Atom-atom correlations induced by resonant coupling with a laser field

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The correlations arising between identical atoms resonantly coupled with the same radiation field are studied within a fully quantized theory. The atoms do not interact with each other and may be widely separated. We study how a modulation of the atom-field parameter of any one atom is transmitted to the others. This exchange of informations between remote systems was predicted in a previous account in the case of two atoms (signal transmission by optical correlations). In this paper, the theory is generalized to more than two atoms in order to model the effect of onlooker atoms. Outlines of experiments enabling investigation of the observability of these correlations and the applications that can be made are presented. [S1050-2947(99)05106-9]

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I. INTRODUCTION

The situation which is the topic of interest of this work is that of identical two-level atoms not interacting with each other but resonantly coupled with an external radiation field (laser). It has been shown [1,2] that a pair of two such atoms coupled with the field via single-photon resonances are correlated. The conditions under which these correlations occur are satisfied by atoms confined within interaction volumes whose size is much smaller than the wavelength of the radiation. We have called the regions, occupied by identical atoms, where the field parameters (phase, polarization, wavelength) can be considered as constant *reciprocal areas*.

In order to generalize what holds inside a single reciprocal area, we note that many identical atom-field systems along the laser beam exist. These hot points are separated from each other by integer multiples of the wavelength and are themselves correlation centers. Such a cloning of reciprocal areas is made possible because the properties of the laser field show a spatial periodicity. Therefore, owing to the indistinguishability of the atom-field systems inside remote reciprocal areas, the events occurring in each of them are influenced by similar events occurring inside the other ones, in the same way as they are within the same area. Then, the modulation of one or several atom-field parameters (intensity, polarization, energy of atomic levels, etc.) in any reciprocal area changes the strength of the correlation with the other ones and can be detected. This mechanism, which enables remote experimental devices to exchange information, has been named *signal transmission by optical correlations* (STOC).

The aim of the present work is (i) to revisit the problem of resonant single-photon emission from a system of two identical two-level atoms initially in the upper state, (ii) to generalize the calculation to the case of three atoms, and (iii) to propose the principles of experiments proving the existence of such correlations and displaying an application which can be made.

The problem is to calculate the probabilities of resonant single-photon emission by the atoms lying inside a reciprocal area. By comparing the values of these probabilities, one deduces the presence and the strength of the correlations. For example, in the case of two atoms, we assume that atom 1 emits the photon while atom 2 is an onlooker and we calculate the probability for this event. Then, without changing the atom-field parameters, we calculate the emission probability for atom 2. One verifies that, as expected, the sum of the probabilities is unity, which implies that if the value of one of them varies, the other is altered correlatively. In other words, the atoms share the emission probability which is unity for an external observer whose sees the two atoms while two local observers, who ignore the presence of each other, will measure the probability rate (≤1) corresponding to the atom they look after. In the case of strictly identical atom-field systems, the correlation is shown to be maximum and the probability is shared at equal rates. Since the atoms are correlated via photon emission this correlation manifests itself in the entanglement of the photons emitted by the atoms. Therefore, one can speak of atom or photon correlations interchangeably.

To study the correlation occurring between two photons emitted at different places (remote reciprocal areas) we resort to an experimental device, a part of which is used to test the nonlocality of quantum mechanics via fourth-order interferometry [3,4]. In such an experiment, two strongly correlated photons are launched on two interferometers (Mach-Zehnder or Michelson) which, in contrast to usual second-order interferences, provide high coincidences rates for path-length differences much larger than the coherence length of the photons. Conversely, in the case where large coincidence rates are observed for path-length differences much greater than the coherence length of the radiation, one deduces that the photons impinging on the detectors are correlated. The experimental arrangement becomes an efficient apparatus for the measurement of correlations of unknown photons.

The emission processes taking place in the reciprocal areas are correlated because the exchanges of photons between the atoms and the field are not located and infinite in number. Thus the entanglement of photon states involving two remote reciprocal areas can be efficiently tested with the above experimental device. The photons are supplied by sub-beams deflected at right angles of a laser beam interacting with a two-level atomic vapor.

An interesting application of STOC is to provide an alter-

nate way for quantum cryptography. The interest in using quantum cryptography to exchange information is that any eavesdropper who tries to break the secret of the cipher perturbs the signal in such a way that the sender and the receiver easily detect the presence of the eavesdropper. To model the consequences of an eavesdropper in STOC, one introduces additional atoms and calculates the resulting distortions of the two-atom results. To this end, some results of the calculations involving a third atom are reported.

II. THEORY

A. Spin-$\frac{1}{2}$ formalism

In what follows, the interaction between the jth atom and the field proceeds via the exchange of a photon labeled k. The index k accounts for all the features of the field in a well-defined region of space. It enables one to compare the values of the field at different places of the beam. The regions of space for which $k = k'$ are indistinguishable and the events occurring at these places are space independent. Since two-level atoms are involved, the Hamiltonian of the atom-plus-field system may be expressed within the spin-$\frac{1}{2}$ formalism. The correlations are related to the commutation relations of the field operators expressed in terms of spin-flip operators.

In order to take into account all the contributions to the correlations, we do not use the rotating wave approximation.

