Topological phases and circulating states of Bose-Einstein condensates

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We show that the quantum topological effect predicted by Aharonov and Casher (AC effect) [Phys. Rev. Lett. **53**, 319 (1984)] may be used to create circulating states of magnetically trapped atomic Bose-Einstein condensates (BECs). A simple experimental setup is suggested based on a multiply connected geometry such as a toroidal trap or a magnetic trap pinched by a blue-detuned laser. We give numerical estimates of such effects within the current atomic BEC experiments, and point out some interesting properties of the associated quantized circulating states. [S1050-2947(99)01501-2]

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Theoretical and experimental research has advanced rapidly since the first experimental evidence of the atomic Bose-Einstein condensation (BEC) in alkali-metal gases [1–3]. Compared with strongly interacting ^{4}He systems, dilute atomic quantum gases opened a unique opportunity to perform detailed comparative studies of the predictions of the many-body theory [4]. In the last several years, experimentalists have demonstrated remarkable success testing theoretical concepts and calculations based on the theory of weakly interacting Bose gases developed many years ago [4]. However, somewhat surprisingly, the realization and the detection of a vortex state [5,6] has not been demonstrated despite the tremendous efforts from several leading experimental groups [7].

Apart from potentially nontrivial physical reasons preventing the direct observation of the vortex state, we believe the lack of robust and effective creation schemes has been a major factor. Several theoretical groups have recently proposed ideas for vortex creation. Walsworth and You [8] proposed a magnetic dipole Franck-Condon coupling scheme. Marzlin, Zhang, and Wright [9] and Bolda and Walls [10] recognized that the circulations of photon mode functions can be coupled into the condensate through the electric dipole Franck-Condon factor. Dum *et al.* [11] developed a robust scheme based on adiabatic following. Rokhsar [12] and Javanainen, Paik, and Yoo [13] have considered interesting properties of the formation and behavior of vortex/circulating states. Goldstein, Wright, and Meystre [14] proposed a possible detection scheme. The interesting connection with superfluidity and persistent current states in a toroidal shaped geometry was addressed in Refs. [15,16].

In this paper we propose a new scheme for the creation of quantized circulating states [17]. Our main idea involves the use of the Aharonov-Casher (AC) effect [18], the magnetic ''dual'' of the Aharonov-Bohm (AB) effect [19]. Our discussion is based on the setup as illustrated in Fig. 1, a three-dimensional magnetically trapped condensate is pierced along the cylindrically symmetric z axis by a far-blue-detuned laser as used in the first MIT BEC experiment [2]. We envision that there exists an imbedded conducting wire inside the laser beam which can be charged to produce a radially directed electric field. Such a setup is reminiscent of the standard discussion of the AB-AC effects [18,19]. We also assume that the atoms are spin polarized by the trap field along the axial direction $\hat{z}$. Such a setup may require the development of new types of magnetic traps. The AC effect utilized here has been extensively discussed in many papers [20,21], and will not be the detailed subject of this paper.

We note that the nonrelativistic effective single atom Hamiltonian takes the form [18]

$$H=\frac{1}{2M}(\vec{p}-\vec{E}\times\vec{\sigma}/c)^2-\frac{\sigma^2E^2}{Mc^2}+V(\vec{r}), \tag{1}$$

where M is the mass of the atom, $\vec{E}$ is the static electric-field strength, $\vec{\sigma}$ denotes the magnetic dipole moment of the trapped atom, and c is the vacuum speed of light. The trapping potential contains two parts: the static magnetic trap $V_t(\vec{r})$ and the optical dipole potential from the piercing laser field $V_L(\vec{r})$. The second quantized form for atoms described by Eq. (1) is

$$\mathcal{H}=\int d\vec{r}\left[\hat{\psi}^{\dagger}(\vec{r})(H-\mu)\hat{\psi}(\vec{r})+\frac{u_0}{2}\hat{\psi}^{\dagger 2}(\vec{r})\hat{\psi}^2(\vec{r})\right]. \tag{2}$$

We have introduced the chemical potential μ to guarantee the conservation of the total number of atoms. We have also used a shape-independent form for the atom-atom interaction with $u_0=4\pi\hbar^2a_{\rm sc}/M$ where $a_{\rm sc}$ is the s-wave scattering length.

