Amplification without inversion in a medium with collisional dephasing

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Amplification without inversion in the presence of collisions in V and Λ systems is studied within the framework of density-matrix formalism. We show that while it is possible to achieve amplification without inversion close to Raman resonance in both systems, it is affected by collisions in a different way. In the V system, collisions enhance quantum interference, whereas in the Λ system they quench it. This effect depends crucially on the collision characteristics, essentially on the relative values of the collisional relaxation rates for the Raman coherence and for the optical coherences. For the Λ system, different incoherent pumping processes are studied: pumping via the upper level and direct pumping between the two low-energy levels. The direct pumping process is interesting for the frequency up-conversion of the coherent and incoherent pump fields. Furthermore, with direct pumping the gain always occurs without inversion. [S1050-2947(99)09101-5]

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I. INTRODUCTION

Extensive recent studies of amplification and lasing without inversion resulted in a good understanding of basic principles and experimental verification of the main theoretical predictions [1]. While the underlying mechanisms of amplification without inversion (AWI) already seem to be well understood, some of its aspects still require further studies. One example is a detailed understanding of the role of collisions in AWI and in lasing without inversion (LWI), because of their importance in any real experiment with a dense medium.

It is the aim of this paper to provide an analysis of the dynamics of two basic three-level models of AWI-LWI, the V and Λ schemes, subjected not only to coherent and incoherent pumps but also to elastic, dephasing collisions. To bring out the salient features of such a phase perturbation, we neglect other possible effects of collisions on atoms, like quenching and/or atomic velocity changes. Quenching collisions could be trivially accounted for in our formalism by appropriate additional terms in the relaxation constants of energy-state populations. The velocity-changing collisions need not be considered when analyzing experiments performed with an atomic beam or with trapped atoms.

Similar to an earlier paper on the V system [2], our analysis will be performed in the bare-state basis as well as in the dressed-state basis. The present paper, however, is a substantial extension of Ref. [2] as it concerns both V and Λ schemes and takes into account an important additional mechanism, i.e., dephasing collisions which are inevitable in many practical situations. We will demonstrate that collisions have a dramatic effect on AWI in the two schemes: while collisionally established quantum interference plays a very important role in the V system, surprisingly this is not the case in the Λ scheme.

One of the main advantages, if not the principal advantage, of AWI-LWI mechanism over standard lasing mechanisms is the possible frequency up-conversion. While it is generally realized that in AWI (LWI) amplified (generated) radiation can have a frequency higher than the coherent pump, the majority of previous models assumed that incoherent pumping supplies energy not smaller than that of amplified (generated) photons. In this work, on the other hand, when discussing various possibilities of incoherent pumping for the Λ scheme, we also consider a system with pumping which does not populate the upper state but couples the low-energy levels directly. Such a process may lead to a full up-conversion with respect to coherent and incoherent pumps.

Our results are not limited only to collisional dephasing. Other phase-destroying processes, like phase diffusion of the coherent pump, could have an impact on AWI which should be described along similar lines as elastic collisions. In some cases, effects due to finite laser linewidths due to phase diffusion can be modeled by appropriate substitution rules for the relaxation constants in a very similar way as for the dephasing collisions [3]. Hence we hope that the present analysis could also be useful for many experiments on AWI performed with nonmonochromatic lasers. For instance, there was recently much interest in experiments with cold atoms, including electromagnetically induced transparency [4]. Though collisions are not very effective at low temperature, the effect of phase diffusion of light fields still could be observed under such conditions.

The paper is organized as follows: in Sec. II we introduce the formalism for the V system, calculate the probe beam absorption for various detailed conditions, and present physical interpretation of the results. In particular, the role of collisionally assisted quantum interference is discussed in detail. In Sec. III we adopt the formalism of Sec. II for the Λ scheme, calculate the probe beam absorption, and analyze various possible schemes of incoherent pumping. We end by summarizing the results in Sec. IV.

II. AMPLIFICATION WITHOUT INVERSION ASSISTED BY DEPHASING COLLISIONS IN A THREE-LEVEL V SYSTEM

We consider a closed three-level V system consisting of a common ground level a and excited levels b and b'. The

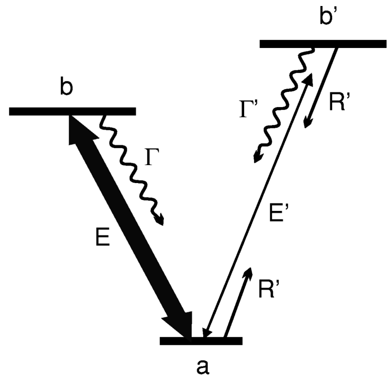

FIG. 1. Scheme of the energy levels of the V system (see the text for notation).

intense field E of frequency ω interacts with the atoms on the a-b transition of frequency ω_0, and the weak probe field E' of frequency ω' interacts with the atoms on the a-b' transition of frequency ω_0' (Fig. 1).

The radiative lifetimes of the excited states b and b' are $1/\Gamma$ and $1/\Gamma'$, respectively. Apart from radiative relaxation, the atoms are exposed to collisions. We assume that the active atoms are perturbed by a buffer gas and that collisions are dephasing in nature, inducing a decay of the atomic optical coherences ρ_{ba} and $\rho_{b'a}$ and of the Raman coherence $\rho_{bb'}$, but not of the atomic state populations. The collisional contribution to the relaxation rate of the atomic state coherence between levels i and j is denoted by γ_{ij}. Furthermore, an incoherent pumping of level b' from level a is admitted with a rate R'. We neglect any movement of atoms, and therefore also the velocity changing collisions. For the analysis of such case we refer the readers to Ref. [5].

The two light fields are detuned from resonances by quantities $\Delta=\omega-\omega_0$ and $\Delta'=\omega'-\omega_0'$. We assume that $|\Delta|$ and $|\Delta'|$ are much larger than the widths Γ and Γ', but $|\Delta-\Delta'|$ remains small compared to $|\Delta|$ and $|\Delta'|$. The difference of detunings $\delta=\Delta-\Delta'$ is the detuning for the Raman resonance between the two levels b and b' in the absence of the pump field E. The resonant Rabi frequency associated with the field E (E') is defined as $\Omega=dE/\hbar$ ($\Omega'=d'E'/\hbar$), where d (d') is the matrix element of the electric dipole moment D between a and b (a and b'). Ω' is assumed to be much smaller than $|\Delta'|$ and Γ'.

We have studied the probe beam absorption per atom, α'', under these conditions. The calculations are carried out using the standard density-matrix approach to all orders in E and to the first order in E'. We have shown that AWI can occur in a V system in the presence of the dephasing collisions. In the perturbative limit ($\Omega\ll|\Delta|$), we have given a physical interpretation for the gain mechanism.

A. Density-matrix approach

1. Probe transmission through a driven V system

The master equation for the case studied is

$$\frac{d}{dt}\rho=\frac{1}{i\hbar}[H_0+H_I+H_I',\rho]+\left\{\frac{d\rho}{dt}\right\}, \tag{1}$$

with

$$H_0=\hbar\omega_0|b\rangle\langle b|+\hbar\omega_0'|b'\rangle\langle b'|,$$

$$H_I=-\tfrac{1}{2}\hbar\Omega(e^{-i\omega t}|b\rangle\langle a|+e^{i\omega t}|a\rangle\langle b|), \tag{2}$$

$$H_I'=-\tfrac{1}{2}\hbar\Omega'(e^{-i\omega' t}|b'\rangle\langle a|+e^{i\omega' t}|a\rangle\langle b'|).$$

