Influence of nonlinear gain and loss on the intensity noise of a multimode semiconductor laser

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We developed a model for the intensity noise of a multimode semiconductor laser which includes both the nonlinear gain and the nonlinear loss. The nonlinear gain stems from spectral hole burning and population pulsation. Saturable absorption by deep trap levels results in the nonlinear loss. We find that the nonlinear gain alone does not enhance the total intensity noise above the quantum efficiency limit. The nonlinear loss, however, does increase the intensity noise when longitudinal side modes have considerable intensities, and thus explains the observed excess intensity noise. In this case the perfect anticorrelation among the longitudinal modes is degraded, which in turn converts the mode-partition noise to the total intensity noise. We remark that larger (smaller) saturable absorption in the quantum-well (transverse-junction-stripe) lasers could explain the larger (smaller) excess noise behavior observed in these lasers. [S1050-2947(99)06201-0]

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I. INTRODUCTION

The quantum-mechanical description of semiconductor lasers led to the prediction of the possibility for intensity squeezed light generation in a constant current driven semiconductor laser [1]. The main aspects of the theory tell us that the sub-shot-noise light is produced when (i) noise of the pumping process is very small (compared to the shot noise level) and (ii) the laser is operating far above the threshold. In this case, the noise characteristics of the total laser emission replicates that of the pumping process and is only limited by the quantum efficiency of conversion from electrons to photons. The noiseless pumping can be achieved in a semiconductor laser pumped by a high-impedance constant current source in a regime where the collective (microscopic) Coloumb blockade is operative [2]. Indeed, the intensity squeezed light was experimentally observed from a semiconductor laser which satisfied the two above-mentioned operating conditions [3].

However, subsequent experiments on different types of semiconductor lasers revealed that not all semiconductor lasers exhibited intensity squeezing when both conditions—quiet pump and operation far above threshold—were satisfied. Moreover, some solitary running semiconductor lasers which did not show intensity squeezing could still produce squeezed light by using injection-locking or external grating feedback techniques [4,5]. These techniques suppress longitudinal side modes in the lasing spectrum. This reduction in the intensity noise by suppressing side modes suggested that it is the multimode operation which destroys squeezing in some semiconductor lasers. The assumptions made in the original theory [1] were single longitudinal mode operation, linear gain, and linear loss. Extension of the theory to multimode operation still predicted intensity squeezing for the total emission in all longitudinal modes [6]. The extended model also predicted that due to the splitting of the common gain among the different longitudinal modes, each individual mode has large intensity noise which is known as mode-partition noise. The mode-partition noise is perfectly anticorrelated with the mode-partition noise in the collection of the rest of the modes. Thereby the mode-partition noise is canceled in the total intensity noise. Although the large anticorrelated noise in longitudinal modes was experimentally verified, this model could not explain the excess noise observed in some lasers.

It was suggested that it is the inhomogeneity and the nonlinearity of the semiconductor gain medium which could result in the excess noise when weak side modes are present [5] (these weak side modes carry large noise due to amplified spontaneous emission). To represent the inhomogeneity of the gain medium, Marin *et al.* [7] introduced a fitting term in the Langevin rate equation model. The fitting term caused small self-saturation of each mode by its own fluctuations. Although their calculations yielded higher noise when side modes were larger, the physical origin of the fitting term was not clarified. This model was extended by Becher *et al.* [8] to include nonlinear gain terms. The nonlinear gain could explain asymmetry in the side mode intensities and noise with respect to the main mode. However, this model could not clarify the physical origin of the self-saturation which resulted in the excess noise in their calculations.

In this paper we report numerical calculations of the intensity noise of a multimode laser in which the nonlinear gain and the nonlinear loss are both included. The physical mechanisms responsible for the nonlinear gain are the spectral hole burning and the gain modulation caused by beating of modes [9]. The nonlinear loss is introduced by saturable absorbers present in the cladding region of a laser.

We obtained two conclusions from our calculations performed with this model. One conclusion is that the practical amount of the nonlinear gain does not enhance noise even if the laser is oscillating in multiple longitudinal modes. This result follows from the conservation of energy. When a laser is operating far above the threshold, electronic excitation created by the noiseless pump is either converted into the laser emission or lost in the internal absorption. The identical loss for all longitudinal modes does not deteriorate anticorrelation among longitudinal modes. Therefore, the mode-partition noise should cancel out in the total intensity noise. The total intensity noise in this case is independent of the relative

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intensities of the side modes. Then, the total intensity noise should only be limited by the quantum efficiency. The other conclusion is that the nonlinear loss does enhance the total intensity noise when side mode intensities are higher. The power in the main mode saturates the nonlinear losses most effectively for the main mode but less effectively for the side modes. This mode-dependent loss degrades the anticorrelation among the longitudinal modes. Consequently, the total intensity noise is enhanced when side mode intensities are large.

We used Langevin rate equations to compute intensity noise of a multimode semiconductor laser. In Sec. II the physical models for the nonlinear gain and loss terms used in the rate equations are described. The method of solution of the rate equations is outlined in Sec. III. Calculation results are presented in Sec. IV and discussed in Sec. V. In Sec. V we also compare the noise characteristics and the saturable loss for two types of lasers, namely quantum-well (QW) and transverse-junction-stripe (TJS) lasers. The experimental results on these two types of lasers support our model.