For alkali-metal atoms under these conditions the atomic spin is $\vec{\sigma}=g_F\mu_B\hat{z}$, (where typically the Lande g_F factor is of order 1). $\mu_B=\hbar e/2m_ec$ is the Bohr magneton for the electron (with mass m_e). Assuming the charged wire to be long and thin compared with all other dimensions in the system, as illustrated in Fig. 1(a), we approximate the electric field with the infinite wire limit $\vec{E}(\vec{\rho})=2n_e\hat{\rho}/\rho$, where n_e is the linear charge density along the wire. This implies that $\vec{E}\times\vec{\sigma}$ is in the azimuthal direction $\hat{\phi}$, effectively acting as a torque term. Such a term will induce a nontrivial ground state of condensed atoms.

At zero temperature ($T=0$) we introduce an order parameter for the condensate $\psi(\vec{r})=\langle\hat{\psi}(\vec{r})\rangle$ (normalized to 1). Minimizing Hamiltonian (2) with respect to $\psi(\vec{r})$, we obtain the standard nonlinear Schrödinger equation

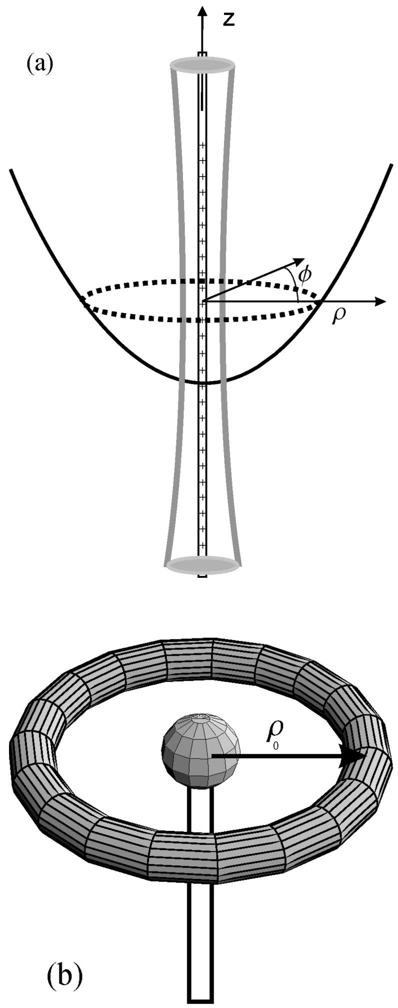

FIG. 1. Two possible setups for vortex and circulating state creation with AC effects. (a) Magnetic trap pinched by a blue detuned laser with an imbedded charged line. (b) A toroidal trap surrounding a charged sphere distribution.

$$\left[-\frac{\hbar^2}{2M}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{\partial^2}{\partial z^2}\right)-\frac{\hbar^2}{2M\rho^2}\left(\frac{\partial}{\partial\phi}-i\eta\right)^2 + Nu_0|\psi(\rho,\phi,z)|^2+V(\rho,\phi,z)-\frac{\hbar^2}{M}\frac{\eta^2}{\rho^2}\right]\psi(\rho,\phi,z) = \mu\,\psi(\rho,\phi,z), \tag{3}$$

where N is the total number of condensed atoms. We define the dimensionless term $\eta=2n_e g_F\mu_B/c\hbar$. Typical numerical values are estimated as

$$\eta=2\times\frac{N_e e}{\hbar/m_e c}\times g_F\times\frac{e\hbar}{2m_e c}\times\frac{1}{c\hbar}=N_e g_F\alpha, \tag{4}$$

where we have expressed the linear charge density in terms of the number of electronic charge units N_e per Compton length $\bar{\lambda}_c=\hbar/m_e c\approx 3.862\times10^{-13}$ m. α is the fine-structure constant: $e^2/\hbar c\approx 1/137$. For the AC effect to be observable, one would require $\eta\sim 1$, which is equivalent to a linear charge density of about

$$n_e\sim 1/(g_F\alpha\bar{\lambda}_c)\sim 3.55\times10^{14}/g_F\ (\mathrm{m}^{-1}) \tag{5}$$

