bell states.

In this Rapid Communication we present a scheme that allows the deterministic generation of all the electronic Bell states of two trapped ions without the requirement of differential laser addressing of the ions. This method is also highly adequate to perform a complete determination of their vibrational quantum state, including the possible entanglement between the two vibrational modes along their alignment direction.

Let us consider the situation in which two trapped ions of mass M are cooled down to very low temperatures and aligned along the z direction, their equilibrium position, at zero temperature, being $z_{10}=d/2$ and $z_{20}=-d/2$. We denote by $\hat{Z}=(\hat{z}_1+\hat{z}_2)/2$ and $\hat{z}=\hat{z}_1-\hat{z}_2$ the center of mass and relative coordinate position operators. Now consider that the

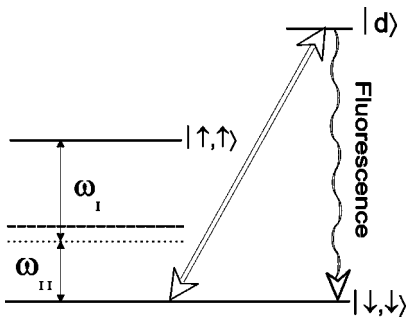

FIG. 1. Energy-level diagram. A stimulated electronic transition is induced by means of two dispersive excitations with frequencies $\omega_I=\omega_0+k\nu-\delta$ and $\omega'_{II}=\omega_0-k\nu+\delta$. The detection of the electronic state, $|\downarrow,\downarrow\rangle$ or $|\uparrow,\uparrow\rangle$, is provided by the fluorescence resulting from optical pumping of the ions with a pulse tuned to the cyclic transition $|\downarrow,\downarrow\rangle\Leftrightarrow|d\rangle$.

ions are excited by two classical homogeneous laser fields, $\vec{E}_I=\vec{E}_{0I}e^{i(\vec{q}_I\cdot\vec{r}-\omega_I t+\phi_I)}$ and $\vec{E}_{II}=\vec{E}_{0II}e^{i(\vec{q}_{II}\cdot\vec{r}-\omega_{II}t+\phi_{II})}$, with $\vec{q}_I$ and $\vec{q}_{II}$ parallel to the z direction, which are quasiresonant with a long living electronic transition among two hyperfine levels $|\downarrow\rangle$ and $|\uparrow\rangle$ of the ions. In the Schrödinger picture the interaction Hamiltonian describing this situation is given by

$$\hat{H}=\sum_{\alpha=I,II}\hbar\Omega_\alpha[e^{i(q_\alpha\hat{Z}-\omega_\alpha t+\phi_\alpha)}\times(\hat{D}_1e^{iq_\alpha\hat{z}/2}+\hat{D}_2e^{-iq_\alpha\hat{z}/2})+\text{H.c.}],\quad(1)$$

where Ω_α ($\alpha=I,II$) are the Rabi classical frequencies corresponding to the one photon transitions and $\hat{D}_j$ is the electric dipole transition operator associated with the ion at the position $\hat{z}_j$ ($j=1,2$). It can be written as $\hat{D}_j=\hat{S}_{+j}+\hat{S}_{-j}$, with $\hat{S}_{+j}=|\uparrow_j\rangle\langle\downarrow_j|$ and $\hat{S}_{-j}=|\downarrow_j\rangle\langle\uparrow_j|$ being the raising and lowering operators associated with the two electronic levels, $|\downarrow\rangle$ and $|\uparrow\rangle$, of frequencies $\omega_\downarrow$ and $\omega_\uparrow=\omega_0+\omega_\downarrow$, respectively (see Fig. 1).

In practice the situation described above could be effectively realized by two pairs of laser fields so that each laser field interaction in Eq. (1) stands for a Raman interaction induced by a pair of laser beams. The lasers belonging to the first (second) pair would have frequencies ω_1 and ω_2 (ω'_1 and ω'_2) so that $\omega_I=\omega_1-\omega_2$ ($\omega_{II}=\omega'_1-\omega'_2$). We notice that the relative phase of the lasers in the first and in the second pair should correspond to ϕ_I and ϕ_{II} in Eq. (1), respectively. To effectively generate the interaction described by Eq. (1) these two laser pairs should independently connect the electronic levels $|\downarrow\rangle$ and $|\uparrow\rangle$ of both ions through a virtual third level $|c\rangle$. This will be the case if the difference $|\Delta-\Delta'|$ in the detunings, $\Delta=\omega_c-\omega_\downarrow-\omega_1$ and $\Delta'=\omega_c-\omega_\downarrow-\omega'_1$, is much larger than the eigenfrequencies of the vibrational motion.