II. PHYSICAL MODELS FOR NONLINEARITY

In the rate equation model, gain and loss are included as lumped terms. We have incorporated nonlinear contributions to both gain and loss, which depend on the laser intensity or photon density. The linear gain is modeled as a linear function of the carrier number and a quadratic function of the wavelength, i.e.,

$$G_{L,i}=A_i(N_c-N_0)=A_0\left(1-\frac{\lambda_i}{\lambda_0}\right)^2(N_c-N_0),$$

where subscript i and 0 refer to the ith longitudinal side mode and the main mode, respectively; N_c is the carrier number (note that it is not carrier density), N_0 is the transparancy carrier number, λ is the wavelength, and $A_i=\beta_i/\tau_{\mathrm{sp}}$ is the gain coefficient, where β_i is spontaneous emission coupling factor and τ_{sp} is the spontaneous emission lifetime. We neglect asymmetry in the gain with respect to the peak wavelength. An asymmetric gain profile would only make quantitative change in the individual mode noise without affecting the fundamental qualitative behavior of the mode correlation and noise. For the nonlinear gain we adopt the following analytical expression derived by Agrawal *et al.* [9]:

$$G_{nl}(\omega_j)=-G_L(\omega_0)\sum_k \zeta_{jk}n_k. \tag{1}$$

Here the superscripts j,k refer to the longitudinal modes, ω is the angular frequency of a mode, and n is the the photon number (note that it is not photon density). The gain saturation coefficients ζ_{jk} are given by the following expression:

$$\zeta_{jk}=\frac{\mu^2\omega_0\tau_{in}(\tau_c+\tau_v)}{V_m 2\epsilon_0\hbar\kappa\kappa_g}\frac{G_L(\omega_k)}{G_L(\omega_0)}\times\frac{C_{jk}}{(1+\delta_{jk})C_k}\left(1+\frac{1+\alpha_k\Omega_{jk}\tau_c}{1+(\Omega_{jk}\tau_c)^2}\right), \tag{2}$$

where $\Omega_{jk}=\omega_j-\omega_k$, and α_k is a dimensionless parameter related to the slope of the gain curve at ω_k. The rate $1/\tau_{\mathrm{in}}=1/\tau_c+1/\tau_v$ contains the conduction band (τ_c) and the valence band (τ_v) relaxation times. C_{jk}/C_k is a factor arising from the spatial structure of the optical mode, $\delta_{jk}=1$ if $j=k$ and zero otherwise, μ is the dipole moment matrix element, κ is the refractive index, κ_g is the group refractive index, and V_m is the optical mode volume. For a GaAs laser, $\mu=4.6\times10^{-29}$ cm. C_{jk}/C_k and α_k are approximately equal to 1 and 0, respectively. The coefficients ζ_{ii} define the self-saturation and $\zeta_{ij}, i\neq j$, define the cross saturation of the gain by the photon field. Substitution of the characteristic parameter values, $\tau_{\mathrm{in}}=0.1$ ps, $\tau_v=0.07$ ps $\tau_c=0.2$ ps, $\kappa=3.4$, and $\kappa_g=4$ in Eq. (2), results in $\zeta_{00}\times V_m=2.7\times10^{-18}$ cm^3. This value is in reasonable agreement with the value inferred from measurements of the self-saturation coefficient [9].

The self-saturation and the cross gain saturation coefficients indicate that the gain at a lasing mode frequency is slightly reduced because of the finite intraband relaxation time of carriers. The amount of the reduction in the gain depends not only on the power of that mode (self-saturation) but also on the power of neighboring modes (cross saturation). Beating between different longitudinal modes causes oscillation in the carrier density. This results in a small asymmetric contribution to the nonlinear gain. Note that in the context of semiclassical laser theory, the gain suppression is referred to as spectral hole burning and dynamic variation in the carrier density is referred to as population pulsation.

Linear loss comes from free-carrier absorption and scattering inside the cavity as well as leakage of power out of the cavity at the end mirrors. Then, the total linear loss coefficient is $\alpha_p=\alpha_{\mathrm{int}}+\alpha_e$, where α_{int} is the internal loss and α_e is the external coupling loss coefficient. $\alpha_e=(1/L)\ln(1/R_1R_2)$, where L is the length of the cavity and R_1,R_2 are facet reflectivities. In addition, saturable absorption caused by the deep trap levels in the band gap has been observed in $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ materials [10,11]. Deep traps, also known as *DX* centers, are formed by the lattice defect created by impurity atoms. Commonly used donor impurities such as Te [10] and Si [12] create *DX* centers in $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$, which usually is the material of choice for the cladding layers in GaAs lasers. The saturable absorption coefficient by the *DX* centers, α_{sa}, derived from rate equations [13] reads

$$\alpha_{\mathrm{sa}}(p)=\frac{\alpha_s}{1+(p/p_s)} \tag{3}$$

with the unsaturated loss coefficient $\alpha_s=\sigma_o N_{DX}$ and the saturation photon density $p_s=N_c\sigma_c v_{\mathrm{th}}/V\sigma_o v_g$. p is the photon density, σ_o is the optical-absorption cross section of the trap, σ_c is the cross section of capturing electrons from the conduction band into the trap, N_{DX} is the density of the *DX* centers, N_c/V is the density of the conduction-band electrons, v_{th} is the thermal velocity of the carriers in the conduction band, and v_g is the group velocity of the light in the material.