The Hamiltonian for an n-atom system is (in atomic units)

$$H = \sum_{j=1}^{n} \omega_{0j} S_3^j + \sum_{j,k} (\alpha_k^j S_-^j + \alpha_k^{j*} S_+^j)(a_k - a_k^\dagger)$$
$$+ \sum_k \omega_k a_k^\dagger a_k, \tag{2.1}$$

where $S_3 = \sigma_3/2$, σ_3 being the Pauli matrix, and $S_\pm^j$ are the spin-flip operators [5] obeying the following commutation relations:

$$[S_\pm^j, S_\mp^{j'}]_- = \pm 2 S_3^j \delta_{jj'} \tag{2.2}$$

and

$$[S_3^j, S_\pm^{j'}]_- = \pm S_\pm^j \delta_{jj'}, \tag{2.3}$$

where, as Dicke did [6], the operators S_3 and $S_\pm$ are labeled by the index j.

In Eq. (2.1), the coefficient α_j^k is expressed in terms of the photon frequency ω_k, the single-photon flux F'/F_0 ($F' =$ flux/photon number, $F_0 = 3.22 \times 10^{34}$ cm^{-2} s^{-1}), and the dipole matrix element corresponding to the jth atom as

$$\alpha_k^j = i \left(\frac{F'}{F_0} \right)^{1/2} \omega_k^{1/2} {}_j \langle \pm | \vec{r}_j \cdot \vec{\varepsilon}_k | \mp \rangle_j, \tag{2.4}$$

where $\vec{\varepsilon}_k$ is the photon polarization while the upper and the lower states of the jth atom are denoted by $|+\rangle_j$ and $|-\rangle_j$, respectively.

The Hamiltonian of Eq. (2.1) can be rewritten in the following form:

$$H = H_0^A + H_F + \sum_{j=1}^{n} \sum_k (V_{j,k}^+ + V_{j,k}^-), \tag{2.5}$$

where

$$H_0^A = \sum_{j=1}^{n} \omega_{0j} S_3^j, \tag{2.6}$$

$$H_F = \sum_k \omega_k a_k^\dagger a_k, \tag{2.7}$$

and

$$V_{j,k}^- = (V_{j,k}^+)^* = (\alpha_k^j S_-^j + \alpha_k^{j*} S_+^j) a_k \quad (j = 1 \cdots n), \tag{2.8}$$

where for more generality, the energy level separations ω_{0j} are assumed to be different.

From Eqs. (2.2) and (2.3) one obtains the commutation relations for the operators $V^\pm$,

$$[V_{j,k}^\pm, V_{j,k}^\pm]_- = 0 \tag{2.9}$$

and

$$[V_{j,k}^\pm, V_{j,k'}^\mp]_- = \pm \{ (\alpha_k^{j*} \alpha_{k'}^{j'} S_+^j S_-^{j'} + \alpha_k^j \alpha_{k'}^{j'} S_-^j S_-^{j'})$$
$$+ \text{c.c.} \} \delta_{k,k'}. \tag{2.10}$$

Equation (2.10) is the condition for correlations to stand between identical atoms labeled by j indices. It shows interesting features which are corroborated by numerical calculations, (i) the occurrence of correlations does not depend on the intensity (absence of field operator) and (ii) the commutator does not vanish for $j \neq j'$, which indicates that absorption-emission processes may involve two identical remote atoms (we note that in contrast to the field operators, the j's designate the atoms, not their states).

B. Nonperturbative model

The effect we are concerned with involves the resonant coupling of two and three atoms with a radiation field. One knows that lowest-order perturbation theory fails when the magnitude of the (intensity-dependent) interaction energy becomes comparable to that of an eigenenergy of the unperturbed system, or when the energy difference between any two atomic levels vanishes (resonances). In this case, the perturbative series representing the solution to the problem under consideration diverges and the lowest-order term is unable to predict the behavior of the system. The relevance of the theory may be restored by making exact resummations of the perturbation series in order to increase their radius of convergence.

Here, we are faced with the complicated problem of handling four (two atoms) and six (three atoms) noncommuting absorption and emission operators. The problem has been exactly solved in the case of two and four operators. For more than two atoms, the number of contributions (diagrams) rapidly increases and the solution to the problem can only be found by isolating a hard core of contributions which account for the effect of additional atoms.

In the S-matrix theory, the behavior of any system can be predicted once the time evolution operator is known. This operator can be calculated from the resolvent operator $G(z)$ by means of the inversion integral

$$U(t) = \frac{1}{2\pi i} \oint e^{-izt} G(z) dz, \qquad (2.11)$$

where

$$G(z) = \frac{1}{z - H}. \qquad (2.12)$$

The resolvent theory is well known and has been widely utilized for many purposes [7]. In particular, it provides a powerful tool within the formalism of the dressed atom. It enables the resummation of diagrams within a time-independent theoretical scheme. For the sake of brevity, only the salient results are recalled below.

According to Eqs. (2.5) and (2.12), $G(z)$ can be expressed as

$$G(z) = G_0(z) + G_0(z) H_I G(z), \qquad (2.13)$$

where

$$H_I = \sum_{j=1}^{n} (V_j^+ + V_j^-), \qquad (2.14)$$

and

$$G_0(z) = \frac{1}{z - (H_0^{AT} + H^F)}. \qquad (2.15)$$

In Eq. (2.14), the subscript k referring to the field state is dropped because it is assumed that the field is the same everywhere. The solution of Eq. (2.13) is obtained by iteration techniques providing infinite series of increasing powers of the interaction H_I.