or larger, i.e., a charge distribution of at least several electronic charges per Compton length. Such a charge distribution would create a large electric field in the trapping area. At a distance of $\bar\rho\ (\bar{\lambda}_c)$ from the wire (where $\bar\rho$ is ρ in units of $\bar{\lambda}_c$), one can estimate the radial electric field to be

$$E(\vec\rho)\sim 2\times N_e e/(\bar\rho\bar{\lambda}_c^2)\sim 2\times(e/a_0^2)\,(a_0^2/\bar{\lambda}_c^2)/(\alpha g_F\bar\rho) \sim 5\times10^6/(g_F\bar\rho)\ \text{(a.u.)}, \tag{6}$$

about $(2/g_F)\times10^{-3}$(a.u. at a distance of 1 mm away from the wire. A toroidal trap with a radius ~ 1 mm, as illustrated in Fig. 1(b), may make these conditions accessible.

In the case of a toroidal trap we consider an alternative charge arrangement consisting of a charged sphere instead of an infinitely long wire [see Fig. 1(b)]. The electric field in the plane of the torus can then be conveniently expressed as $N_e e/\rho_0^2$, with ρ_0 the radius of the torus. Assuming the width of the torus tube to be much smaller than its radius, one can effectively find that

$$\eta=\frac{N_e e}{\bar\rho_0(\hbar/m_e c)}\times g_F\times\frac{e\hbar}{2m_e c}\times\frac{1}{c\hbar} = N_e g_F\alpha/2\bar\rho_0. \tag{7}$$

For η to be of the order of 1, one would require

$$N_e\sim 2\bar\rho_0/g_F\alpha. \tag{8}$$

We estimate the magnitude of the electric field inside the torus tube to be

$$E(\vec\rho_0)\sim\frac{N_e e}{\rho_0^2}=\frac{2}{\alpha g_F}\times\frac{1}{\bar\rho_0}\times\left(\frac{a_0}{\bar{\lambda}_c}\right)^2\times\frac{e}{a_0^2} = \frac{2}{\alpha g_F}\times\frac{1}{\bar\rho_0}1.885\times10^4\ \text{(a.u.)}. \tag{9}$$

$a_0=\hbar^2/m_e e^2$ is the Bohr radius. For a torus with a radius of about 1 mm, the electric field is about $(5/g_F)10^{-4}$ a.u. We note the atomic unit for the field strength is 5.142×10^9 V/cm. One may hope that such field strength could be realized in the future [22].

Assuming the toroidal trap can confine the condensate to a width much narrower than the radius ρ_0, we approximate the three-dimensional problem as described above in Eq. (3) by an effective one-dimensional (1D) one along the azimuthal direction ϕ. To continue our analysis, we take the following ansatz for the transverse structure of the toroidally confined condensate,

$$\psi(\rho,\phi,z)=\Phi(\rho,z)\psi(\phi)/\sqrt{\rho_0}, \quad \Phi(\rho,z)=\frac{1}{\sqrt{\mathcal{N}}}\exp\left[-\frac{1}{4}\left(\frac{(\rho-\rho_0)^2}{\sigma_\rho^2}+\frac{z^2}{\sigma_z^2}\right)\right], \tag{10}$$

with the normalization $\mathcal{N}=2\pi\sigma_\rho\sigma_z$. σ_ρ and σ_z are the effective ground state width in ρ and z, respectively.

The effective 1D equation of motion for $\psi(\phi)$ is

$$\left[-\left(\frac{\partial}{\partial\phi}-i\eta\right)^2+\tilde{u}_0|\psi(\phi)|^2\right]\psi(\phi)=\mu_{\rm eff}\psi(\phi), \quad (11)$$

where

$$\tilde{u}_0=Nu_0\times\frac{1}{4\pi\rho_0^2\,\sigma_\rho\sigma_z}\bigg/\left(\frac{\hbar^2}{2M\rho_0^2}\right),$$

$$\mu_{\rm eff}=\frac{\eta^2}{2}+\tilde{u}_0+\langle V(\rho,\phi,z)\rangle\bigg/\left(\frac{\hbar^2}{2M\rho_0^2}\right) \quad (12)$$

$$-\rho_0^2\left\langle\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{\partial^2}{\partial z^2}\right\rangle.$$