In Eq. (1), term $\{d\rho/dt\}$ describes the relaxation and incoherent pumping of the populations [Eqs. (3)] and the coherences [Eqs. (4)] of the density matrix ρ:

$$\left\{\frac{d}{dt}\rho_{bb}\right\}=-\Gamma\rho_{bb},$$

$$\left\{\frac{d}{dt}\rho_{b'b'}\right\}=-\Gamma'\rho_{b'b'}+R'(\rho_{aa}-\rho_{b'b'}), \tag{3}$$

$$\left\{\frac{d}{dt}\rho_{aa}\right\}=-R'(\rho_{aa}-\rho_{b'b'})+\Gamma\rho_{bb}+\Gamma'\rho_{b'b'},$$

$$\left\{\frac{d}{dt}\rho_{ij}\right\}=-\Gamma_{ij}\rho_{ij}, \tag{4}$$

with

$$\Gamma_{ba}=(\Gamma+R')/2+\gamma_{ba},$$

$$\Gamma_{b'a}=\Gamma'/2+R'+\gamma_{b'a}, \tag{5}$$

$$\Gamma_{bb'}=(\Gamma+\Gamma'+R')/2+\gamma_{bb'}.$$

To simplify the calculations we assume that the pressure-dependent contributions to the overall relaxation rates γ_{ab}, $\gamma_{ab'}$, and $\gamma_{bb'}$ are real. Then we can write

$$\gamma_{ab}=\beta p,$$

$$\gamma_{ab'}=\beta' p, \tag{6}$$

$$\gamma_{bb'}=(\beta+\beta'-\gamma)p,$$

where p is the buffer gas pressure and γ is limited to the range $0\leq\gamma\leq2\sqrt{\beta\beta'}$ [6]. It is important to notice that due to the minus sign in the last of Eqs. (6), elastic dephasing collisions do not always increase relaxation of the Raman coherence $\rho_{bb'}$. In particular, when $\beta=\beta'$ and γ has its maximum value, the collisional contribution to the relaxation rate of $\rho_{bb'}$ is zero. This has profound consequences for electromagnetically induced transparency and amplification without inversion of the systems considered in this paper. Below, we perform our calculations for $\beta=\beta'$ and study two limiting cases $\gamma=0$ and $\gamma=2\beta$. When $\gamma=0$, the Raman coherence $\rho_{bb'}$ is destroyed just as the optical coherences ρ_{ab} and $\rho_{ab'}$, and this case is referred to as the case of collisional dephasing of the Raman coherence, whereas for $\gamma=2\beta$ the rate $\gamma_{bb'}$ is null and the coherence $\rho_{bb'}$ not affected by collisions (in this case there is no collisional dephasing of the Raman coherence). The assumption $\beta=\beta'$ is just for the sake of simplicity, and by no means limits the applicability of our treatment. The specific value of γ, on the other hand, depends on the particular collisional perturbations of the given atomic states. The more closely the wave functions of states

b and b' resemble each other, the closer the value of γ is to its maximum, $2\sqrt{\beta\beta'}$. Cases of a very weak collisional dephasing of the Raman coherence are quite frequent, e.g., in the cases of Zeeman coherences, as demonstrated in numerous Hanle-effect and pressure-induced-extra-resonance experiments [7].

In the absence of the probe field $E'=0$, the zero-order solutions $\rho^{(0)}$ of the master equation are

$$\rho_{aa}^{(0)}=\frac{A(1+s/2)}{1+s/2(1+A)},$$

$$\rho_{bb}^{(0)}=\frac{As/2}{1+s/2(1+A)}, \tag{7}$$

$$\rho_{b'b'}^{(0)}=\frac{(1-A)(1+s/2)}{1+s/2(1+A)},$$

$$\rho_{ba}^{(0)}=\frac{i\Omega}{2[\Gamma_{ba}-i\Delta]}(\rho_{aa}^{(0)}-\rho_{bb}^{(0)})e^{-i\omega t}, \tag{8}$$

with

$$A=1-B'=\frac{R'+\Gamma'}{2R'+\Gamma'} \tag{9}$$

and

$$s=\frac{\Gamma_{ba}}{\Gamma}\frac{\Omega^2}{\Delta^2+\Gamma_{ba}^2}. \tag{10}$$

A is the population of the ground state a, B' is the population of level b' in the absence of the intense field ($E=0$), and s is the saturation parameter.

To first order in the probe field amplitude E', the density matrix can be written as $\rho^{(0)}+\rho^{(1)}$, where $\rho^{(1)}$ is linear in E'. The master equation [Eq. (1)] for $\rho^{(1)}$ then becomes

$$\frac{d}{dt}\rho^{(1)}=\frac{1}{i\hbar}[H_0+H_I,\rho^{(1)}]+\frac{1}{i\hbar}[H_I',\rho^{(0)}]+\left\{\frac{d}{dt}\rho^{(1)}\right\}. \tag{11}$$

We are particularly interested in coherence $\rho_{b'a}^{(1)}$, which gives the linear probe absorption. Solving Eq. (11) we obtain, in the steady state,

$$\rho_{b'a}^{(1)}=-\frac{\Omega'}{2\tilde{\Delta}'\tilde{\delta}_R}\left[(\rho_{aa}^{(0)}-\rho_{b'b'}^{(0)})\tilde{\delta}+(\rho_{aa}^{(0)}-\rho_{bb}^{(0)})\frac{\Omega^2}{4\tilde{\Delta}^*}\right]e^{-i\omega' t}, \tag{12}$$

with

$$\tilde{\Delta}=\Delta+i\Gamma_{ba}, \tag{12a}$$

$$\tilde{\Delta}'=\Delta'+i\Gamma_{b'a}, \tag{12b}$$

$$\tilde{\delta}=\delta-i\Gamma_{bb'}, \tag{12c}$$

$$\tilde{\delta}_R=\left(\delta+\frac{\Omega^2}{4\tilde{\Delta}}\right)-i\Gamma_{bb'}. \tag{12d}$$

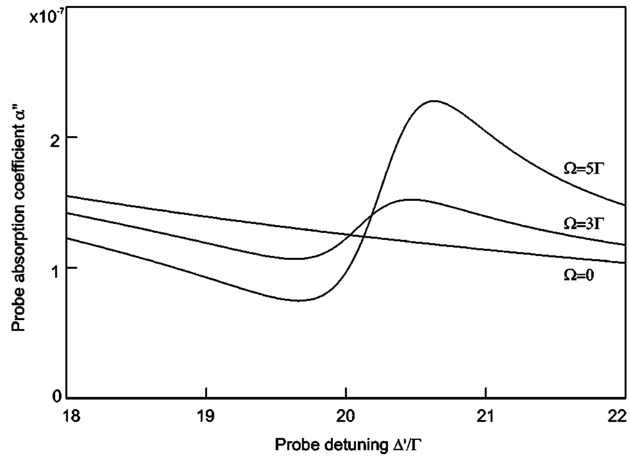

FIG. 2. Metastable b' case: $\Gamma'=10^{-3}\Gamma$. Probe absorption coefficient vs the probe detuning Δ'/Γ for three values of Ω/Γ (0, 3, and 5) and no incoherent pumping. Pump detuning $\Delta=20\Gamma$. This figure is drawn for collisional dephasing of optical coherences $\beta p=\beta' p=0.1\Gamma$ and no collisional destruction of the Raman coherence ($\gamma=2\beta$).

Quantities $\tilde{\delta}$ and $\tilde{\delta}_R$ are the resonant denominators for the Raman resonance between levels b and b' in the absence and in presence of the driving field E, respectively. In the limit considered in this paper, $|\Delta|,|\Delta'|\gg\Gamma_{ba},\Gamma_{b'a},|\delta|$, and in the perturbative limit $\Omega\ll|\Delta|$ we find $\tilde{\delta}_R\approx\delta'-i\Gamma_{bb'}$ with $\delta'=\delta+\Omega^2/4\Delta$.

The absorption of the probe field per atom, α'', is related to $\rho_{b'a}^{(1)}$:

$$\alpha''=\mathrm{Im}\frac{\Gamma'}{\Omega'}\rho_{b'a}^{(1)}e^{i\omega' t}. \tag{13}$$

Equations (12) and (13) are valid whatever the respective values of Γ' and Γ are. In the following we distinguish two cases: the first case, when level b' is much longer living than level b, $\Gamma'\ll\Gamma$ ($\Gamma'=10^{-3}\Gamma$), is called the metastable b' case; the second case, when the lifetimes are equal, $\Gamma'=\Gamma$, is called the short-living b' case.

2. Electromagnetically induced transparency (EIT)

We first study the situation without incoherent pumping ($R'=0$), i.e., when all the atoms are initially in the ground state in the absence of any coherent field.