From now on we will consider only the effective interaction described in Eq. (1), taking into consideration that ϕ_I and ϕ_{II} may be set at will. Furthermore, for simplicity, we take $\Omega_I=\Omega_{II}=\Omega$, $q_I=q_{II}=q$, and $\phi_I=\phi_{II}=\phi$. We discuss a situation where the effective transitions described in Eq. (1) are quasiresonant to one of the vibrational modes, say, the center-of-mass one, which we take as having frequency ν. Dispersive interactions quasiresonant to a vibrational mode have already been considered in Refs. [11,10]. Assuming that $\omega_I=\omega_0+k\nu-\delta$ and $\omega_{II}=\omega_0-k'\nu+\delta'$, where $\delta,\delta'\ll\nu$, and discarding the rapidly oscillating terms in Eq. (1), the Hamiltonian in the interaction picture may be written as

$$\hat{H}=\hbar\Omega[(\hat{S}_{+1}e^{i\phi_0/2}+\hat{S}_{+2}e^{-i\phi_0/2})[(i\eta)^k\hat{a}^{\dagger k}\hat{F}_k(\hat{n}_c,\hat{n}_r)e^{i\delta t}+(i\eta)^{k'}\hat{F}_{k'}(\hat{n}_c,\hat{n}_r)\hat{a}^{k'}e^{-i\delta' t}]e^{i\phi}+\text{H.c.}].\quad(2)$$

Here, $\phi_0=qd$ is the phase difference due to the spacing between the ions, $\hat{a},\hat{b}$ and $\hat{a}^\dagger,\hat{b}^\dagger$ are the annihilation and creation operators associated with the excitations of the center of mass and relative motion modes, respectively. The corresponding number operators are $\hat{n}_c=\hat{a}^\dagger\hat{a}$ and $\hat{n}_r=\hat{b}^\dagger\hat{b}$ and

$$\hat{F}_k(\hat{n}_c,\hat{n}_r)=\sum f_k(n_c,n_r)|n_c,n_r\rangle\langle n_c,n_r|\quad(3)$$

with

$$f_k(n_c,n_r)=e^{-(\eta^2+\eta_r^2)/2}\frac{n_c!}{(n_c+k)!}L^k_{n_c}(\eta^2)L^0_{n_r}(\eta_r^2).\quad(4)$$

$\eta=q\sqrt{\hbar/4M\nu}$ and $\eta_r=q\sqrt{\hbar/4M\nu_r}$ are the Lamb-Dicke parameters associated to the center of mass and relative vibrations, respectively, and $L^k_m(x)$ are associated Laguerre polynomials. For the Coulomb interaction among the two ions it can be easily shown that $\nu_r=\sqrt{3}\nu$, so that $\eta_r=\eta/\sqrt[4]{3}$. When $k=k'$ and $\delta'=\delta$, so that $\omega_I+\omega_{II}=2\omega_0$ [12], Eq. (2) reduces to the simpler result

$$\hat{H}=\hbar\Omega[(i\eta)^k(\hat{S}'_{+1}+\hat{S}'_{+2})e^{i\phi}+\text{H.c.}]\times[\hat{a}^{\dagger k}\hat{F}_k(\hat{n}_c,\hat{n}_r)e^{i\delta t}+\hat{F}_k(\hat{n}_c,\hat{n}_r)\hat{a}^k e^{-i\delta t}],\quad(5)$$