The problem under consideration consists of calculating the resonant emission probability for atom 1 in the presence of atoms $2,3,\ldots n$ (which also can resonantly absorb and emit photons of the field). Initially, all the atoms are assumed to be in their upper states. The initial and final states of the system are $|a_1,a_2,\ldots,a_n;n_k\rangle$ and $|b_1,a_2,\ldots,a_n;n_k +1\rangle$, respectively, i.e., atom 1 emits a photon by making a transition from the upper state a_1 toward the lower state b_1, by the time atoms $2,3,\ldots,n$ remain in their initial states labeled $a_2,a_3,\ldots,a_n$ while n_k, the number of photons in the mode k is increased by one unit. Notice that the influence of atoms 2, 3, etc. on atom 1 is independent of the choice of the states. It can be shown that the entanglement of the photon states still holds if atoms 2, 3, etc. are initially in their lower states.

In the general case of n atoms, the matrix element one has to calculate is

$$^{(n_k+1)}G_{b_1,a_2,\ldots,a_n;a_1 a_2,\ldots,a_n}(z)$$

$$= \langle b_1 a_2,\ldots,a_n;n_{k+1}| \, ^{(n_k+1)}G(z)|a_1 a_2,\ldots,a_n;n_k\rangle, \qquad (2.16)$$

where $^{(n_k+1)}G(z)$ is the operator describing the resonant emission of a photon by atom 1. As a result of the resummation, this operator can be expressed as

$$^{(n_k+1)}G(z) = G(z) B G^+(z), \qquad (2.17)$$

where

$$G(z) = \frac{1}{1 - \rho^+(z) - \rho^-(z)}, \qquad (2.18)$$

$$G^{\pm}(z) = \frac{1}{1 - \rho^{\pm}(z)}, \qquad (2.19)$$

and

$$\rho^+(z) = \sum_{j=1}^{n} B_j G^+(z) A_j, \qquad (2.20)$$

$$\rho^-(z) = \sum_{j=1}^{n} A_j G^-(z) B_j, \qquad (2.21)$$

where the subscript k has been dropped since the field is assumed to be the same everywhere.

The absorption-emission operators A_j, B_j of the jth atom appearing in Eqs. (2.17), (2.20), and (2.21) are defined by $A_j = G_0 V_j^-$, $B_j = G_0 V_j^+$, respectively. To get the computational formulas, we replace $G^{\pm}(z)$ in Eqs. (2.20) and (2.21) by their values obtained from Eq. (2.19) by iteration and then one substitutes the obtained expressions into Eq. (2.18).

The operators entering into the calculation of $G(z)$ are much simplified compared to those obtained from the general theory [1]. In particular, to make the problem tractable, we only retain the terms where the operators $G^{\pm}(z)$ are switched between the lowest-order absorption (emission) and emission (absorption) operators $A(B)$ and $B(A)$, respectively. Higher-order irreducible absorption and emission operators have been discarded. The consequences of this approximation can be evaluated via the total emission probability, which must be unity. The numerical results we have obtained in most cases show that this last condition is fulfilled to a high degree of accuracy. Therefore, the approximation which consists of discarding certain classes of higher-order diagrams does not affect the final conclusion.

III. QUANTITATIVE ANALYSIS

A. Analytic formulation

Equations (2.16)–(2.21) enable us to compute self-consistently, to all orders, the probability for the resonant net emission of a photon by atom 1 in the case where $n-1$ other atoms, submitted to the same field, are also able to emit a photon. The formulation of the problem is done in the case where atom 1 is isolated while the $n-1$ remaining ones are grouped at a remote place in the same reciprocal area, but other situations can be studied within the same framework. In particular, the numerical results of Sec. III C are obtained in the case where the onlooker atom lies inside the reciprocal area containing atom 1 (active atom). The computational codes we handle are made tractable by limiting the number

of iterations of the continued fractions to that ensuring the stability of the results. The most convenient way to formulate the problem is to put the matrix element of Eq. (2.16) into a form characteristic of two-level problems. In doing so, the matrix element corresponding to the net emission of a photon by atom 1 is

$$
^{(n_k+1)}G_{b_1,a_2,\ldots,a_n;a_1,a_2,\ldots,a_n}(z)=\frac{\alpha_1}{\left[z-\omega_{01}/2-\sum_{j=2}^{n}\omega_{0j}/2-R_{a_1a_1}(z)\right]\left[z-\omega+\omega_{01}/2-\sum_{j=2}^{n}\omega_{0j}/2-R_{a_2a_2}(z)\right]-|\alpha_1|^2},
\tag{3.1}
$$

where it is assumed that the atoms are initially in the upper state. This formula is called the *two-level formula* because it has the same form as the one encountered in the theory of two-level atoms.

In Eq. (3.1), the origin of energies has been changed by subtracting the quantity $n_k\omega$ everywhere. On the other hand, for each atom this origin is half the distance of the (naked) levels, i.e., $\omega_{0(1,2)}=|\omega_{a(1,2)}-\omega_{b(1,2)}|/2$. The atom-field parameters α_j are given by Eq. (2.4) where the subscripts and the superscripts are replaced by a single subscript. The α's are related to the intensity by the relation

$$
I=\frac{14.038\times10^{16}}{\langle|\vec{\varepsilon}\cdot\vec{r}|\rangle^2}\alpha_J^2,
\tag{3.2}
$$

which, in the case of the $1S$-$2P$ transition in hydrogen reduces to $I=2.53\times10^{17}\times\alpha^2$, where I is in W/cm^2 and α is in a.u. The operator $R(z)$ is called the effective operator or the shift operator because it provides the contributions to the shifts of the levels a_1 and b_1. Its matrix elements are expressed in terms of continued fractions of α_j, ω, and ω_{0j} whose number and complexity rapidly increase. For brevity, we do not write the most general expressions of the R-matrix elements. We display the formulas obtained in the case of three atoms and we write only the beginning of the continued fractions. One obtains