(a) *Metastable b' case* ($\Gamma'\ll\Gamma$): In Figs. 2 and 3 the frequency dependences of α'' versus Δ' are presented for $\Delta=20\Gamma$. Figure 2 applies to the case of no collisional dephasing of the Raman coherence $\gamma=2\beta$, i.e., $\gamma_{bb'}=0$. It should be noticed in this figure that a dispersive feature appears in the absorption coefficient around the Raman resonance $\Delta=\Delta'$, resulting in a reduction of the absorption (EIT) for $\delta'>0$ (when $\Delta>0$). A similar dependence of α'' versus δ' has been predicted in other papers on collisional effects in three-level systems [6,8]. It has also been predicted for the case without collisions, but then the presence of the intense field increases overall absorption even in the minimum of the dispersive shape [2]. Figure 3 applies to the case of collisional dephasing of the Raman coherence, $\gamma=0$. This

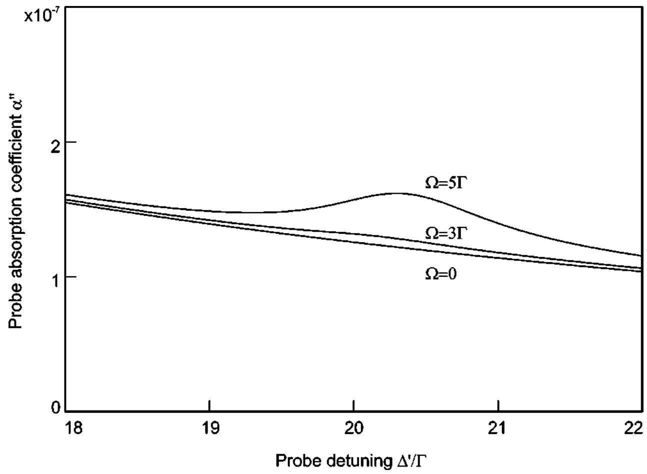

FIG. 3. Same parameters as in Fig. 2, but for equal dephasing of the optical and Raman coherences ($\gamma=0$).

time, absorption is modified around the Raman resonance, but α'' does not depend on the sign of δ' and increases with Ω.

(b) *Short-living* b' *case* ($\Gamma'=\Gamma$): Without collisions, only an independent of δ' decrease of the absorption can be observed for the $\Gamma'=\Gamma$ case, as shown in Fig. 4 for the pump detuning $\Delta=20\Gamma$. Recall that, for the metastable b' case, under the same collisionless conditions, a dispersive feature is observed around the Raman resonance [2]. The presence of collisions causes that this dispersive feature appears also for $\Gamma'=\Gamma$. In Fig. 5 we present a variation of α'' around the Raman resonance, assuming a collisional dephasing of optical coherences but no collisional dephasing of the Raman coherence. We observe a dispersive feature of the amplitude comparable to the linear absorption, and that the presence of the intense beam reduces absorption for $\delta'>0$ and leads to EIT. In cases when γ takes some value between the two limits ($\gamma=0$ and $\gamma=2\sqrt{\beta\beta'}$), the Raman resonance of α'' takes some intermediate form corresponding to reduced EIT.

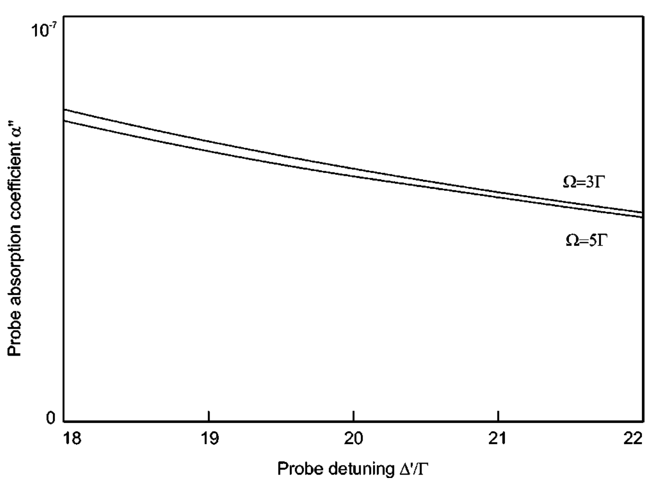

FIG. 4. Short-living b' case: $\Gamma'=\Gamma$. Probe absorption in the absence of collisions vs detuning Δ'/Γ for two values of Ω/Γ (3 and 5) and no incoherent pumping; pump detuning $\Delta=20\Gamma$.

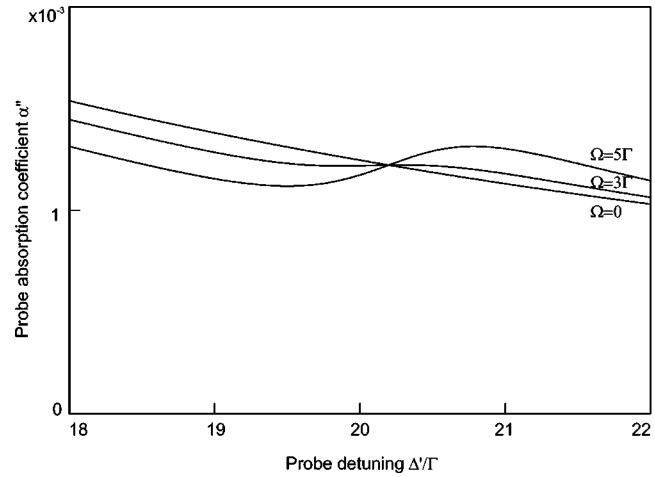

FIG. 5. Short-living b' case: $\Gamma'=\Gamma$. Probe absorption in the presence of collisions ($\beta p=\beta' p=0.5\Gamma$) vs detuning Δ'/Γ for three values of Ω/Γ (0, 3, and 5), no incoherent pumping ($R'=0$), pump detuning $\Delta=20\Gamma$, and no collisional destruction of the Raman coherence ($\gamma=2\beta$).

3. Amplification without inversion in the bare-state basis

To obtain not merely transparency of the medium, but some gain, incoherent pumping of level b' must be present ($R'\neq 0$).

(a) *Metastable* b' *case* ($\Gamma'\ll\Gamma$): The pump beam detuning Δ is set to 20Γ, and we plot α'' versus Δ' for the pump intensity $\Omega=6\Gamma$ and for a varied initial population B' of level b' (Fig. 6). Optical coherences are destroyed by collisions, but the Raman coherence is not. We search for the amplification of the probe beam, i.e., for negative α''. Indeed, when we increase the initial population B' of level b' over a certain threshold, an amplification occurs. For $\Omega=6\Gamma$ this threshold value is $B'_{\rm th}=6.27\times10^{-3}$. The condition $B'_{\rm th}\ll 1$ clearly indicates amplification without inversion in the bare-atom basis.

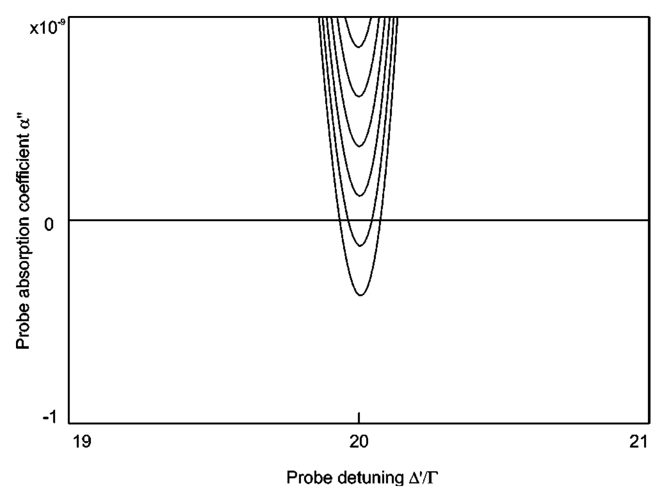

FIG. 6. Metastable b' case: $\Gamma'=10^{-3}\Gamma$. Probe absorption vs detuning Δ'/Γ in the presence of collisions ($\beta p=\beta' p=0.1\Gamma$) and for no collisional destruction of the Raman coherence ($\gamma=2\beta$); $\Omega=6\Gamma$ and $\Delta=20\Gamma$. The effect of incoherent pumping is shown: the curves are drawn for the population of level b' incremented from 6.2×10^{-3} (uppermost) to 6.3×10^{-3} (lowest), with a step of 2×10^{-5}. There is gain for $B'>B'_{\rm th}=6.27\times10^{-3}$.