To obtain the saturable loss for a longitudinal mode, we assume that the saturation of traps comes mainly from one dominant mode. This is a good assumption when the side modes are a few times smaller than the main mode and the power in the main mode itself is a few times larger than the saturation intensity. The standing wave optical field of the main mode results in the spatial variation of the saturation of traps inside the cavity. At the positions where the main mode intensity is minimum, absorption by the traps is locally large. Since different longitudinal modes have a slightly different spatial profile, the average saturable loss for a side mode is slightly larger than the saturable loss of the main mode. The average saturable loss in a mode can be estimated by averaging the local loss weighted by the spatial intensity distribution of the mode [13]. When the main mode intensity is about two to ten times the saturation intensity, the saturable absorption loss of the nearest side mode could easily be about 10–20 % larger than the main mode loss.

The experimental value of σ_o for Si doping in $Al_{0.4}Ga_{0.6}As$ is $\sigma_o \approx 10^{-18}$ cm^2 and has negligible dependence on temperature. On the other hand, σ_c for Si in $Al_{0.4}Ga_{0.6}As$ strongly depends on temperature. The measured values are $\sigma_c \approx 10^{-26}$ cm^2 and $\sigma_c \approx 10^{-17}$ cm^2 at 80 K and 300 K, respectively. For the carrier density $N_c/V = 10^{17}$ cm^{-3}, the saturation photon density is $p_s \approx 7 \times 10^4$ photons/cm^3 and $p_s \approx 7 \times 10^{14}$ photons/cm^3 at 80 K and 300 K, respectively.

III. NOISE ANALYSIS USING LANGEVIN RATE EQUATIONS

In the following we briefly describe the ac analysis of rate equations for three longitudinal modes, one main mode and two side modes symmetrically placed about the main mode. Using Eqs. (1) and (2) and approximations on some parameters mentioned in the preceding section, we rewrite the nonlinear gain as follows:

$$G_{nl,i} = -A_0 N_c \sum_k \zeta_{ik} n_k, \tag{4}$$

$$\zeta_{ik} \approx -\frac{A_k}{A_0}\zeta_{00}\left\{1+\frac{1}{1+(\Omega_{ik}\tau_c)^2}\right\}. \tag{5}$$

We also define linear gain of the side modes as $A_{1,-1} = \beta_{1,-1}/\tau_{\mathrm{sp}} = m\beta_0/\tau_{\mathrm{sp}} = mA_0$.

We assume that the nonlinear loss is saturated only by the main mode. We use a spatially averaged value of the saturable loss in Langevin rate equations and define the averaged expressions as follows:

$$\alpha_{\mathrm{sa},i}^{\mathrm{av}} = \frac{\alpha_{s,i}^{\mathrm{av}}}{1+(n_0/n_s)}, \tag{6}$$

$$\alpha_{s,\pm 1}^{\mathrm{av}} = \alpha_{s,0}^{\mathrm{av}}(1+\Delta\alpha),$$

where $\Delta\alpha$ is the relative difference in the loss of the side mode and the main mode. The effective loss rate due to the saturable absorber then reads

$$\frac{1}{\tau_{\mathrm{sa},i}} = \alpha_{\mathrm{sa},i}^{\mathrm{av}} v_g = \frac{1}{\tau_p}\frac{\alpha_{s,i}^{\mathrm{av}}/\alpha_p}{1+(n_0/n_s)} = \frac{1}{\tau_p}\frac{\alpha_{r,i}}{1+(n_0/n_s)}, \tag{7}$$

where $1/\tau_p = \alpha_p v_g$ is the total linear loss rate. Then, $1/\tau_p = (\alpha_{\mathrm{int}}+\alpha_e)v_g = 1/\tau_{\mathrm{int}} + 1/\tau_e$. On this basis, the Langevin rate equations for the carrier number N_c and the photon number n_i in the ith mode ($i=0, \pm 1$ for the main and side modes, respectively) are

$$\frac{dN_c}{dt} = P - \frac{N_c}{\tau_{\mathrm{sp}}} - \sum_i A_i N_c n_i + A_0 N_c \sum_k \zeta_{ik} n_k n_i + \Gamma^p + \Gamma^{\mathrm{sp}} + \Gamma^{\mathrm{st}}, \tag{8}$$

$$\frac{dn_i}{dt} = -\frac{n_i}{\tau_p} - \frac{n_i}{\tau_p}\frac{\alpha_{r,i}}{1+n_0/n_s} + A_i N_c(n_i+1) - A_0 N_c \sum_k \zeta_{ik} n_k n_i + f_i^g + f_i^l + f_i^e, \tag{9}$$

where P is the pumping rate. Γ^p, Γ^{sp}, and Γ^{st} are Langevin noise sources for the carrier number due to fluctuations in the pump, the spontaneous emission, and the stimulated emission, respectively. Similarly f_i^g, f_i^l, and f_i^e are Langevin noise sources for the photon number due to fluctuations in the gain, the internal loss, and the external coupling loss, respectively.