$\langle V(\rho,\phi,z)\rangle$ is the potential averaged over the transverse profile $\Phi(\rho,z)/\sqrt{\rho_0}$. It is constant since the original potential $V(\rho,\phi,z)$ is azimuthally symmetric and independent of ϕ. The last term of $\mu_{\rm eff}$ is the transverse kinetic energy, and can be approximated as

$$\left\langle\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{\partial^2}{\partial z^2}\right\rangle\approx\left\langle\frac{\partial^2}{\partial\rho^2}+\frac{\partial^2}{\partial z^2}\right\rangle \quad (13)$$

over the state $\Phi(\rho,z)/\sqrt{\rho_0}$. This is also constant, determined by the detailed transverse profile of the trapping potential.

Now it is easy to appreciate the consequences of the topological phase term η as described by Eq. (11). Thermodynamically the ground state should be the state with the minimum value of $\mu_{\rm eff}$. Because of the required periodic boundary condition $\psi(\phi)=\psi(\phi+2\pi)$, the uniform state in ϕ has to be of the form $\psi(\phi)\sim e^{im\phi}$, where m is the integer of quantized circulation, or the angular momentum along the z axis. For such a state one obtains

$$\mu_{\rm eff}=(m-\eta)^2+\frac{\tilde{u}_0}{2\pi}. \quad (14)$$

Minimizing the above with respect to m, we obtain that $m=[\eta]$, where $[\eta]$ denotes the nearest integer of η. Since η can be controlled experimentally, one may expect the m value of the circulating state to jump whenever η crosses half-integer values. This result is illustrated in Fig. 2 as solid steps for $T=0$. We note that the dot-dashed line for the case of $0<T<T_c$ (the condensation temperature) is simply a weighted average of the result for $T=0$ with that of the thermal atoms (denoted by the dotted line). This result is similar to the Figs. 3 and 4 of Ref. [23], except now there is no rotation of the trap.

We can also check the local stability of the above circulating states. In the case of a superposition state

$$\psi(\phi)=\frac{1}{\sqrt{2\pi}}[\sqrt{1-x}\,e^{im\phi}+\sqrt{x}\,e^{i\theta}e^{i(m+1)\phi}], \quad (15)$$

where x is a variational mixing parameter and θ a random phase factor. One then finds

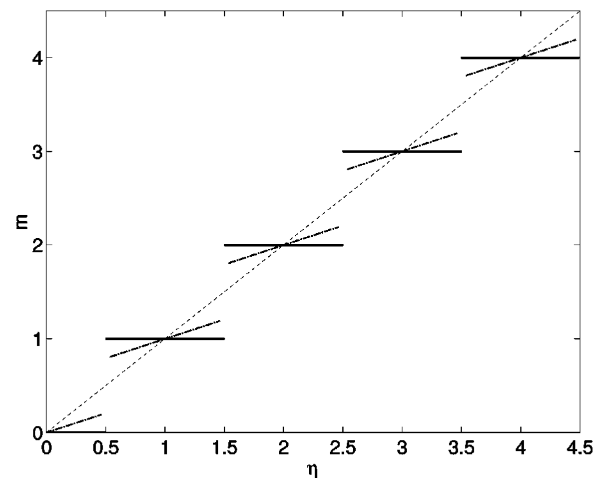

FIG. 2. The ground state index m, i.e., the angular momentum per atom at zero temperature ($T=0$). The dotted line is the result for a classical (noncondensed) ensemble of atoms whose individual atomic angular momentum is not quantized. The tilted dot-dashed line is the result for $T\neq 0$, but $<T_c$.

$$\mu_{\rm eff}=(1-x)\,(m-\eta)^2+x\,(m+1-\eta)^2$$
$$+\frac{\tilde{u}_0}{2\pi}[1+2x(1-x)]. \quad (16)$$