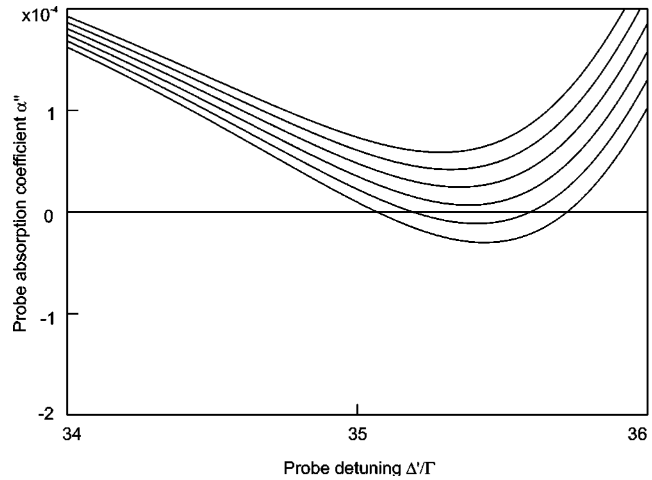

FIG. 7. Short-living b' case: $\Gamma'=\Gamma$. Probe absorption vs detuning Δ'/Γ in the presence of collisions ($\beta p=\beta' p=\Gamma$) and for no collisional destruction of the Raman coherence ($\gamma=2\beta$); $\Omega=13\Gamma$ and $\Delta=35\Gamma$. The effect of incoherent pumping is shown: the curves are drawn for the population of level b' incremented from 6.8×10^{-2} (uppermost) to 7.8×10^{-2} (lowest), with a step of 2×10^{-3}. The gain occurs for $B'>B'_{\rm th}=7.48\times10^{-2}$.

(b) *Short-living b' case* ($\Gamma'=\Gamma$): In this case a higher pump detuning and higher Rabi frequency are required to achieve gain without inversion for a small initial population in level b'. The parameters $\Delta=35\Gamma$ and $\Omega=13\Gamma$ are taken to plot absorption (Fig. 7) and, again, the initial population B' of the level b' is varied. Optical coherences have collisional relaxation rates $\beta' p=\beta p=\Gamma$, and no dephasing of the Raman coherence is assumed. Gain without inversion in the bare-state basis is obtained in the presence of collisions, this time even for a broader detuning range than in the previously described metastable case.

4. Amplification without inversion in the dressed-state basis

It is interesting to check whether the gain discussed above also occurs without population inversion in the dressed-atom basis. The eigenstates of the dressed-atom Hamiltonian [9] in a given multiplicity are denoted $|1,N,N'\rangle$, $|2,N,N'\rangle$, and $|3,N,N'\rangle$ and are linear combinations of the bare-atom states $|i,N,N'\rangle$, where $i=a,\ b,\ b'$ and N (N') is the number of photons in the mode of the pump (probe) beam. In the perturbative limit, $\Omega\ll|\Delta|$, and for $\Delta>0$ the dressed eigenstates are expressed in terms of bare states as

$$|1,N,N'\rangle=|a,N,N'\rangle+\frac{\Omega}{2\Delta}|b,N-1,N'\rangle+\frac{\Omega'}{2\Delta'}|b',N,N'-1\rangle,$$

$$|2,N,N'\rangle=-\frac{\Omega}{2\Delta}|a,N,N'\rangle+|b,N-1,N'\rangle, \tag{14}$$

$$|3,N,N'\rangle=-\frac{\Omega'}{2\Delta'}|a,N,N'\rangle+|b',N,N'-1\rangle.$$

The energies of the dressed-atom levels inside the multiplicity are then, respectively, equal to

$$E_{|1,N,N'\rangle}=\hbar\left(N\omega+N'\omega'+\frac{\Omega^2}{4\Delta}\right),$$

$$E_{|2,N,N'\rangle}=\hbar\left((N-1)\omega+N'\omega'+\omega_0-\frac{\Omega^2}{4\Delta}\right), \tag{15}$$

$$E_{|3,N,N'\rangle}=\hbar(N\omega+(N'-1)\omega'+\omega'_0).$$

The difference of energy (in $\hbar$ units) between levels $|1,N,N'\rangle$ and $|3,N,N'\rangle$ is nearly equal to Δ', and the difference between the levels $|3,N,N'\rangle$ and $|2,N,N\rangle$ is nearly equal to $\delta'=\delta+\Omega^2/4\Delta$ ($\Omega\ll|\Delta|$). In the situation considered here, $|3,N,N'\rangle$ and $|2,N,N'\rangle$ are nearly degenerate.

Since the amplification of the probe beam can be attributed to the Raman process starting from level $|3,N,N'\rangle$ and ending in level $|2,N,N'\rangle$, with stimulated emission of a photon ω' and absorption of a photon ω, amplification without inversion occurs in the dressed-state basis if population π_3 of level $|3,N,N'\rangle$ is smaller than π_2, the population of level $|2,N,N'\rangle$. To calculate π_2 it is necessary to calculate the pumping term due to collisions. We assume that before a collision the atom is in state $|1,N,N'\rangle$, which corresponds to the ground state, and we want to calculate the state of the atom after the collision. In the impact limit, the effect of the collision is just the dephasing of the amplitude of each atomic state [10,11], so that after the collision, the dressed atom is in the state $|\Psi_c\rangle$ equal to

$$|\Psi_c\rangle=|a,N,N'\rangle+e^{-i\Phi}\frac{\Omega}{2\Delta}|b,N-1,N'\rangle+e^{-i\Phi'}\frac{\Omega'}{2\Delta'}|b',N,N'-1\rangle. \tag{16}$$

State $|\Psi_c\rangle$ differs from $|1,N,N'\rangle$, in particular there is some probability of finding the atom in state $|2,N,N'\rangle$. The pumping rate due to collisions is equal to $\langle R_{22}\rangle$, the average of $R_{22}=|\langle 2,N,N'|\Psi_c\rangle|^2$ over all possible collisions. For γ_{ab} real, this rate is equal to $\langle R_{22}\rangle=(\Omega^2/2\Delta^2)\gamma_{ab}$. The total pumping rate is then given by the sum of $\langle R_{22}\rangle$ and the term $\Gamma(\Omega/2\Delta)^4$ which is due to the radiative cascade from $|1,N+1,N'\rangle$ to $|2,N,N'\rangle$. Because $\pi_1\approx1$, π_2, the population of level $|2,N,N'\rangle$ is almost equal to $(\Omega^2/2\Delta^2)(\beta p/\Gamma)+(\Omega/2\Delta)^4$ and $\pi_3\approx B'$. Thus amplification occurs without inversion if we have

$$B'\leqslant\frac{\Omega^2}{2\Delta^2}\frac{\beta p}{\Gamma}+\left(\frac{\Omega}{2\Delta}\right)^4. \tag{17}$$

For the conditions of Fig. 6, π_2 is equal to 5×10^{-3}, which means that, according to Eq. (17), gain occurs with inversion in the dressed-atom basis. For a higher pump intensity $\Omega=8\Gamma$, the amplification threshold value is $B'_{\rm th}=3.88\times10^{-3}$, population π_2 is equal to 9.6×10^{-3}, and thus gain occurs without inversion. Similarly, for the conditions of Fig. 7 ($\beta p=\Gamma$), π_2 is equal to 7×10^{-2}, the thresh-

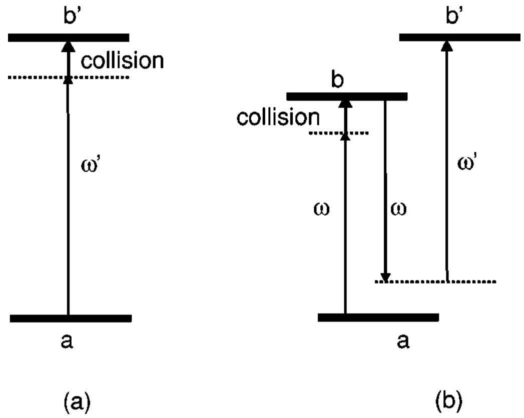

FIG. 8. (a) Direct collisional excitation of level b' with absorption of a photon of the probe field. (b) Collisional excitation of level b followed by a two-photon transition from b to b'.

old population $B'_{\rm th}=7.48\times10^{-2}$, and we observe gain with inversion, however, already for $\beta p=1.3\Gamma$, π_2 increases to 9.1×10^{-2} whereas $B'_{\rm th}=8.92\times10^{-2}$, and this is the case of gain without inversion also in the dressed-state basis.