We have not solved the rate equation for deep traps simultaneously with photon and carrier rate equations because the dynamics of traps is much slower than the dynamics of photons. The trap lifetime (≈ 10 ns) is three to four orders of magnitude longer than photon lifetime (≈ 1 ps). The carrier dynamics is of the same order as that of photons.

The correlation functions for all these six noise sources as derived in Ref. [6] are as follows:

$$\langle \Gamma^p(t)\Gamma^p(t')\rangle = 0,$$

$$\langle \Gamma^{\mathrm{sp}}(t)\Gamma^{\mathrm{sp}}(t')\rangle = \delta(t-t')\langle N_c\rangle/\tau_{\mathrm{sp}}, \tag{10}$$

$$\langle \Gamma^{\mathrm{st}}(t)\Gamma^{\mathrm{st}}(t')\rangle = \delta(t-t')\sum_i (\langle G_i^l\rangle + \langle G_i^{nl}\rangle)\langle n_i\rangle,$$

$$\langle f_i^g(t) f_i^g(t')\rangle = \delta(t-t')(\langle G_i^l\rangle + \langle G_i^{nl}\rangle)\langle n_i\rangle,$$

$$\langle f_i^l(t) f_i^l(t')\rangle = \delta(t-t')\langle n_i\rangle/\tau_{lo,i}, \tag{11}$$

$$\langle f_i^e(t) f_i^e(t')\rangle = \delta(t-t')\langle n_i\rangle/\tau_e,$$

and the cross terms

$$\langle \Gamma^{\mathrm{st}}(t) f_i^g(t')\rangle = -\delta(t-t')(\langle G_i^l\rangle + \langle G_i^{nl}\rangle)\langle n_i\rangle, \tag{12}$$

where $\langle\ \rangle$ denotes ensemble average, and $1/\tau_{lo,i} = (\alpha_{\mathrm{int}} + \alpha_{\mathrm{sa},i})v_g = 1/\tau_{\mathrm{int}} + 1/\tau_{\mathrm{sa},i}$ is the total internal loss rate. The remaining combinations of noise sources that have not been included in the above list are uncorrelated. The pump fluctuation $\Gamma^p(t)$ is taken to be zero because of the assumption of a noiseless pump source. Since both $\Gamma^{\mathrm{st}}(t)$ and $f_i^g(t)$ originate from the same source—fluctuations in the stimulated emission in which the loss of a carrier results in the emission of a photon and vice versa—these two noise sources are perfectly anticorrelated.

Steady-state values of the photon number in all three modes, $\langle n_i\rangle$, were calculated from Eqs. (8) and (9) by substituting zero for the time derivative term and noise terms. The photon number noise was derived by performing small-signal ac analysis of the rate equations, employing the method used in Ref. [6]. In summary, variable decompositions $N_c = \langle N_c \rangle + \Delta N_c$ and $n_i = \langle n_i \rangle + \Delta n_i$ were substituted in the rate equations. Then the resulting equations were linearized in fluctuating components and Fourier transformed. The Fourier analysis frequency was taken to be zero in our calculations. This derivation gave fluctuations in the photon number inside the cavity, Δn_i, which were then expressed in terms of the photon field amplitude, a_i $(n_i = a_i^2)$, as $n_i = \langle n_i \rangle + \Delta n_i = \langle a_i \rangle^2 + 2\langle a_i \rangle \Delta a_i$ or $\Delta n_i = 2\langle a_i \rangle \Delta a_i$. The noise in the photon flux outside the cavity, Δr_i, can be expressed in terms of Δa_i as follows [3]:

$$\Delta r_i = \frac{1}{\sqrt{\tau_e}}(\Delta a_i) - \sqrt{\tau_e}\,\frac{f_i^e}{2\langle a_i \rangle}. \tag{13}$$

The second term in the above equation is the vacuum field reflected from the cavity mirrors. We then combined the noise of all three modes in the output flux and calculated the total intensity noise. This corresponds to the noise which is measured in the experiments.

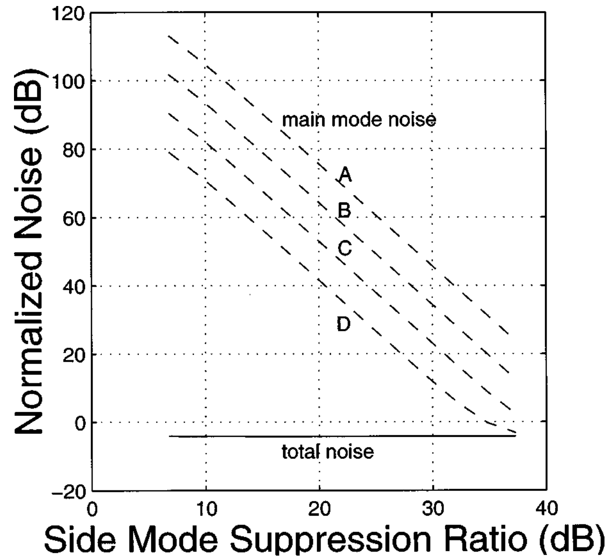

FIG. 1. Intensity noise vs SMSR for the case with only linear gain and loss. Noise was normalized by the shot noise. The spontaneous emission coupling factor, β_0, was varied as (A) 2×10^{-6}, (B) 7.36×10^{-6}, (C) 2.7×10^{-5}, and (D) 1×10^{-4} while other parameters were fixed as the pumping level $I/I_{\mathrm{th}} = 10$, the external quantum efficiency $\eta_e = 0.75$, and the total linear loss $\alpha_p = 120\ \mathrm{cm}^{-1}$.