Since $\mu_{\rm eff}=(m-\eta)^2+(\tilde{u}_0/2\pi)$ at $x=0$ (m state) and $\mu_{\rm eff}=(m+1-\eta)^2+(\tilde{u}_0/2\pi)$ at $x=1$ ($m+1$ state), we see that the lower energy state is indeed determined by the value $[\eta]$. The curves connecting x values are inverted parabolas (for $\tilde{u}_0>0$) peaked at $x=\frac{1}{2}+(m+\frac{1}{2}-\eta)/(\tilde{u}_0/\pi)$ where $\mu_{\rm eff}$ takes the value of

$$\mu_{\rm eff}=\left(1+\frac{\pi}{\tilde{u}_0}\right)(m-\eta)(m+1-\eta)+\frac{1}{2}\left(1+\frac{\pi}{2\tilde{u}_0}+\frac{3\tilde{u}_0}{2\pi}\right). \quad (17)$$

To reach the lower side of $[\eta]$ from m to $m+1$, or vice versa, the potential barrier in between has to be overcome. In the present case, this is achieved first by adiabatically climbing the potential barrier to the top when η is experimentally adjusted, and then sliding down to the new minimum of $\mu_{\rm eff}$. This is illustrated in Fig. 3.

An important practical concern for the AC effect as applied here is the effect of the electric field to polarize atoms. As previously shown by several authors, a polarized atom inside a crossed electric and magnetic field experiences additional AB-AC-like topological effects [21]. In particular, the effective contribution is of the same form as discussed above, but

$$\eta_{E\times B}=\alpha(0)n_eB/(2\hbar c) \quad (18)$$

for a linear charge distribution. B is the magnetic trapping field in the $\hat{z}$ direction, and $\alpha(0)=\overline{\alpha}(0)(a_0^3)$ is the polarizability, with $\overline{\alpha}(0)$ typically of the order of a few hundred for alkali metal atoms. In dimensionless units, one obtains

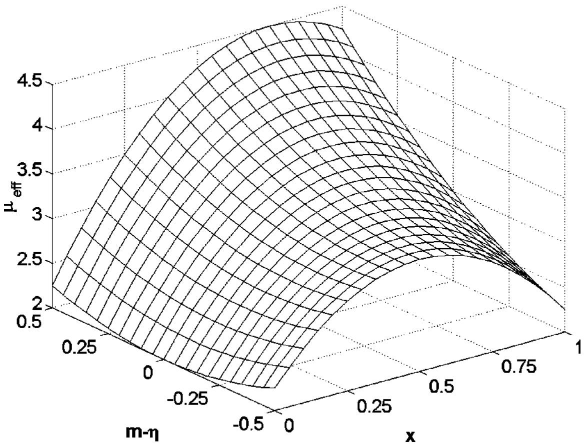

FIG. 3. A plot of $\mu_{\rm eff}$ showing the local stability of the $m=[\eta]$ states in the free-energy functional space. We have taken the parameter $\tilde{u}_0/2\pi=2$.

$$\eta_{E\times B}=\overline{\alpha}(0)\times a_0^3\times\frac{N_e e}{a_0}\times B\times\frac{1}{2\hbar c}$$

$$=\overline{\alpha}(0)\times N_e\left(\mu_B B\bigg/\frac{e^2}{a_0}\right), \qquad (19)$$

where N_e now is the number of charges per Bohr radius. The last term in parentheses is the Bohr magneton interaction energy with the external field, divided by the atomic unit for energy. It is $\sim 2\times 10^{-10}$/(G). With typical values for N_e, $\overline{\alpha}(0)$, and B, we conclude that the same topological effect due to the applied electric field and the magnetic trapping field is much smaller, $\eta_{E\times B}\ll 1$.

In conclusion, we have estimated Aharonov-Casher effects on the quantum degenerate Bose gas. We have shown that due to the presence of a topological phase term, the ground state of a degenerate quantum gas will display quantized flux states, i.e., circulating states of definite angular momentum. Our numerical estimates have shown that these would be difficult to realize within the current BEC experiments, but are potentially observable in the future. We want to emphasize that although our model is based on an effectively toroidal shaped trap, the topological effects discussed require only a multiply connected geometry, i.e., independent of the details of the transverse shape of the condensate profile. Therefore the results as illustrated in Figs. 2 and 3, should be qualitatively valid even when a more rigorous approach is taken. Our proposal makes the crucial first link between the topological phases and atomic Bose-Einstein condensates, and can also be further explored along the direction of using Bose Gases as a system in which the AC effect can be realized.

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