$$
R_{a_1a_1}(z)=\cfrac{|\alpha_1|^2}{z+\omega+\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}-\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z+2\omega-\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z+2\omega+\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}+\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z+2\omega+\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}}
$$

$$
+\cfrac{|\alpha_2|^2}{z+\omega-\cfrac{\omega_{01}}{2}+\cfrac{\omega_{02}}{2}-\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z+2\omega+\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z+2\omega-\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z+2\omega-\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}+\frac{\omega_{03}}{2}-\cdots}}
$$

$$
+\cfrac{|\alpha_3|^2}{z+\omega-\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}+\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z+2\omega+\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}+\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z+2\omega-\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}+\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z+2\omega-\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}}
\tag{3.3}
$$

and

$$
R_{b_1b_1}(z)=\cfrac{|\alpha_1|^2}{z-2\omega-\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}-\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z-3\omega+\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z-3\omega-\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z-3\omega-\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}+\frac{\omega_{03}}{2}-\cdots}}
$$

$$
+\cfrac{|\alpha_2|^2}{z+\cfrac{\omega_{01}}{2}+\cfrac{\omega_{02}}{2}-\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z+\omega-\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z+\omega+\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z+\omega+\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}+\frac{\omega_{03}}{2}-\cdots}}
$$

$$
+\cfrac{|\alpha_2|^2}{z-2\omega+\cfrac{\omega_{01}}{2}+\cfrac{\omega_{02}}{2}-\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z-3\omega-\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z-3\omega-\frac{\omega_{01}}{2}-\frac{\omega_{02}}{2}-\frac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z-3\omega+\frac{\omega_{01}}{2}+\frac{\omega_{02}}{2}+\frac{\omega_{03}}{2}-\cdots}}
$$

$$+\cfrac{|\alpha_3|^2}{z+\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}+\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z+\omega-\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}+\cfrac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z+\omega+\cfrac{\omega_{01}}{2}+\cfrac{\omega_{02}}{2}+\cfrac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z+\omega+\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}-\cfrac{\omega_{03}}{2}-\cdots}}$$

$$+\cfrac{|\alpha_3|^2}{z-2\omega+\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}+\cfrac{\omega_{03}}{2}-\cfrac{|\alpha_1|^2}{z-3\omega-\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}+\cfrac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_2|^2}{z-3\omega+\cfrac{\omega_{01}}{2}+\cfrac{\omega_{02}}{2}+\cfrac{\omega_{03}}{2}-\cdots}-\cfrac{|\alpha_3|^2}{z-3\omega+\cfrac{\omega_{01}}{2}-\cfrac{\omega_{02}}{2}-\cfrac{\omega_{03}}{2}-\cdots}}. \quad (3.4)$$

One must note that these matrix elements of $R(z)$ contain the whole physics of the process we are concerned with. Thus, the correlations come from crossed processes where one or several photon emissions or absorptions by any one atom is followed by an equivalent number of absorptions or emissions by the other ones. More precisely, the atoms are correlated via the mixing of different species of atom-field quantities in the continued fractions. This process seems to have no classical equivalent.

B. Diagrammatic representation

The operator $R(z)$ is represented in Fig. 1 by diagrams of increasing orders inspired by the many-body theory. According to the usual rules, these diagrams are read upward. For
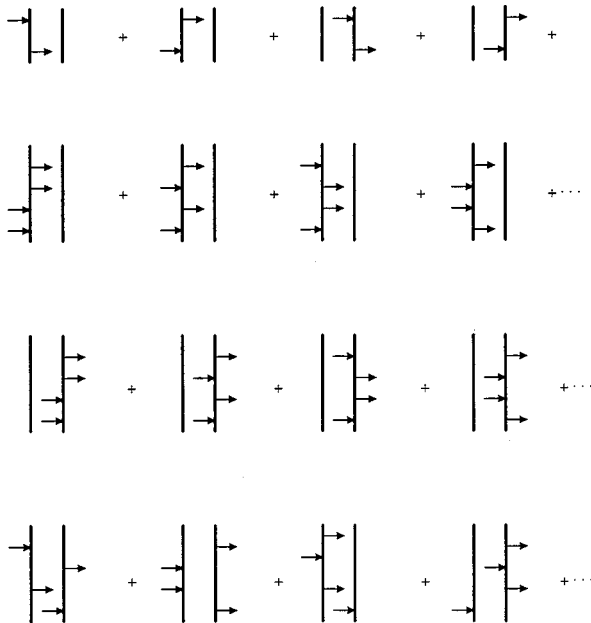

FIG. 1. Diagrammatic representation of the shift operator $R(z)$ in the case of two correlated atoms. The left- and the right-hand side vertical lines represent atoms 1 and 2, respectively. The arrows arriving at a fermion line are the photon absorptions and the arrows leaving the fermion lines are the photon emissions. The diagrams are read upward. The diagrams displayed in the first three lines are the contributions to level shifts of atoms 1 and 2. The diagrams of the last line concern atom-atom correlations. They mix the events occurring in the two atoms. No Coulomb interaction line appears between the fermion lines since we assume that the wave functions never overlap.

each diagram, the fermion line on the left-hand side represents atom 1 while those on the right-hand side are assigned to atoms 2, 3, etc. The photon absorptions and emissions are represented by horizontal lines located on the left-hand side and on the right-hand side of each fermion line, respectively. There is no Coulomb interaction between the fermion lines since it is assumed that the wave functions of the atoms do not overlap. Concerning the topology, we encounter to each order diagrams containing (i) only absorption and emission operators of atom 1, atom 2, etc., and (ii) mixings of absorption and emission operators coming from different atoms. This last class of crossed diagrams is responsible for the correlations because they mix the events corresponding to atoms 1, 2, etc. Obviously these diagrams do not factorize. Their occurrence is the consequence of the equivalence of the atom-field systems.