IV. NUMERICAL RESULTS

A. Numerical parameters

The parameters used in calculations were $\tau_{\mathrm{sp}} = 1$ ns, $\tau_p = 1$ ps, $\tau_{\mathrm{int}} = 1.33$ ps, and $\tau_e = 4$ ps. The pumping level was ten times the threshold. The relative gain parameter, $m = \beta_i/\beta_0$, was varied in the range 0.99–0.999 99 to change the ratio of the power in the side modes with the power in the main mode. In the following we will refer to this ratio as the side-mode-suppression ratio (SMSR). The value of β_0 was either fixed as 2.6×10^{-6} or varied in the range $10^{-6} - 10^{-4}$ to change the mode-partition noise for the same SMSR. For the self-saturation coefficients we used $\zeta_{00}\times V_m = 2.7\times10^{-18}\ \mathrm{cm}^3$ [9] and deduced the remaining nonlinear gain coefficients from Eq. (5). The optical mode volume V_m was $6.2\times10^{-10}\ \mathrm{cm}^3$. The average unsaturated nonlinear loss coefficient α_s^{av} was approximated as $\alpha_s^{\mathrm{av}} \approx \sigma_o N_{DX}$. The optical-absorption cross section of a DX center (σ_o) was taken from measurements on $Al_xGa_{1-x}As$ doped with Si [12], which is commonly used as donor impurity in the $Al_xGa_{1-x}As$ lasers. The measured value was $\sigma_o = 10^{-18}\ \mathrm{cm}^2$ for 0.9 μm laser light. This value was found to be insensitive to the Si concentration and alloy composition change. It has also been observed that the concentration of the DX center (N_{DX}) is almost the same as the concentration of the Si dopant. To see the influence of the doping concentration on the intensity noise, we varied the value of N_{DX} in the range $10^{17}\ \mathrm{cm}^{-3} - 10^{18}\ \mathrm{cm}^{-3}$. For $N_{DX} = 10^{17}\ \mathrm{cm}^{-3}$, if we assume $N_c/V = N_{DX}$, the saturation photon density $p_s \approx 10^{16}\ \mathrm{mW/cm}^3$ at 300 K (for 0.9 μm light). Then the average loss for the main mode is $\alpha_{\mathrm{sa},0}^{\mathrm{av}} \approx 0.01\ \mathrm{cm}^{-1}$. We assumed the relative absorption parameter, $\Delta\alpha = 0.09$, which is close to the estimate obtained from the physically reasonable parameters [11].

B. Results

To elucidate the influence of the nonlinear gain and saturable loss on the intensity noise, we started our analysis by calculating noise in the absence of both the nonlinear gain and the saturable loss. Figure 1 displays that, as expected, total intensity noise did not depend on the size of the side modes (SMSR) but the noise of an individual (main) mode strongly depended on the SMSR. The total intensity noise was determined by the internal loss (or the quantum efficiency) and the pumping level. This value is referred to as the quantum efficiency limit of the intensity noise. The noise of the main mode was about 80 dB larger than the total intensity noise when SMSR was about 20 dB.

In the next step, we repeated the calculations with only the nonlinear gain (Fig. 2) term. The total intensity noise remained insensitive to the SMSR. The total intensity noise and the main mode noise were the same as in the linear case (Fig. 1). Figure 3 shows noise when only saturable absorption loss was included. In this case, the excess total intensity noise was present when the SMSR was small. The total intensity noise decreased and approached the quantum efficiency limit as the SMSR was increased. Evidently, the saturable absorption enhances the total intensity noise when the mode-partition noise is large. When we finally included both the nonlinear gain and the saturable loss in our calculations, the total intensity noise had the same dependence on the SMSR as obtained with the saturable loss alone (Fig. 4).

These results are in contrast with the common hypothesis that the gain nonlinearity increases the total intensity noise of a multimode laser [8]. Our calculations show that nonlinear gain does not enhance the total intensity noise of the laser. It is the nonlinear loss which introduces excess noise into a multimode laser.