B. Physical interpretation

1. Amplification

We first consider the process in which the probe beam is amplified. In the bare-state basis, this process corresponds to the stimulated Raman transition from b' to b followed by the spontaneous emission of a photon ω_0 on the transition from b to a. In the dressed-state basis the same process corresponds to the transition from $|3,N,N'\rangle$ to $|1,N-1,N'\rangle$, connected with the spontaneous emission of a photon ω_0. For $\Omega'\ll\Omega$, the gain connected with this process for a single atom in the b' state is

$$\alpha''_{\rm gain}=\left(\frac{\Omega}{\Delta}\right)^2\frac{\Gamma\,\Gamma'}{4\delta'^2+\Gamma^2}. \tag{18}$$

2. Absorption

There are several processes which lead to absorption of light of a given frequency. The first process is the residual linear absorption of a far-detuned light (Rayleigh scattering process) with the single-atom absorption coefficient

$$\alpha''_{\rm Rayleigh}=\left(\frac{\Gamma'}{\Delta}\right)^2. \tag{19}$$

There are two absorption processes aided by collisions. The first one, represented in Fig. 8(a) in the bare-state basis, corresponds to a direct collisional excitation of level b' with absorption of a photon ω'. In the dressed-atom basis the same process corresponds to the collisionally induced transition from $|1,N,N'\rangle$ to $|3,N,N'\rangle$. There is a second process which corresponds to the collisionally aided excitation of level b, followed by a two-photon transition from b to b' [Fig. 8(b)]. In the dressed-atom basis this corresponds to a collisional transfer from level $|1,N,N'\rangle$ to level $|2,N,N'\rangle$, followed by a transition from $|2,N,N'\rangle$ to $|3,N,N'\rangle$ due to coupling with the probe field. The transition amplitude of the second process has a Raman resonance when the two levels $|2,N,N'\rangle$ and $|3,N,N'\rangle$ have the same energy, i.e., for $\delta'=0$. Since the quantum states of the two fields and the internal degrees of freedom are the same in the initial and final states for the two pathways of Fig. 8, one must consider the possibility of interference when calculating the total transition probability. As the atom undergoing collision is not isolated, this must also involve its external degrees of freedom, in particular the kinetic energy exchanged between the active atom and its collision partner [8]. In the first pathway the energy received by the atom is equal to $E_{|3,N,N'\rangle}-E_{|1,N,N'\rangle}$. In the second pathway, which involves photons of an intense field, the atom receives an energy equal to $E_{|2,N,N'\rangle}-E_{|1,N,N'\rangle}$. In order to have the same exchange of kinetic energy for each pathway, we must have $E_{|2,N,N'\rangle}=E_{|3,N,N'\rangle}$, i.e., $\delta'=0$. In that way the independent conditions for interference and the Raman transition happen to be the same.

These two pathways, which have comparable amplitudes for $\Omega^2/\Gamma\Delta\approx1$, must be included in a calculation of the total transition amplitude. The transition probability for a given collision is then equal to the square of the modulus of the sum of the amplitudes associated with each pathway. The total single-atom coefficient of absorption aided by collisions is obtained by summing such transition probability over all collisions, and by normalizing the result to the photon flux. We find

$$\alpha''_{\rm coll}=\frac{2\Gamma'}{\Delta}\left(\frac{\beta'p}{\Delta}+\frac{\Omega^4}{4\Delta^2(4\delta'^2+\Gamma^2)}\frac{\beta p}{\Delta}-\frac{\Omega^2\delta'}{\Delta(4\delta'^2+\Gamma^2)}\frac{\gamma p}{\Delta}\right). \tag{20}$$

The comparison between the nonlinear cross section obtained in the radiative limit [2] and the preceding result shows that the process induced by collision dominates over the purely radiative processes if $\Omega^2/(4\Delta^2)\ll\beta p/\Gamma$. In the following, we will assume that this condition is fulfilled, and will call this case *the collisional limit*. The last term in the above expression, proportional to δ' and γp, results from the interference between the pathways of Fig. 8.

3. Outcome: Amplification without inversion

We now calculate the average absorption in the case when a small fraction B' of all atoms ($B'\ll1$) is pumped into the level b' and the rest remains in state a. In the collisional limit this is

$$\alpha''=\alpha''_{\rm Rayleigh}+\alpha''_{\rm coll}-B'\alpha''_{\rm gain}, \tag{21}$$

$$\alpha''=\left\{\left(\frac{\Gamma'}{\Delta}\right)^2+\frac{2\Gamma'\beta'p}{\Delta^2}+\left(\frac{\Omega}{\Delta}\right)^2\Gamma\Gamma'\frac{\frac{\Omega^2}{2\Delta^2}\frac{\beta p}{\Gamma}-B'}{4\delta'^2+\Gamma^2}-\left(\frac{\Omega}{\Delta}\right)^2\frac{2\Gamma'\delta'\gamma p}{\Delta(4\delta'^2+\Gamma^2)}\right\}. \tag{22}$$

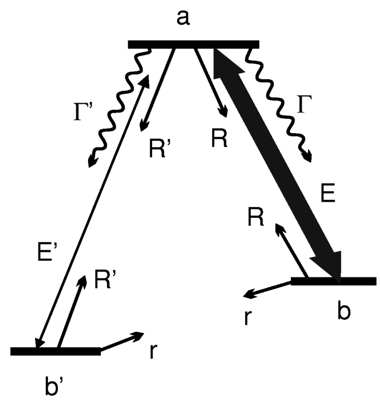

FIG. 9. Scheme of the energy levels of the Λ system (the notation is defined in the text).

All the terms in this equation have a simple interpretation: the first one corresponds to the Rayleigh scattering, the second one corresponds to the linear absorption aided by collision [Fig. 8(a)], and the third one corresponds to the Raman process between the dressed-atom levels, the numerator being the difference between the population of the level $|2,N,N'\rangle$ (the dressed level associated with level b), which is equal to $(\Omega^2/2\Delta^2)(\beta p/\Gamma)$ (in the collisional limit), and the population of the level b', which is equal to B'. When $(\Omega^2/2\Delta^2)(\beta p/\Gamma)<B'$, the Raman process leads to amplification of the probe beam. The third, Raman term, however, is not the only relevant contribution around the Raman resonance. An important role is also played by the last term in Eq. (26), resulting from the collisionally induced quantum interference, and having a dispersive line shape. When $\gamma p \neq 0$, the absorption is reduced for $\Delta\delta'>0$, and amplification can occur even if there is no population inversion. The last term is most important when there is no collisional dephasing of the Raman coherence, i.e., when γ takes its largest possible value, $\gamma=2\sqrt{\beta\beta'}$. However, even for other, intermediate, values of γ, the role of collisionally induced quantum interference terms in Eqs. (20) and (22) is not negligible.

We have thus shown that amplification can be achieved when a small fraction of the atoms is pumped in level b'. This amplification occurs for large detunings and close to the Raman resonance. There is a range of parameters where this amplification occurs without inversion either in the bare-state basis or in the dressed-state basis. Using an approach similar to the one used in the radiative case [2], we have related this amplification without inversion to a destructive interference between two absorption pathways induced by collisions.

III. AMPLIFICATION WITHOUT INVERSION IN THE PRESENCE OF COLLISIONS IN A THREE-LEVEL Λ SYSTEM

In this section we discuss conditions of amplification without inversion in a three-level, Λ-like system (Fig. 9) consisting of two ground levels b and b' and an excited level a. The atomic system interacts with an intense pump beam E on the b-a transition, and with a probe beam E' on the coupled b'-a transition. The field detunings are defined as in the V case. The atoms can decay from state a to b and from a to b' by spontaneous emission with rates equal, respectively, to Γ and Γ', and there is an incoherent pumping process with rate R (R') from state b (b') to the excited state and/or directly between states b and b' with rate r. Similarly to the V scheme, we neglect velocity effects.

Such a configuration with two coherent fields, $r=0$, and no dephasing collisions was considered by Imamoglu, Field, and Harris [12] who showed that AWI is possible for certain values of the parameters. Agarwal [13], Lounis and Cohen-Tannoudji [14], and Cohen-Tannoudji, Zambon, and Arimondo [15] gave a physical interpretation of the mechanisms responsible for AWI in this configuration. They showed that, in the system of Fig. 9, gain is due to a destructive interference between two absorption amplitudes of the probe field, associated with the transitions between state b' and two states of the atom dressed by the E field. The gain is due to a stimulated Raman process between the two differently populated ground levels, the difference of population being caused by the asymmetry between the population and depopulation rates of b and b'.

We study EIT and AWI in the Λ system in the off-resonant case, and show how collisions destroy the interference, which is the essence of the observed amplification. We show, nevertheless, that close to the Raman resonance, gain without inversion in the bare- and dressed-state bases is also possible in the presence of collisions.