where $\hat{S}'_{+1}=\hat{S}_{+1}e^{i\phi_o/2}$ and $\hat{S}'_{+2}=\hat{S}_{+2}e^{-i\phi_o/2}$. A similar result may be obtained if we choose to excite the stretch mode. Note that for $k\neq0,1$ we may have $k\nu-\delta=m\sqrt{3}\nu-\overline{\delta}$, with $\delta,\overline{\delta}\ll\nu$, $\eta_r^m/\overline{\delta}\approx O(\eta^k/\delta)$, and both modes could be excited simultaneously. This will not be the case if we exchange the roles of the stretch and center of mass modes in Eq. (5). When δ is much larger than the vibronic Rabi frequency, $\eta^k\Omega f_k(n_c,n_r)$, and for interaction times $\delta t\gg1$, the Hilbert space of the electronic states effectively decouples into two subspaces: one containing only one ion in the excited state and the other containing both ions either in the excited or in the ground state. In this case the effective Hamiltonian reads

$$\hat{H}_{\text{eff}}=\hbar\Omega_k\left[\hat{S}'_{+1}\hat{S}'_{+2}e^{2i\phi}+(-1)^k\left(\hat{S}'_{+1}\hat{S}'_{-2}+\frac{1}{2}\right)\right]\times\hat{F}_k^2(\hat{n}_c,\hat{n}_r)\left[\frac{\hat{n}_c!}{(\hat{n}_c-k)!}-\frac{(\hat{n}_c+k)!}{\hat{n}_c!}\right]+\text{H.c.},\quad(6)$$

where $\Omega_k=2|\Omega|^2(i\eta)^{2k}/\delta$. The first term and its Hermitian conjugate describe two-photon processes leading to the simultaneous excitation or deexcitation of the electronic states of the two ions. The second term and its Hermitian conjugate describe processes where one ion undergoes a transition from

the ground to the excited electronic state and the other ion makes a transition in the inverse direction, both processes taking place simultaneously. The third term is the contribution from the interaction to the self-energy of the ions. Note that processes leading to independent excitation of the ions are not present anymore.

Assume that we have prepared the two-ion system in the state $|\downarrow,\downarrow\rangle_{n_c,n_r}\equiv|\downarrow,\downarrow\rangle\otimes|n_c,n_r\rangle$. Applying the two laser pairs simultaneously, this initial state evolves after a time t to

$$e^{-(i/\hbar)\hat{H}_{\rm eff}t}|\downarrow,\downarrow\rangle_{n_c,n_r}=e^{-i(-1)^k\Omega^k_{n_cn_r}t}[\cos(|\Omega^k_{n_cn_r}|t)|\downarrow,\downarrow\rangle + i(-1)^k e^{2i\phi}\sin(|\Omega^k_{n_cn_r}|t)|\uparrow,\uparrow\rangle] \otimes|n_c,n_r\rangle, \tag{7}$$

where

$$\Omega^k_{n_cn_r}=\Omega_k f_k(n_c,n_r)^2\left[\frac{n_c!}{(n_c-k)!}-\frac{(n_c+k)!}{n_c!}\right] \tag{8}$$

are effective Rabi frequencies for transitions in the Hilbert subspace spanned by the states $|\downarrow,\downarrow\rangle_{n_c,n_r}$ and $|\uparrow,\uparrow\rangle_{n_c,n_r}$. Note that, in this approximation, the interaction is purely dispersive, so that it does not change the number of vibrational excitations, leaving vibrational Fock states $|n_c,n_r\rangle$ unchanged. Also, the two-ion state with one ion in the excited state and the other in the ground state is never populated, if the initial state is any combination of states where the ions are both in their excited or ground electronic states. Therefore, the time evolution described by Eq. (7) corresponds to an ''effective two-photon transition'' among the two states $|\downarrow,\downarrow\rangle$ and $|\uparrow,\uparrow\rangle$. In this sense they could be considered as a collective two-level system. This fact can be exploited for measuring the vibrational state of the two-ion system, as will be shown below.

Equation (7) shows that we may generate any combination of the ground, $|\downarrow,\downarrow\rangle$, and the doubly excited states, $|\uparrow,\uparrow\rangle$, and, in particular, the two orthogonal Bell states

$$|\Phi^{(\pm)}\rangle=\frac{1}{\sqrt{2}}[|\downarrow,\downarrow\rangle\pm i(-1)^k e^{2i\phi}|\uparrow,\uparrow\rangle], \tag{9}$$

if we let the lasers interact with the ions for either a time $t_+=\pi/(4|\Omega^k_{n_cn_r}|)$ (to generate the state $|\Phi^{(+)}\rangle$) or a time $t_-=3\pi/(4|\Omega^k_{n_cn_r}|)$ (to generate the state $|\Phi^{(-)}\rangle$). Alternatively, we could maintain the same time of interaction and change the value of ϕ by $\pi/2$.