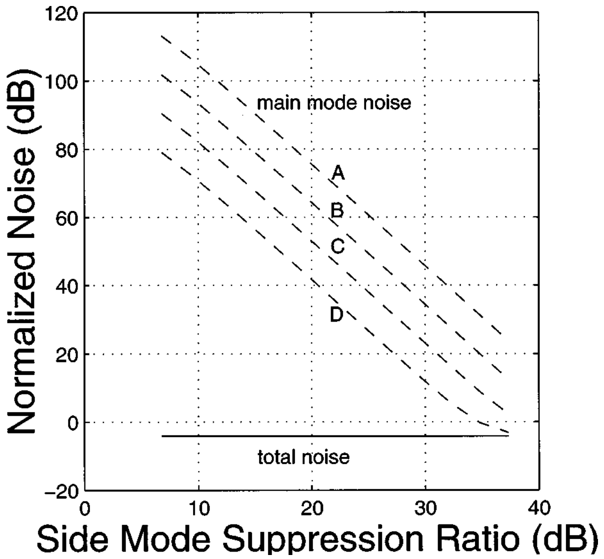

FIG. 2. Intensity noise vs SMSR for the case with nonlinear gain and linear loss. The self-gain saturation coefficient was $\zeta_{00} = (1/V_m)2.7\times10^{-18}$ (V_m is the optical mode volume in cm^3) and other parameters were the same as in Fig. 1.

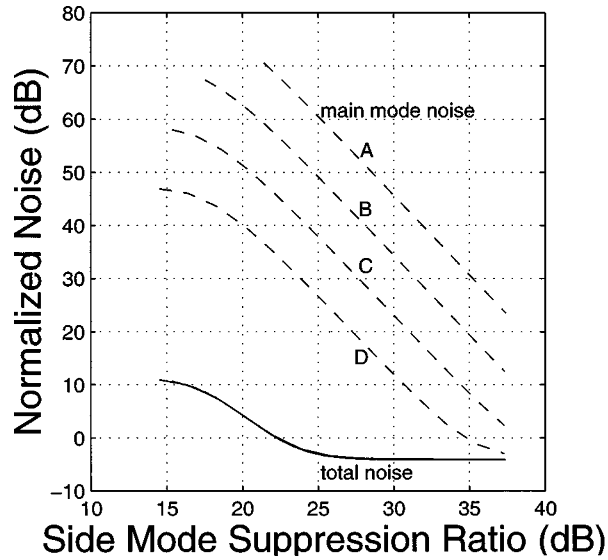

FIG. 4. Intensity noise vs SMSR for the case with both nonlinear gain and nonlinear loss included. The spontaneous emission coupling factor, β_0, was varied as (A) 2×10^{-6}, (B) 7.36×10^{-6}, (C) 2.7×10^{-5}, and (D) 1×10^{-4}. The self-gain saturation coefficient was $\zeta_{00} = (1/V_m)2.7\times10^{-18}$ (V_m is the optical mode volume in cm^3), the saturable loss coefficient was $\alpha_{sa,0}^{av} = 0.5\times10^{-3}\alpha_p$, the relative excess saturable loss for the side modes was $\Delta\alpha = 0.09$, and the rest of the parameters were the same as in Fig. 1.

V. DISCUSSION

A. Effect of saturable loss

The results presented in the preceding section can be intuitively interpreted as follows. The nonlinear gain saturates the conversion of electronic excitation supplied via a quiet pump to the cavity mode photons. However, when the dominant mode is far above threshold, photons either go out of the cavity in any of the cavity modes or get deleted equally from all the modes because of the linear loss. Since it is the same gain which is shared by all the longitudinal modes, the mode-partition noise of each mode should still be perfectly anticorrelated with the rest of the modes, collectively. Hence the mode-partition noise does not contribute to the noise of the total photons generated inside the cavity. Identical loss for the modes does not affect this null contribution. Therefore the total laser emission detected outside the cavity should approach the quantum efficiency limit, being independent of the SMSR. Alternatively, for intuitive understanding the fluctuations in the photon number of the ith mode inside the cavity, Δn_i, can be loosely defined as $\Delta n_i = \Delta n_i^p + \Delta n_i^m$, where the Δn_i^p contribution is determined by the pumping level and linear loss and Δn_i^m is determined by the mode competition or the mode-partition noise. In this context, anticorrelation means that $\Sigma_i \Delta n_i^m = 0$. The noise observed outside the cavity with the external quantum efficiency, η_e, is thus $\eta_e \Sigma_i \Delta n_i = \eta_e \Sigma_i \Delta n_i^p$. This result is the same as that of a single mode laser for the same pumping level.

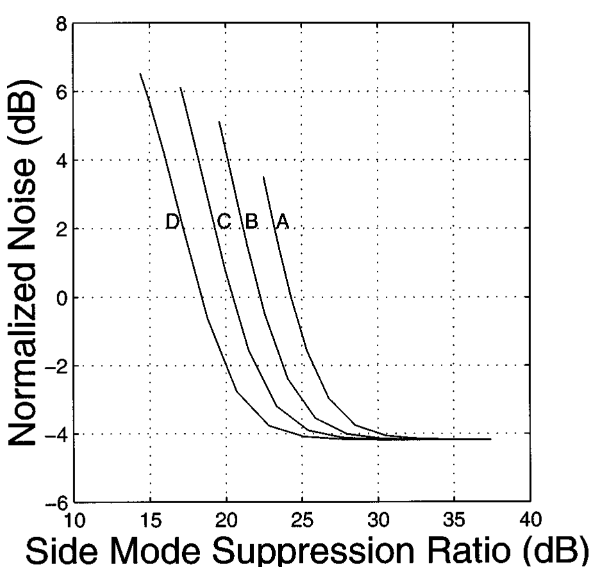

FIG. 3. Intensity noise vs SMSR for the case with linear gain and nonlinear loss. The saturable loss coefficient $\alpha_{sa,0}^{av}$ was varied as (A) $10^{-3}\alpha_p$, (B) $0.5\times10^{-3}\alpha_p$, (C) $0.25\times10^{-3}\alpha_p$, and ($D$) $0.125\times10^{-3}\alpha_p$, where $\alpha_p = 120$ cm^{-1} was the total linear loss. The relative excess saturable loss for the side modes was $\Delta\alpha = 0.09$, the spontaneous emission coupling factor was $\beta_0 = 2.6\times10^{-6}$, and the rest of the parameters were the same as in Fig. 1.