We consider various models of the incoherent processes. First we concentrate on the case analyzed in Ref. [12], with incoherent pumping on b-a and b'-a transitions. Then we consider other incoherent population schemes, in particular pumping via a fourth level in a double-Λ system, and direct transfer of population between levels b and b', which can be accomplished either by magnetic dipole transitions or nonradiatively. The interest of such pumping schemes consists in the use of low-energy incoherent pumping. This allows energy up-conversion with respect to both coherent and incoherent pumping processes. The calculations are carried out in the density-matrix approach.

A. Density-matrix evolution

The evolution of the density matrix in the Λ configuration is deduced from Eq. (1), given above for the V configuration, by changing the signs of ω, ω', ω_0, and ω_0'. In this operation the complex detunings $\tilde{\Delta}$, $\tilde{\Delta}'$, and $\tilde{\delta}$ are changed into the opposite of their conjugate. Term $\{d\rho/dt\}$ describes the relaxation and incoherent pumping of the populations and coherences:

$$\left\{\frac{d}{dt}\rho_{aa}\right\}=-(\Gamma+\Gamma')\rho_{aa}+R(\rho_{bb}-\rho_{aa})+R'(\rho_{b'b'}-\rho_{aa}),$$

$$\left\{\frac{d}{dt}\rho_{bb}\right\}=\Gamma\rho_{aa}-R(\rho_{bb}-\rho_{aa})-r(\rho_{bb}-\rho_{b'b'}), \quad (23)$$

$$\left\{\frac{d}{dt}\rho_{b'b'}\right\}=\Gamma'\rho_{aa}-R'(\rho_{b'b'}-\rho_{aa})+r(\rho_{bb}-\rho_{b'b'}),$$

$$\left\{\frac{d}{dt}\rho_{ij}\right\}=-\Gamma_{ij}\rho_{ij}, \quad (24)$$

with

$$\Gamma_{ab}=(\Gamma+\Gamma'+2R+R'+r)/2+\beta p,$$

$$\Gamma_{ab'}=(\Gamma+\Gamma'+R+2R'+r)/2+\beta' p, \tag{25}$$

$$\Gamma_{bb'}=(R+R'+2r)/2+(\beta+\beta'-\gamma)p.$$

In the above equations, both the rates included in the model of Ref. [12] (R, R') and in the direct pumping model (r) are taken into account. Solving for optical coherence $\rho^{(1)}_{b'a}$, in the Λ system we obtain

$$\rho^{(1)}_{ab'}=-\frac{\Omega'}{2\tilde{\Delta}'\tilde{\delta}_R}\left[(\rho^{(0)}_{b'b'}-\rho^{(0)}_{aa})\tilde{\delta}+(\rho^{(0)}_{bb}-\rho^{(0)}_{aa})\frac{\Omega^2}{4\tilde{\Delta}^*}\right]e^{-i\omega' t} \tag{26}$$

with the notation defined in Eq. (12).

B. Electromagnetically induced transparency

In the absence of a probe beam but in the presence of an intense beam, atomic populations depend on pumping and relaxation processes. In the case considered here, in the presence of an intense beam but for no incoherent pumping ($R=R'=r=0$), all the atoms are in level b': $\rho^{(0)}_{b'b'}=1$, $\rho^{(0)}_{aa}=\rho^{(0)}_{bb}=0$. Consequently, $\rho^{(1)}_{ab'}$ is equal to

$$\rho^{(1)}_{ab'}=-\frac{\Omega'}{2\tilde{\Delta}'\tilde{\delta}_R}\tilde{\delta}\,e^{-i\omega' t}. \tag{27}$$

If we explicitly write $\tilde{\delta}=\tilde{\delta}_R-\Omega^2/4\tilde{\Delta}'$ in the preceding formula, the optical coherence can be expressed as the sum of two terms:

$$\rho^{(1)}_{ab'}=-\left(\frac{\Omega'}{2\tilde{\Delta}'}-\frac{\Omega'\Omega^2}{8(\tilde{\Delta}')^2\tilde{\delta}_R}\right)e^{-i\omega' t}. \tag{28}$$

Similarly to Ref. [14], the first term in Eq. (28) can be attributed to the direct Rayleigh process, and the second one to the Raman transition between levels b' and b displaced by the presence of the intense beam, followed by the b-a transition. In the absence of collisions ($p=0$) and for Raman resonance ($\Delta=\Delta'$), $\tilde{\delta}$ is equal to zero; hence, in view of Eq. (27), coherence $\rho^{(1)}_{ab'}=0$. The curves of Fig. 10 show α'' as a function of Δ' for $\Gamma'=0.5\Gamma$, $\Delta=1.5\Gamma$, and $\Omega=0.3\Gamma$. The solid curve corresponds to $p=0$. As expected, absorption disappears at the Raman resonance due to the destructive interference between two quantum pathways. The dashed curve in Fig. 10 shows the effect of dephasing collisions which affect both the optical and Raman coherence ($\beta p=\beta' p=6\times10^{-3}\Gamma$, and $\gamma=0$). Absorption is positive for any value of δ in the presence of collisions. We define the effective excitation rate from level b as $\Gamma_p=\frac{1}{2}s\Gamma$. For the values of the parameters of Fig. 10, the collision rate is much smaller than Γ, but is comparable to $\Gamma_p=1.2\times10^{-2}\Gamma$. Thus the Rayleigh process is not affected by collisions, but the Raman contribution is reduced, because the relaxation rate $\Gamma_{bb'}$ is different from zero and of the order of Γ_p. Consequently, the fully destructive interference between the two contributions cannot occur on Raman resonance. On the other hand, if there is no collisional influence on the Raman coherence ($\gamma=2\beta$), the absorption curve does not differ from the case without collisions. This shows clearly that the effect of collisions on the electromagnetically induced transparency depends critically on the value of γ, similarly to the case of V system analyzed above.

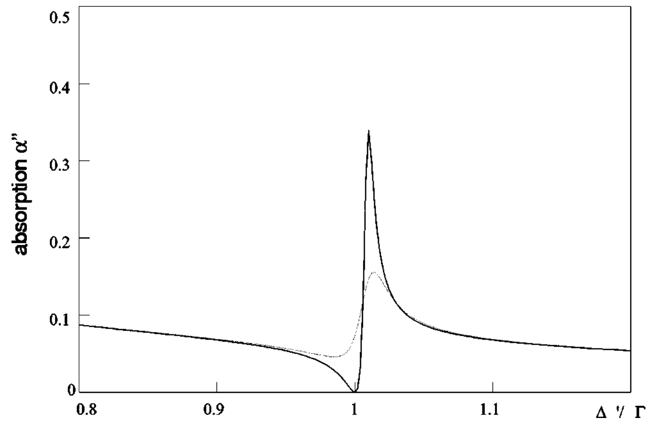

FIG. 10. Probe absorption vs detuning Δ'/Γ in the Λ system without collisions (solid line) and with collisions (dashed line; $\beta p=\beta' p=6\times10^{-3}\Gamma$, and $\gamma=0$). $\Omega=0.3\Gamma$ and $\Delta=1.5\Gamma$. In the absence of collisions, the absorption decreases to zero for the Raman resonance condition, $\Delta=\Delta'$. In the case of collisions that do not dephase the Raman coherence ($\gamma=2\beta$), the absorption curve does not differ from the collisionless case.

C. Amplification without inversion—no collisions, different pumping schemes

The probe-beam absorption coefficient α'', corresponding to the model of Ref. [12], i.e., for $r=0$, $R\neq0$, and $R'\neq0$, is presented in Fig. 11(a), and the results for the alternative direct pumping process $R=R'=0$ and $r\neq0$ are presented in Fig. 11(d). As can be seen by a comparison of these plots, the process of direct pumping yields gain comparable to the case of pumping via state a.