We will show now that it is possible to prepare the other two Bell states, applying a laser pulse resonant to the electronic transition $|\uparrow\rangle\leftrightarrow|\downarrow\rangle$ (carrier pulse) to one of the $|\Phi^{(\pm)}\rangle$ states prepared with the dispersive interaction. For a carrier pulse we just need to deal with one effective laser $\vec{E}=\vec{E}_0e^{i(\vec{q}_0\cdot\vec{r}-\omega_0t+\varphi)}$, with $\vec{q}_0$ in the z direction. In this case the effective Hamiltonian is

$$\hat{H}=\hbar\Omega[(S_{+1}e^{i\varphi_0/2}+S_{+2}e^{-i\varphi_0/2})e^{i\varphi}+\text{H.c.}]\hat{F}_0(\hat{n}_c,\hat{n}_r), \tag{10}$$

where Ω is the Rabi frequency associated with the Raman transition and $\varphi_0=q_0d$.

After we have prepared one of the states $|\Phi^{(\pm)}\rangle|n_c,n_r\rangle$ we apply the carrier pulse during a time t_0. The time evolution of $|\Phi^{(\pm)}\rangle|n_c,n_r\rangle$ is then given by

$$\frac{1}{\sqrt{2}}\{[\cos^2(\Omega_0t_0)\mp\sin^2(\Omega_0t_0)e^{-2i\varphi}]|\downarrow,\downarrow\rangle - \frac{i}{2}\sin(2\Omega_0t_0)(e^{i\varphi}\pm e^{-i\varphi})(e^{i\varphi_0/2}|\uparrow,\downarrow\rangle+e^{-i\varphi_0/2}|\downarrow,\uparrow\rangle) +[\pm\cos^2(\Omega_0t_0)-\sin^2(\Omega_0t_0)e^{2i\varphi}]|\uparrow,\uparrow\rangle\otimes|n_c,n_r\rangle\}, \tag{11}$$

where $\Omega_0=\Omega f_0(n_c,n_r)$. If we let the pair of Raman lasers interact with the ions during $t_0=\pi/(4|\Omega_0|)$ and choose their relative phases equal to 0 (modulus π) or $\varphi=\pi/2$ (modulus π), accordingly, if we start with $|\Phi^{(+)}\rangle$ or $|\Phi^{(-)}\rangle$, we generate the Bell state

$$\frac{1}{\sqrt{2}}[|\uparrow,\downarrow\rangle+e^{i\varphi_0}|\uparrow,\downarrow\rangle]. \tag{12}$$

The state orthogonal to this one may be generated by changing φ_0 to $\varphi_0+\pi$, as was done in Ref. [6].

It is remarkable that, in generating these Bell states, we neither need to be precise about which sideband k is excited nor to claim the validity of the Lamb-Dicke approximation. This is a consequence of the fact that the interactions we are considering do not change the values of the vibrational quanta, either because they are purely dispersive [Eq. (6)] or correspond to a carrier pulse [Eq. (10)].

Until now we have considered the ions to be in a vibrational two-mode Fock state. The general state that describes a motional thermal state, when the ions are cooled down to a temperature close to zero, is a density operator of the form

$$\hat{\varrho}_v=\sum\Pi_{n_c,n_r}|n_c,n_r\rangle\langle n_c,n_r|, \tag{13}$$

where $\Pi_{n_c,n_r}=\langle n_c,n_r|\hat{\varrho}_v|n_c,n_r\rangle$ is the vibrational population. As $\Omega^k_{n_cn_r}$ depends on the values of n_c and n_r our proposal to generate the Bell states will not work in general. In this case, there are several different contributions to the probability of exciting the vibronic transitions, which may interfere without control when the temperature departs from zero. Notice that thermal states at very low temperatures, such that the probability of finding $n_r\approx 99\%$ and $n_c\approx 90\%$, have already been reported in Ref. [7]. On the other hand, if we consider excitations in the first sideband ($k=1$) and if the Lamb-Dicke limit is valid [$f_k(n_c,n_r)\to 1$], the effective Rabi frequencies are independent of the vibrational state of the ions. This fact has been noticed before by Sørensen and Møllmer [10]. It allows the generation of electronic Bell states even if the ions are in a thermal state with equilibrium mean vibrational numbers much less than the inverse of the Lamb-Dicke parameters.