However, the saturable absorber makes the loss of each longitudinal mode dependent on the power of the dominant mode. When the main mode fluctuates to a higher power level, the gain is reduced and consequently the emission in all the modes decreases. This is the origin of the intensity noise anticorrelation among the longitudinal modes. The fluctuations of the main mode also reduce the saturable loss of the side modes. The reduction in the internal loss in turn increases the output coupling efficiency of power in the side modes. This process partly compensates for the decrease caused by the smaller gain. Since the nonlinear loss for the main mode is heavily saturated by its large intensity, the fluctuation in the main mode power has a negligible effect on its output coupling efficiency. Thus the fluctuation in the main mode power modulates the output coupling efficiency for the side modes but not for the main mode. Consequently, outside the laser cavity the overall anticorrelation between the main mode and the side modes becomes less perfect. Therefore, the mode-partition noise does not cancel out when all the modes are detected.

When the laser is operating far above threshold and pumping is noiseless, because of the conservative coupling between photons and carriers the total photon flux generated internally should be noiseless. However, some photons get lost internally (by absorption and scattering) and only those photons which escape the laser cavity are detected. One might expect that these events are random and therefore contribute Poissonian noise to the output photon flux measured in the experiments. Thus the total intensity noise would be limited only by the quantum efficiency. This assumption is no longer valid for any loss which depends on the intensity of the photon flux. In this case loss of photons no longer is a truly independent event. Therefore the fluctuations in the output photon flux and the photons which are lost (we call it lost photon flux) are correlated. This correlation changes the intrinsic anticorrelation among longitudinal modes (originating from the shared gain) in the output photon flux and results in excess noise when the mode-partition noise is large. Following is a simple mathematical description for this argument. If we assume all modes have identical saturable loss $(\Delta\alpha=0)$, the average rate of photon loss (lost photon flux), Φ_l, may be expressed as

$$\Phi_l=\sum_i \left(\frac{1}{\tau_{\text{int}}}+\frac{1}{\tau_{\text{sa}}}\right)n_i=\sum_i \left(\frac{1}{\tau_{\text{int}}}+\frac{\alpha_r}{\tau_p[1+(n_0/n_s)]}\right)n_i . \tag{14}$$

Then the fluctuations in the lost photon flux, $\Delta\Phi_l$, read

$$\begin{aligned}\Delta\Phi_l&=\sum_i \left(\frac{1}{\tau_{\text{int}}}+\frac{\alpha_r}{\tau_p[1+(n_0/n_s)]}\right)\Delta n_i-\sum_i \frac{\alpha_r}{\tau_p[1+(n_0/n_s)]^2}\frac{\Delta n_0}{n_s}n_i\\&=\left(\frac{1}{\tau_{\text{int}}}+\frac{\alpha_r}{[1+(n_0/n_s)]}\right)\sum_i \Delta n_i-\frac{1}{\tau_p}\frac{\alpha_r}{[1+(n_0/n_s)]^2}\frac{\Delta n_0}{n_s}\sum_i n_i .\end{aligned} \tag{15}$$

The first term in the above expression is proportional to the noise of the total internally generated photons $\Sigma_i\Delta n_i$ which depends only on the pumping level. It should be insensitive to the SMSR because the mode-partition noise in each mode gets canceled by the anticorrelated noise in the other modes. The second term is proportional to the internal noise in the main mode, which is large when the side mode intensities are bigger. The negative sign of this term should be noted. It implies that the noise of the lost photon flux is anticorrelated with the main mode noise.

This simple description was verified by more exact calculations using rate equations (8) and (9). Figure 5 shows that the noise of the lost photon flux is large when the SMSR is large. The noise of the combined flux including the output flux (the total external intensity noise) and the lost photon flux is also plotted in Fig. 5. The noise of the combined flux is smaller than the noise of both the lost photon flux and the output photon flux, which indicates that the noise of the lost photon flux is anticorrelated with the noise of the output photon flux. Again, we emphasize that it is the total external intensity noise which is measured in an experiment. The observed noise is larger than the quantum efficiency limit because of the deletion of the anticorrelated noise which is carried by the lost photon flux. The noise of the combined flux or internal photon flux is independent of the SMSR and is determined by the pumping level only.

These calculation results imply that the so-called excess noise (noise higher than the quantum efficiency limit) measured in many lasers stems from the nonlinear loss processes. The amount of the excess noise depends on the strength of the nonlinear loss and the intensity of the side modes.