We find it instructive to expand the gain curves of Fig. 11(a) into the purely absorptive contribution $\alpha''_{abs}\propto(\rho_{b'b'}-\rho_{aa})$ and the one due to the Raman scattering $\alpha''_{\rm Raman}\propto(\rho_{bb}-\rho_{aa})$. In Figs. 11(b) and 11(c), we present such an expansion for $\Omega=\Gamma$ and $\Omega=4\Gamma$. The two contributions to the total absorption α'' have their extreme values at different frequencies: the gain is centered on the real part of $\tilde{\delta}_R$ [vertical lines in Figs. 11(b), 11(c), 11(e), and 11(f)], whereas the absorption contribution is given by a slightly asymmetrical curve. The asymmetry has its origin in the destructive interference which reduces the absorption at some intermediate frequency between ω_1 and ω_2, where $\omega_{1,2}$ are frequencies of the transitions between b' and the states $|1,N\rangle$ and $|2,N\rangle$, dressed by the strong field E. For $\Delta>0$, these dressed states are equal to

$$|1,N\rangle=\cos\left(\frac{\varphi}{2}\right)|a,N\rangle+\sin\left(\frac{\varphi}{2}\right)|b,N+1\rangle,$$
$$\tag{29}$$
$$|2,N\rangle=-\sin\left(\frac{\varphi}{2}\right)|a,N\rangle+\cos\left(\frac{\varphi}{2}\right)|b,N+1\rangle,$$

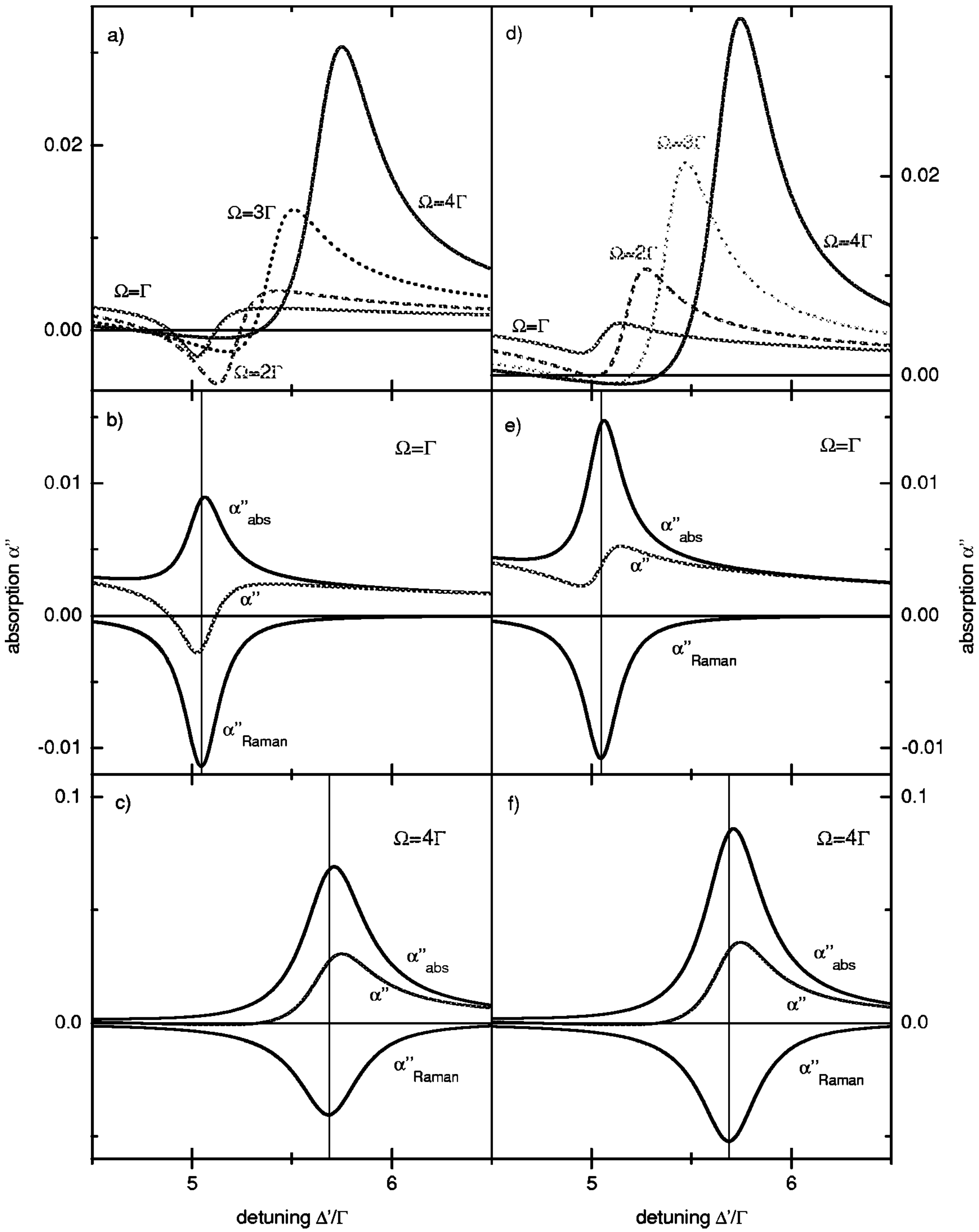

FIG. 11. Probe absorption vs detuning Δ'/Γ for $\Delta=5\Gamma$ and $\Gamma'=0.5\Gamma$, and for different incoherent pumping schemes. Plots (a), (b), and (c) are drawn for pumping via the upper state with $R=R'=0.1\Gamma$ and $r=0$, whereas plots (d), (e), and (f) refer to direct incoherent pumping with $R=R'=0$ and $r=0.1\Gamma$. Plots (a) and (d) show dependences of the $\alpha''(\Delta')$ curves on Ω, plots (b) and (e) present expansions of α'' into $\alpha''_{\rm abs}(\Delta')$ and $\alpha''_{\rm Raman}(\Delta')$ for $\Omega=\Gamma$, and plots (c) and (f) show the same for $\Omega=4\Gamma$. For $\Omega=\Gamma$ when pumping via the upper state [plot (b)], the net gain is dominated by the Raman gain *with inversion* between b and b', and there is no gain when pumping directly [plot (e)]. For $\Omega=4\Gamma$ in both pumping schemes [plots (c) and (f)] the gain is weaker than in plot (b), but is *inversionless* and centered at some frequency between ω_1 and ω_2 (see text), rather than on the Raman resonance.

where N is the number of photons of the E field, and φ is defined by $\tan\varphi=\Omega/\Delta$. Here we neglect the effect of dressing by the E' field since $E'\ll E$.

When Ω increases, the splitting between the dressed states increases, and the whole gain curve $\alpha''_{\rm Raman}$ shifts toward larger values of Δ'. There is also a shift of the intermediate frequency at which absorption $\alpha''_{\rm abs}$ is minimal, but this shift is different from the gain curve shift. This difference results in the asymmetry of the net gain α'', e.g., for $\Omega=4\Gamma$ in Fig. 11(c). For still higher Ω the gain contribution decreases. This can be explained by the effect of optical pumping $b\rightarrow a\rightarrow b'$ by the driving field which empties the b state and populates the b' state with a rate proportional to Γ_p and hence increases the absorption $\alpha''_{\rm abs}$ for high Ω.

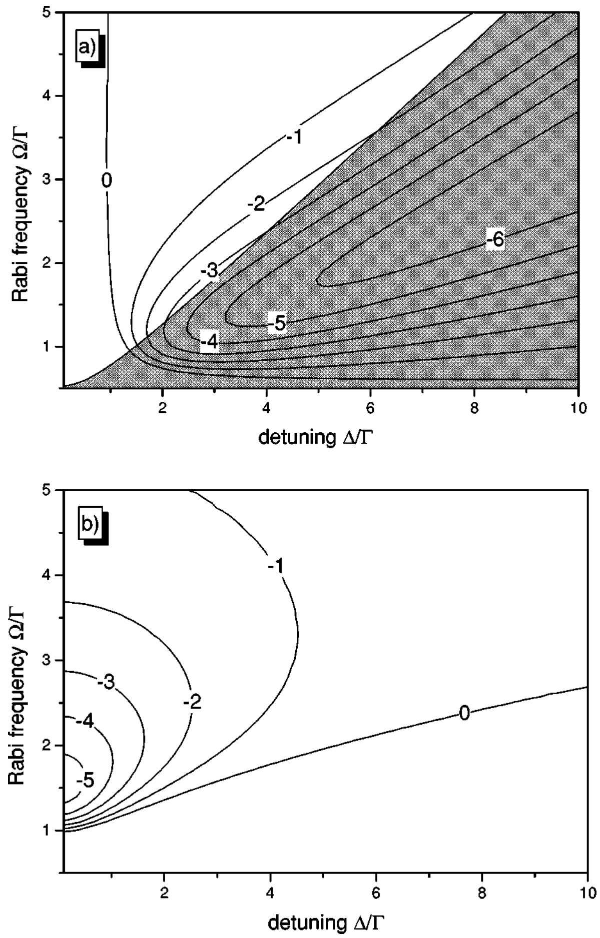

FIG. 12. Map of maximum gain ($\times 10^{-3}$) depending on the detuning and Rabi frequency of the driving field for $\Gamma' = 0.5\Gamma$ and for different pumping schemes: (a) represents pumping via the upper state with $R = R' = 0.1\Gamma$ and $r = 0$ [as in Figs. 11(a)–11(c)]. The shadowed area corresponds to the case *with inversion* between state b' and other states, $\rho_{b'b'} < 0.5$. (b) refers to direct incoherent pumping with $R = R' = 0$ and $r = 0.1\Gamma$ [as in Figs. 11(d)–11(f)], where all gain (negative α'') is *inversionless*.