Let us now turn our attention to the determination of a general vibrational quantum state, $\hat{\varrho}_v$, of the ions. Our method is based on the determination of the two-mode

Wigner function $W(\alpha_1,\alpha_2)$, characterizing the joint state of two harmonic quantum oscillators at the point (α_1,α_2) of their extended phase space, which can be written as [14]

$$W(\alpha_1,\alpha_2)=\frac{4}{\pi^2}\sum_{n_1,n_2}^{\infty}(-1)^{n_1+n_2}\Pi_{n_1,n_2}(-\alpha_1,-\alpha_2), \quad (14)$$

where $\Pi_{n_1,n_2}(-\alpha_1,-\alpha_2)$ is the population corresponding to the two-mode harmonic-oscillator state displaced coherently in the extended phase space by $-\alpha_1$ and $-\alpha_2$, respectively. To determine the vibrational Wigner function, $W(\alpha_c,\alpha_r)$, using the above relation [Eq. (14)], we first displace coherently the state $\hat{\varrho}_v$ in the extended phase space,

$$\hat{\varrho}_v(-\alpha_c,-\alpha_r)=\hat{D}_c^{\dagger}(\alpha_c)\hat{D}_r^{\dagger}(\alpha_r)\hat{\varrho}_v\hat{D}_r(\alpha_r)\hat{D}_c(\alpha_c). \quad (15)$$

The displacement of the center of mass and the relative modes of vibration can be done, independently of each other, by using the excitation mechanism applied in Ref. [15]. Our problem, then, is to find a way to measure the displaced vibrational population, $\Pi_{n_c,n_r}(-\alpha_c,-\alpha_r)$. For this purpose, we use the same dispersive interaction scheme that led to the generation of the $|\Phi^{\pm}\rangle$ Bell states. Consider that, after the vibrational displacements, the state of the ions is described by $\hat{\rho}(0)=\hat{\varrho}_v(-\alpha_c,-\alpha_r)\otimes|\downarrow,\downarrow\rangle\langle\downarrow,\downarrow|$. From Eqs. (6) and (7), the probability $P_{\downarrow\downarrow}(\tau)$ of finding both ions in their lower electronic states, after an interaction time τ with the laser fields, is

$$P_{\downarrow\downarrow}(\tau)=\langle\downarrow,\downarrow|\mathrm{Tr}_v\{e^{-(i/\hbar)\hat{H}_{\mathrm{eff}}\tau}\hat{\rho}(0)e^{(i/\hbar)\hat{H}_{\mathrm{eff}}\tau}\}|\downarrow,\downarrow\rangle$$

$$=\sum_{n_c,n_r}\cos^2(|\Omega^k_{n_cn_r}|\tau)\Pi_{n_c,n_r}(-\alpha_c,-\alpha_r), \quad (16)$$

where Tr_v means the partial tracing over the vibrational degrees of freedom. From Eq. (8) one can see that, for $k\neq 0$, the dependence of the vibronic Rabi frequencies $\Omega^k_{n_cn_r}$ on n_c and n_r discriminate efficiently the different two-mode Fock states $|n_c,n_r\rangle$ (see Fig. 2). For this reason the vibrational populations $\Pi_{n_c,n_r}(-\alpha_c,-\alpha_r)$ may be easily extracted from $P_{\downarrow\downarrow}(\tau)$ [13].

We can measure $P_{\downarrow\downarrow}(\tau)$ by monitoring the fluorescence produced in driving the auxiliary cycling transition $|\downarrow\rangle\leftrightarrow|d\rangle$ of the ions with resonant laser light, once the interaction leading to Eq. (16) has been turned off (see Fig. 1). The presence of fluorescence assures us that the ions are in the joint electronic state $|\downarrow,\downarrow\rangle$, and its absence indicates the occupation of the state $|\uparrow,\uparrow\rangle$. This is only the case because the states $|\downarrow,\uparrow\rangle$ and $|\uparrow,\downarrow\rangle$ are not populated before the cycling transition is driven. That is, the use of the dispersive interaction [Eq. (6)] opens the possibility of measuring $P_{\downarrow\downarrow}(\tau)$ in as simple and highly efficient manner as that used to measure the corresponding quantity in a single ion [15].