B. Comparison of two laser structures with respect to the saturable loss

The numerical results can explain some of the differences in the noise characteristics of different types of lasers. Specifically we consider the difference in the behavior of transverse-junction-stripe (TJS) and quantum-well (QW) edge emitting lasers. A detailed comparison along with the measurements on two lasers will be published elsewhere [14]. Free-running TJS lasers usually exhibit squeezing in the correct operating conditions. On the contrary, the inten-

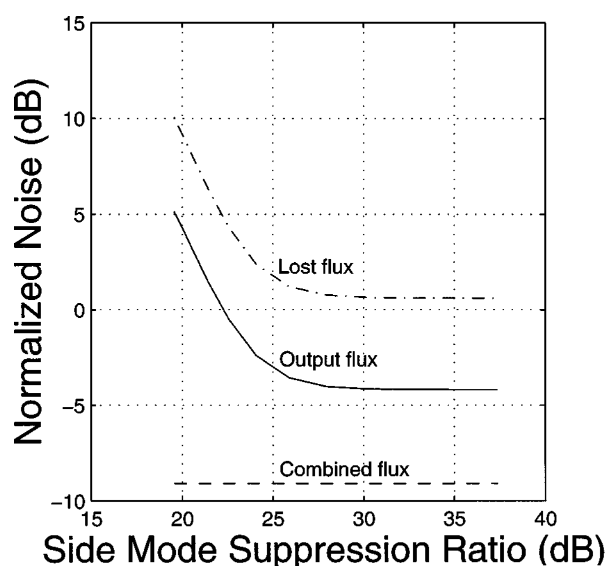

FIG. 5. Intensity noise of the lost photon flux (dot-dashed), the output flux (solid), and the combined flux (dashed) vs SMSR.

sity noise of a typical free-running QW laser was found to be larger than the shot noise value.

The first important difference between the two types of lasers is in the current injection scheme. In a QW laser, the active junction is in the growth plane (or epitaxial layer) and the electric current is injected perpendicular to this plane by the cladding layer. This current injection scheme requires heavy doping of the cladding layer. Doping concentrations of $\approx 10^{18}$ cm^{-3} are commonly used in the cladding layer of a QW laser. As we mentioned in Sec. II, donor impurities in the cladding layer of $Al_xGa_{1-x}As$ lasers create DX centers which behave like a saturable absorber.

In a TJS laser, the active junction is a homojunction, perpendicular (transverse) to the growth plane, and the pump current is injected parallel to this plane by the active layer itself. The doping concentration of the cladding layers in a TJS laser is typically $\approx 10^{17}$ cm^{-3}. This is considerably lower than QW lasers. Due to the higher doping levels of the cladding layer, DX center concentration in a QW laser is higher than a TJS laser. The saturable absorption by a DX center increases monotonically with the concentration of DX centers (N_{DX}).

The saturation photon density increases with the concentration of free carriers which can be trapped by these centers. In a QW laser, the free-carrier concentration in the cladding layer, which injects carriers into the active junction, is similar to the doping concentration. On the other hand, part of the cladding layer of a TJS laser where the optical field spreads is also a transverse p-n junction. Since free carriers are depleted from this region, the concentration of the conduction-band electrons is significantly lower than the doping concentration which is already an order of magnitude smaller compared to a QW laser. Consequently, the strength of saturable absorption in a TJS laser should be more than two orders of magnitude smaller than in a QW laser. When we calculated noise using the amount of saturable loss estimated from Si and the free-carrier concentration in these two types of lasers, results predicted excess noise in QW lasers but the quantum efficiency limited squeezing in TJS lasers.

We obtained experimental evidence of the difference in the saturable loss in these two types of lasers from their spectral characteristics. The saturable loss makes the side mode suppression higher because the weaker modes suffer slightly larger loss than the stronger mode. It also causes larger hysteresis in the tuning of the wavelength with temperature and pump current change.

The side modes in QW lasers are typically about 20 dB smaller than the main mode. On the contrary, side modes in the TJS laser are only 2 to 5 dB below the main mode. We observed large hysteresis in the wavelength tuning characteristics of several QW lasers. In contrast, TJS lasers did not show any hysteresis characteristics.

VI. SUMMARY

In summary, we calculated the intensity noise on the basis of Langevin rate equations for a semiconductor laser with three longitudinal modes. We incorporated both nonlinear gain and nonlinear loss terms in our model. The mechanisms considered for the nonlinear gain are spectral hole burning and population pulsations. The saturable absorption by the DX center is included as the nonlinear loss mechanism.

Our calculations show that for a multimode laser the nonlinear gain alone does not lead to the enhanced total intensity noise in comparison to the linear model. This is because the nonlinear gain preserves the anticorrelation among the longitudinal modes. Hence, the mode-partition noise cancels out in the total intensity noise.

It is the nonlinear loss which leads to the excess noise observed in a multimode laser. In the case of saturable loss, the fluctuations in the laser output are anticorrelated with that of absorbed photons. The removal of this anticorrelated noise increases the noise measured at the laser output. In the case of QW lasers which show excess intensity noise, the noise enhancement can be attributed to the higher saturable loss arising from the higher concentrations of DX centers and free carriers in the cladding layer of these lasers.

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