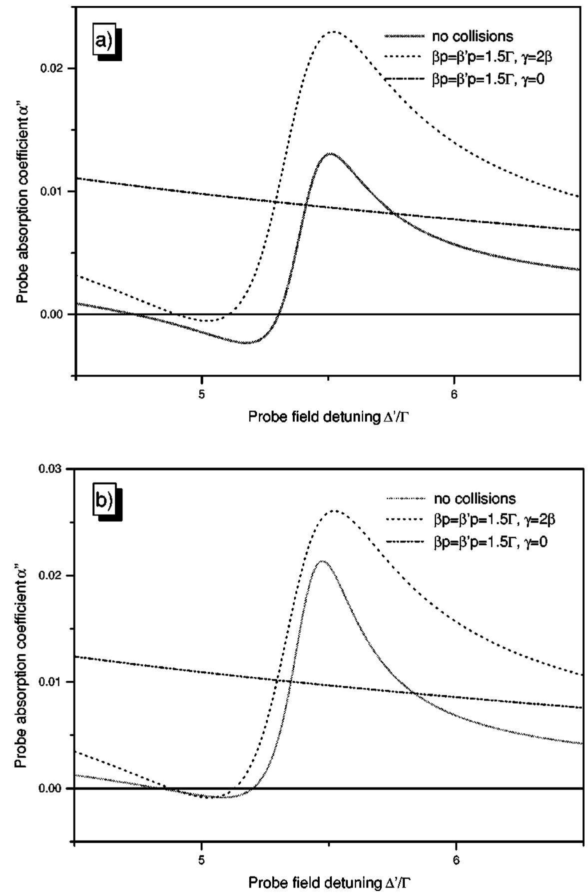

FIG. 13. Probe absorption vs probe detuning in the presence of collisions for $\Delta = 5\Gamma$, $\Omega = 3\Gamma$, and $\Gamma' = 0.5\Gamma$, and different incoherent pumping schemes: (a) Pumping via the upper state with $R = R' = 0.1\Gamma$, and $r = 0$ [as in Figs. 11(a)–11(c)]; (b) direct incoherent pumping with $R = R' = 0$ and $r = 0.1\Gamma$ [as in Figs. 11(d)–11(f)].

In the case of direct pumping [Figs. 11(d)–11(f)] the absorption contribution α''_{abs}, shown in Figs. 11(e) and 11(f), remains large, because, in contrast to the previous case, incoherent pumping is not able to deplete the b' level below $B' = 0.5$. It can also be noted that the gain contribution is similar in both cases.

We have investigated the dependence of the maximum possible gain on the driving field Rabi frequency and detuning for the two pumping mechanisms. In Figs. 12(a) (pumping via the upper state) and 12(b) (direct pumping) we plot these dependences in the form of contour plots. For each point of these plots the probe-beam frequency is chosen to maximize the gain (gain maxima correspond to the minima of absorption in Fig. 11).

The whole area of Figs. 12(a) and 12(b) is inversionless if one considers inversion just between states b' and a. In fact, since it is the two-photon Raman transition that is responsible for gain, it is more appropriate to consider a different criterion of inversion than in the earlier parts of this work. We take the ratio between population of b' and the sum of populations of a and b. The same ratio is obtained when one considers inversion between state b' and dressed states $|1,N\rangle$ and $|2,N\rangle$ [Eqs. (29)]. For a negligible population of state $|1,N\rangle$, the inversion criterion is the same as of the V scheme. In Fig. 12(a) the limiting value of $\rho_{b'b'} = 0.5$ is marked by a solid line, and the part of the plot which corresponds to the inversion, $\rho_{b'b'} < \rho_{aa} + \rho_{bb}$, is grayed out. For incoherent transfer via another excited level a', which also forms a Λ system with the b and b' states, but lies lower than a (double-Λ scheme) the contour plots are very similar to those of Fig. 12(a), so we do not display them here.

In Fig. 12(b) (direct ground state pumping), one can notice that the standard Raman gain, associated with inversion $\rho_{b'b'} < \rho_{bb} + \rho_{aa}$, is absent; hence the whole gain area is inversionless. There is also another, qualitative difference between Figs. 12(a) and 12(b): in Fig. 12(b) the area of the largest amplification is centered around $\Delta = 0$ for relatively small Rabi frequency ($\Omega \approx 1.4\Gamma$). This can be explained by the properties of the interference mechanism in the absorption profile: the generalized Rabi frequency $\sqrt{\Omega^2 + \Delta^2}$ must not be too large, otherwise the dressed states are split too far and there is no interference (in the former, Raman, case, the interference plays no such crucial role, so the gain can also be achieved at large Rabi frequencies).

D. Amplification without inversion—influence of collisions

Similarly to the V scheme, we have investigated the influence of the dephasing collisions on the amplification in the Λ system. In this case, however, no principally interesting features appear with respect to the V system. The collisions that destroy only the optical coherences (but not the Raman $\rho_{bb'}$ coherence) cause only a small change of amplification, even if the collision rate βp is comparable with the width of level a (Fig. 13). This collisional gain reduction is larger for pumping via the upper state [Fig. 13(a)], but very small for direct pumping [Fig. 13(b)]. In both cases collisions shift the frequency of maximum gain toward smaller Δ'. The gain vanishes completely when collisions also destroy the Raman coherence ($\gamma=0$), so that $\gamma_{bb'}$ becomes of the order of Γ_p.

IV. CONCLUSIONS

We have shown that under conditions of elastic dephasing collisions, AWI can be achieved in V and Λ systems, determined the conditions under which this is possible, and discussed detailed relevant mechanisms. We analyzed the off-resonant situation, where gain occurs close to the Raman resonance. In the presence of incoherent pumping the gain can be without inversion due to the destructive quantum interference in absorption. The criteria of inversion are discussed in the bare-atom basis as well as the dressed-atom basis.

In both systems dephasing collisions, in general, reduce the gain, with an efficiency which is controlled by the values of the pressure-related relaxation coefficients β, β', and γ. Nevertheless, we found profound differences between the effect of collisions on the V and Λ systems. For example, in the short-living b' case ($\Gamma'=\Gamma$) of the V system and when $\gamma=2\beta=2\beta'$, AWI is possible in the presence of collisions for low values of Ω. Without collisions, as shown in Ref. [2], this occurs only for the metastable b' case ($\Gamma'\ll\Gamma$). Additionally, in the V system the gain, which without collisions was associated with inversion, becomes inversionless in presence of collisions. This indicates that collisionally assisted quantum interference of various absorption channels plays an important role in such lasing systems, similarly to earlier described pressure-induced extra resonances [7].

When $\gamma=2\beta=2\beta'$, i.e., for dephasing collisions which destroy only the optical coherences and do not contribute to the relaxation of the Raman coherence, and when βp becomes comparable to the total relaxation rates of optical coherences, the effect of collisions is more important for V systems than for Λ systems.

Since, in the three-level systems considered in this paper, it is the Raman scattering that is responsible for the gain, the upper-state population plays an entirely different role than in standard lasing systems. In particular, its population does not directly influence the gain. The b'-b flow of population, necessary for sustaining the gain in the Λ system, need not be via the upper level a, but via any of lower-lying excited levels, or even direct. This is of considerable practical importance for achieving frequency up-conversion with respect to both coherent and incoherent pumps, i.e., the situation when each of the pumps has an energy lower than the amplified radiation. We have shown that these low-energy pumping configurations indeed yield gain, which turns out to be inversionless for any values of relevant parameters. It should be remembered that such a possibility of incoherent pumping by using routes other than level a has also been noticed by other authors [16], but, as far as we know, its practical potentials for frequency up-conversion of incoherent optical pumping have not been appreciated so far.

ACKNOWLEDGMENTS

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