C. Numerical results

To calculate the integral of Eq. (2.11) by the technique of residues, one needs the poles of $^{(n_k+1)}G_{b_1,a_2,\ldots,a_n;a_1,a_2,\ldots,a_n}(z)$. They are determined to any desired accuracy by a method which consists of searching the position of the divergences induced by these poles. Once the value of the poles is known one calculates the residues in the usual way by isolating the regular part of $^{(n_k+1)}G_{b_1,a_2,\ldots,a_n;a_1,a_2,\ldots,a_n}(z)$.

One finds that the residue corresponding to the pole of energy ω_i is

$$\mathrm{Re}\, s(\omega_i,t) = {}^{(n_k+1)}H_{b_1,a_2,\ldots,a_n;a_1,a_2,\ldots,a_n}(z)e^{-i\omega_i t}, \quad (3.5)$$

where $^{(n_k+1)}H_{b_1,a_2,\ldots,a_n;a_1,a_2,\ldots,a_n}(z)$ is the regular part of $^{(n_k+1)}G_{b_1,a_2,\ldots,a_n;a_1,a_2,\ldots,a_n}(z)$ around the pole ω_i.

The time-dependent probability is

$$P(t) = \left| \sum_i \mathrm{Re}\, s(\omega_i,t) \right|^2. \quad (3.6)$$

The expression of the probability (3.6) contains many oscillating terms. Most of them oscillate at very high frequencies while few of them exhibit low time variations. The sum of these contributions leads to probability curves displaying slowly varying functions of time modulated by high-frequency oscillations. These oscillations can be eliminated by calculating the probability per unit time or by averaging over a large number of systems. These procedures which are
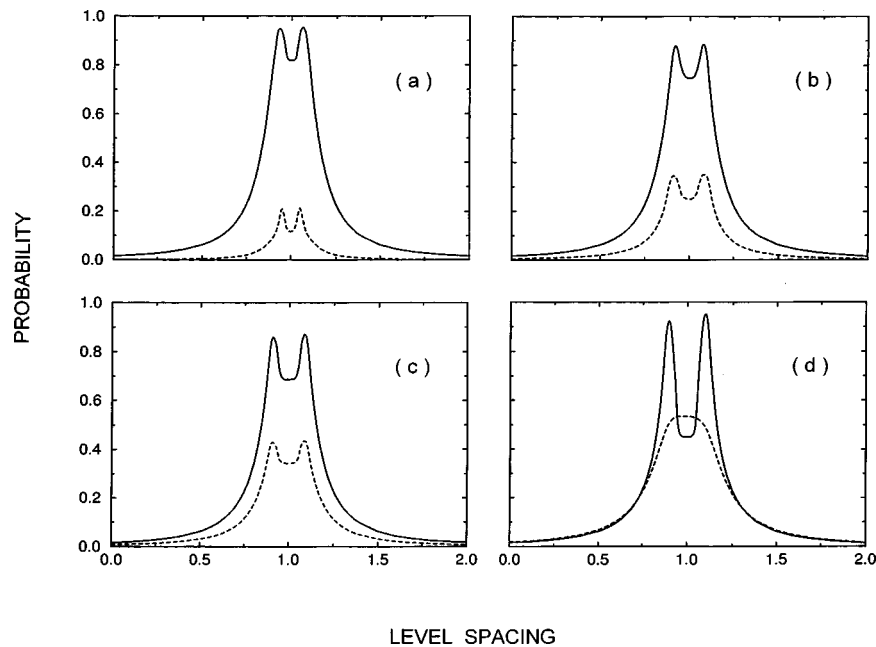
FIG. 2. Single-photon emission by two correlated two-level atoms. The solid lines and the dashed lines represent the probabilities that would be measured by a detector placed near atoms 1 and 2, respectively. The intensity I_1 at the place of atom 1 is kept constant and equal to 0.045 a.u. (5.123×10^{14} W/cm^2). The corresponding intensity for atom 2 is I_2. Curves (a), (b), (c), and (d) are calculated for intensity ratios I_2/I_1 equal to 0.4, 0.8, 1, and 1.5, respectively. Near resonance the two probabilities represented by the solid and the dashed lines sum to unity according to the probability conservation principle.

easily handled analytically are not appropriate to numerical analysis, especially if no rotating wave approximation is used. Since only the low-frequency energy-conserving terms have a physical sense, they provide the main support to the discussion. Accordingly, the quantity which is plotted in Figs. 2–4 as a function of ω_{01} is the maximum probability that atom 1 emits a photon. Notice that the behavior of the

correlations with respect to intensity and frequency does not depend on the way the probability is defined.