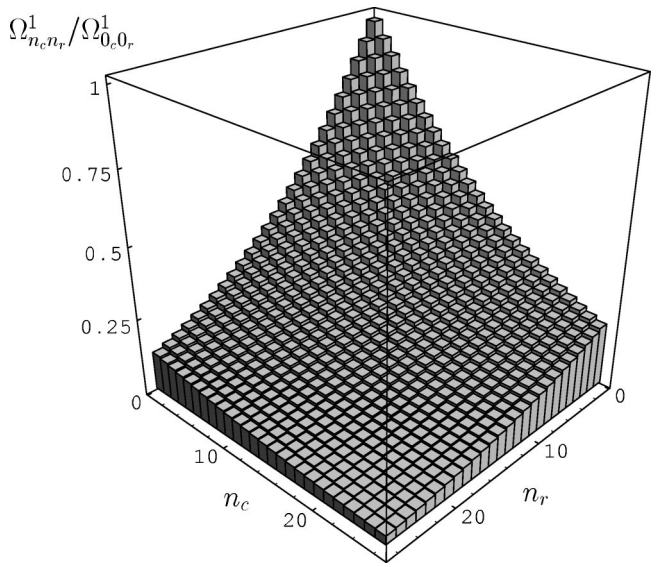

FIG. 2. Scaled Rabi frequency $\Omega^k_{n_cn_r}$ plotted as a function of n_c and n_r for $k=1$ and $\eta=0.23$. We stress the fact that we have a different positive Rabi frequency for each different pair (0,0) $<(n_c,n_r)<$(25,25).

In summary, we have presented a method that allows the deterministic generation of all the electronic Bell states of two trapped ions. This is achieved by combining a purely dispersive with a resonant laser excitation of vibronic transitions of the two ions. It is important to stress that, in the Lamb-Dicke limit, the generation of the Bell states does not require previous cooling of the ions to their motional ground state. Moreover, we have shown that the two-photon dispersive interaction discussed in this Rapid Communication is highly adequate for the implementation of a procedure to completely determine the motional state of the ions. In contrast to other methods presented up to now, our proposal does not require the differential laser addressing of the individual ions and, therefore, may be easily implemented with present available techniques.

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[1] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, England, 1987); A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).

[2] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. Wootters, Phys. Rev. Lett. **70**, 1895 (1993); L. Davidovich, N. Zagury, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **50**, R895 (1994).

[3] S. Wiesner, SIGACT News **15**, 78 (1983); A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991); C. H. Benett, G. Brassard, and N. D. Mermin, *ibid.* **68**, 557 (1992); C. H. Benett, *ibid.* **68**, 3121 (1992); A. K. Ekert, J. G. Rarity, P. R. Tapster, and G. M. Palma, *ibid.* **69**, 1293 (1992).

[4] D. Deutsch, Proc. R. Soc. London, Ser. A **400**, 97 (1985); A.

Berthiaume and G. Brassard, in *Proceedings of the Seventh Annual IEEE Conference on Structure in Complexity Theory, Boston, 1992* (IEEE, New York, 1992), p. 132.

[5] E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **79**, 1 (1997).

[6] Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. Lett. **81**, 3631 (1998).

[7] B. E. King, C. S. Wood, C. J. Myatt, Q. A. Turchette, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. Lett. **81**, 1525 (1998).

[8] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).

[9] J. J. Bollinger *et al.*, Phys. Rev. A **54**, R4649 (1996).

[10] A. Sørensen and K. Møllmer, Phys. Rev. Lett. **82**, 1971 (1999).

[11] J. C. Retamal and N. Zagury, Phys. Rev. A **55**, 2387 (1997).

[12] A similar technique was used by J. Steinbach, J. Twamley, and P. L. Knight, Phys. Rev. A **56**, 4815 (1997)

[13] D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **77**, 4281 (1996).

[14] S. Wallentowitz, R. L. de Matos Filho, S.-C. Gou, and W. Vogel (unpublished).

[15] D. M. Meekhof, C. Monroe, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **76**, 1796 (1996).