The field frequency ω whose value is unity in the calculations is used as a scaling parameter. Three iterations of the continued fractions are enough to ensure a good stability of the solution. There exist a great number of poles in the energy plane which come from higher-order iterations. How-
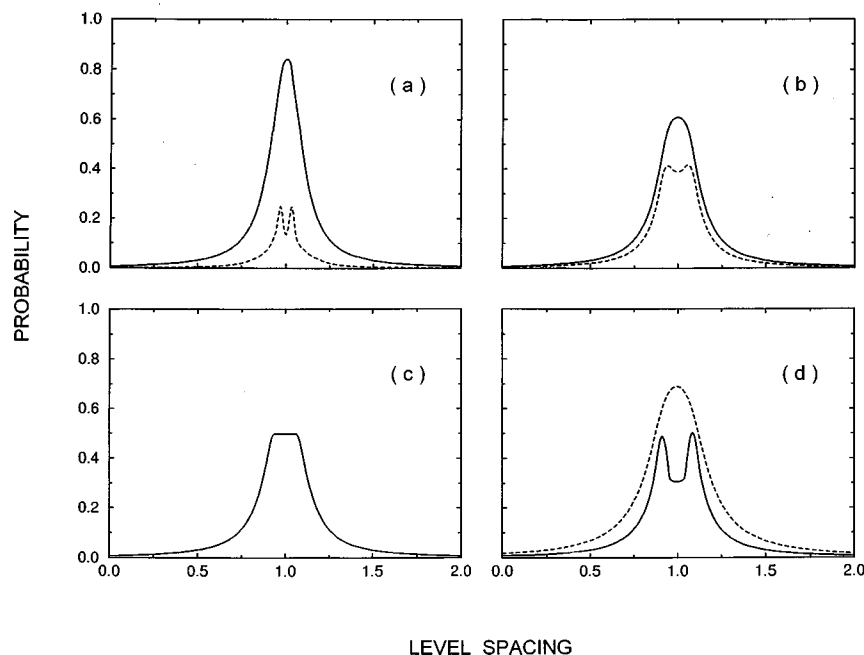

FIG. 3. Same as Fig. 2 except that the photon emission concerns three atoms. Atoms 1 and 2 are confined within the same reciprocal area located at a place where the intensity is I_1. The probabilities that atom 1 or 2 emits a photon are represented by solid lines. The dashed lines in (a), (b), (c), and (d) are the probabilities that atom 3 emits a photon at places where the intensity ratio $I_2/I_1 = 0.4$, 0.8, 1, and 1.5, respectively. As in Fig. 2, the probabilities sum to unity near resonance.
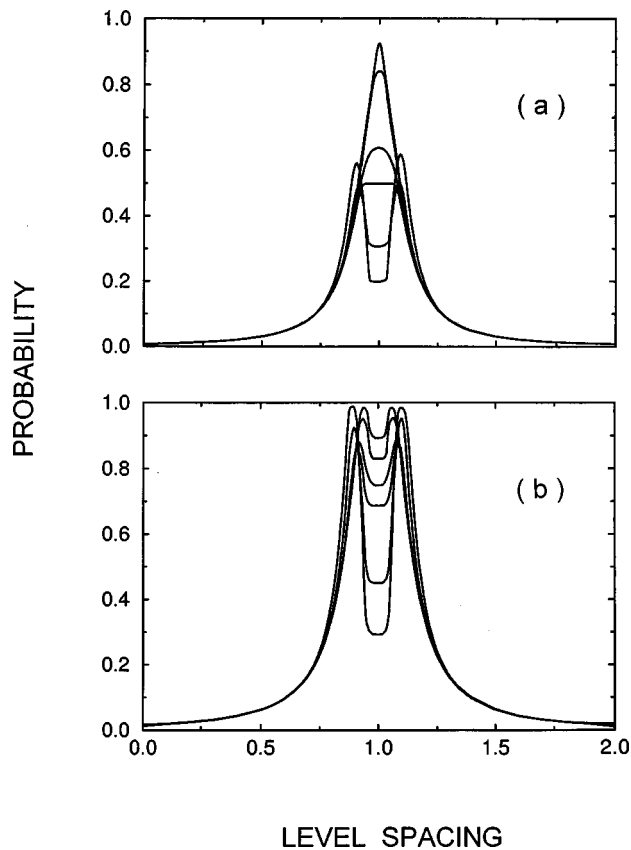
FIG. 4. (a) Single-photon emission from two correlated atoms. The curves correspond to the emission probability of atom 1 submitted to intensity $I_1 = 0.045$ a.u. in the presence of atom 2, which is affected by the intensity I_2. From top to bottom, the six curves correspond to intensity ratios $I_2/I_1 = 0.2$, 0.4, 0.8, 1, 1.5, 2, successively. (b) Single-photon emission from three correlated atoms. Atom 2 lies in the same reciprocal area as atom 1. The curves have the same meaning as in (a) and are calculated for the same values of the intensity ratio.

ever, the contribution of such poles to the probability is much smaller than that provided by the three principal poles lying within the energy range $[z=0, z=2\omega]$.

As was previously mentioned, one obtains the maximum correlation when the atom and the field parameters are identical at the places of the atoms. Conversely one may alter the indistinguishability of the two atom-field systems by changing either the field or the atom parameters. In what follows, we study the value of the emission probability of atom 1 when the intensity at the places of atom 2 (two atoms) or 3 (three atoms) is varied. According to what it is usually done, the resonance curves are scanned by varying the $1S$-$2P$ energy gap of atom 1, i.e., by varying ω_{01}.

The intensity value at the place of atom 1 is chosen to be 0.045 a.u., which corresponds to $I = 5.123 \times 10^{14}$ W/cm^2. This intensity, which is required to get a good precision of the calculations concerning the $1S$-$2P$ transition in hydrogen, can be lowered by considering other transitions characterized by larger oscillator strengths. In this respect, multilevel atoms provide an interesting investigation field because there exist many possibilities for selecting transitions characterized by large oscillator strengths. The introduction of nonresonant levels in the theory does not modify the behav-

ior of the system near resonance because any multilevel atom behaves like a two-level atom. In contrast, it is expected that the additional levels change the probability curves far from resonance, i.e., in regions where nonresonant transitions are not negligible.

The situation would be somewhat different in the case of multiphoton transitions. I plan to discuss this problem, which is out of the scope of the present work, in future.

In Figs. 2 and 3 are displayed the resonant single-photon probabilities of a system of two and three correlated atoms, respectively. In the case of two atoms, the solid line represents the single-photon emission probability corresponding to atom 1 and intensity $I_1 = 0.045$ a.u. while the dashed line is the probability that the photon emission comes from atom 2. In Figs. 2(a), 2(b), 2(c), and 2(d) the intensity ratio I_2/I_1 is equal to 0.4, 0.8, 1, and 1.5, respectively. I_2 is the value of the intensity at the place of atom 2.

In the case of three atoms, the curves have the same meaning except that it is assumed that atoms 1 and 2 belong to the same reciprocal area where the field intensity is $I_1 = 0.045$ a.u. while atom 3 lies inside another (remote) area where the intensity is I_2. Both areas are submitted to measurements performed by two observers named Alice and Bob, respectively. Since it is assumed that Alice and Bob do not exchange any information with each other, the emission probabilities they observe (which are not independent of each other) are to be calculated separately. In the absence of losses, these probabilities must sum to unity because sooner or later the photon will be emitted and detected by an observer looking at both experimental devices. On the other hand, since Alice cannot determine which atom emits the photon, one has to make symmetric the final states of atoms 1 and 2 in the case of three atoms. As a result of elementary algebra, one finds a probability that is twice the emission probability for a single atom.

The probability predicted by our calculations for Alice's and Bob's measurements is represented by the solid and the dashed lines, respectively. Near resonance, their sums depart from unity by factors less than 1%. These small discrepancies come from the truncation of the continued fractions and from the lack of certain classes of higher-order absorption-emission contributions which have been discarded.

We note that the probabilities measured by Alice and Bob sum to unity only near resonance. This is a consequence of the assumptions which have been made. It is well known that the probability conservation requires the summation of the probabilities of all the processes occurring in a well-defined system. In the presence of a resonance, some terms prevail over the remaining ones. They are sufficient to ensure the probability conservation to a high degree of accuracy. In contrast, far from resonance many (nonresonant) processes become equally probable and are to be included in the sum. In the present case, the discussion holds only near resonance since many processes ensuring probability conservation far from resonance are ignored.

Figures 4(a) and 4(b) display the resonant single-photon emission probabilities for atom 1 in the case of two and three atoms, respectively. Alice performs measurements in its reciprocal area containing atom 1 or atom 1 and 2 according to whether two atoms or three atoms are involved. The intensity at Alice's place is $I_1 = 0.045$ a.u. while the intensity in Bob's

device is I_2. The curves displayed in Figs. 4 correspond, from top to bottom, to values of the intensity ratio I_2/I_1 equal to 0.2, 0.4, 0.8, 1, 1.5, and 2.

We note that the perturbations suffered by Alice's resonance curves are important when the intensity ratio becomes equal to or greater than unity. These alterations do not vary linearly with intensity as they would in single-photon emission. For example, a variation of the intensity value by a factor of 10 changes the probability by factors of 4.5 and 3 for systems composed of two and three atoms, respectively. This indicates that the variations of the probability come from more complex mechanisms than mere intensity modulations. Such a behavior reveals the presence of atom-atom correlations which play an essential role in the effect named STOC [2].

This discussion holds for more than three atoms confined within more than two reciprocal areas.

IV. EXPERIMENTS

The first step of experimental investigations consists of proving the entanglement of photons emitted by an ensemble of two-level atoms not interacting with each other but resonantly coupled with a radiation field. To this end, we proposed, in Fig. 5, the outline of an experiment, a part of which is widely encountered in problems dealing with nonlocality of quantum theory [3,4]. A cell containing a vapor of two-level atom is placed inside a laser beam. The atoms are resonantly coupled with the laser field. Two photon beams are deflected at right angles and are sent separately at the entry of two Mach-Zehnder (or Michelson) interferometers, each of them containing a long and a short path. The difference in transit times over the two paths is the same for the two interferometers and is much larger than the coherence time of the photons. Under these conditions, each interferometer is unable to give rise to second-order interferences, i.e., the signals delivered separately by the two counters are independent of path-length difference between the long and the short paths. It is not so if the rate of simultaneous arrival of photons upon the detectors is measured. It has been shown [3,4] that when the photons which are launched at the entry of the interferometers are correlated, the coincidence counting rate shows a cosine variation with the path-length difference. The fourth-order interferences which appear come from photons having traveled via long-long and short-short paths. This experiment, involving strongly correlated photons generated by down-conversion or cascade decay, has been extensively involved for testing nonlocality of quantum mechanics. The difference between this experiment and the one we propose consists of replacing correlated photons by photons whose correlation is unknown. Since the counting rate is directly related to the degree of correlation of the photons impinging upon the interferometers, the experimental device of Fig. 5 becomes a correlation detector which discriminates among the different species of photons contained in the beams. Only photons coming from reciprocal areas exhibit, as expected, large coincidence rates (maximum correlation).

The random motion of atoms in the vapor has little influence on the emission yield because the probability that two atoms lie at places where the field is the same, by the time the emission takes place, is very large.
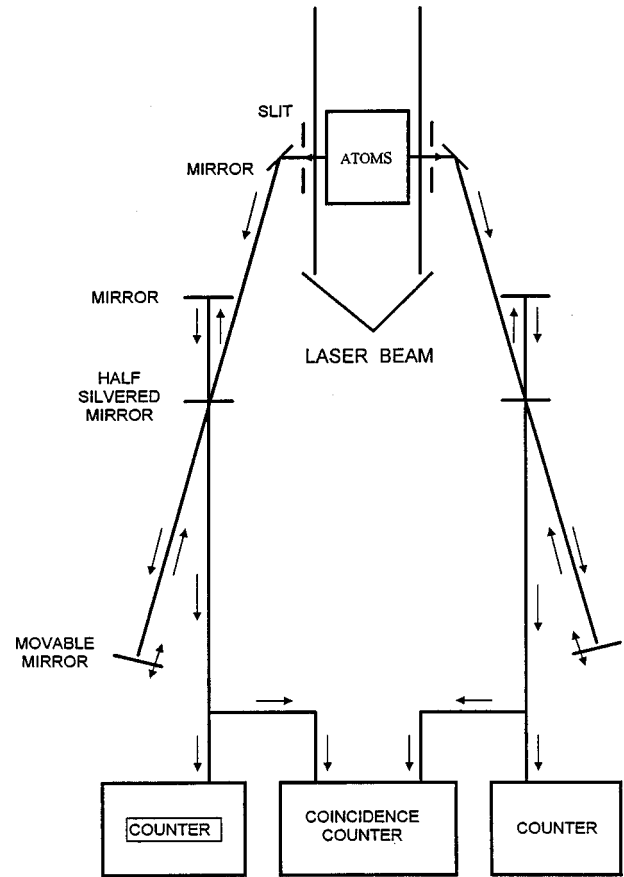

FIG. 5. Outline of an experiment enabling investigation of the correlations of photons emitted by two-level atoms resonantly coupled to a laser beam. The photons are collected at right angles of the beam and are launched at the entry of an experimental device identical to that of Ref. [4]. From the dependence of the coincidence counting rate with respect to the path difference in the two interferometers, one can deduce the degree of correlations of unknown photons impinging upon this correlation detector.

The problem of collector location becomes crucial when few atoms are concerned. In this case, to increase the coincidence rates one has to slow down (cooling) the atom motion and use subnanometer displacement devices to scan the hot points of the interaction volume.

Another interesting experiment to be done consists of inserting a removable screen inside a laser beam interacting with a two-level vapor, according to Fig. 6. The measurement of large coincidence counting rates (high fringe visibility) of photons emitted on both sides of the screen will prove that correlations result from nonlocal interactions. As a consequence, the signals provided by Alice's and Bob's detectors will depend on the field parameters on both sides of the screen. If one observes the photon emission taking place in the lower part of the screen, one expects that, according to the probability curves previously displayed, any intensity variation of the upper (Bob's) sub-beam will modify the lower (Alice's) detector yield. This enables Bob to transmit information to Alice via atom-atom correlations (STOC).

By examining the curves of Figs. 2(c) and 3(c), ones sees that the presence of an eavesdropper is easily detected. If only Alice and Bob are concerned and for identical atom-field parameters, the corresponding two-atom curves [Fig.
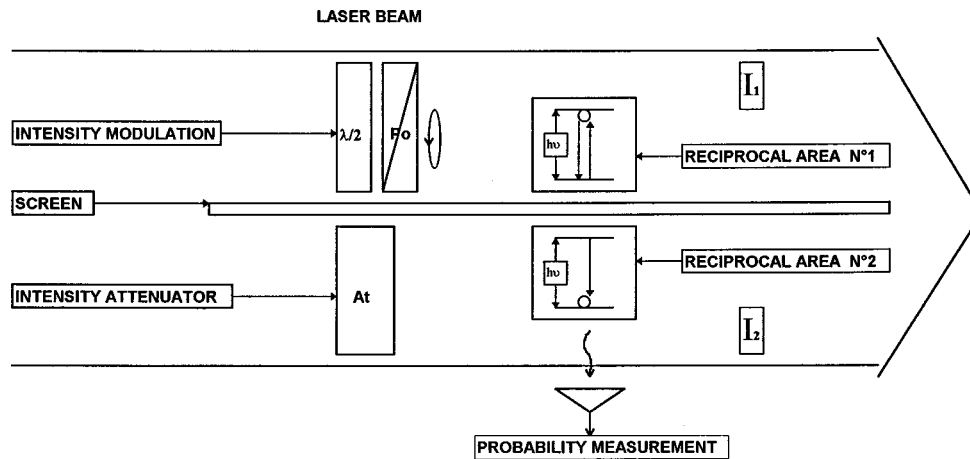
FIG. 6. Principle of an experiment enabling testing of the ability for a signal to be transmitted from a point to another point of a laser beam via atom-atom correlations. The intensity modulation at the place of reciprocal area 1 ''crosses'' the screen and influences the emission rate of atom 2 in the lower part of the beam. The role of the beam attenuator is to compensate the losses coming from the modulator.

2(c)] share at equal rates the probability of detecting the emitted photon (the two curves are superposed). The presence of a third observer is modeled by an additional atom. In this case, the curves of Fig. 3(c) show that the probabilities measured by Alice (solid curve) and Bob (dashed curve) will never be equal. Therefore, they know that there exists an eavesdropper without exchanging any information via a classical channel.

V. CONCLUDING REMARKS

The correlations between photons emitted by atoms interacting with the same radiation field have been discussed within an all-order quantum theory. The process considered is the single-photon emission by two-level atoms confined inside a single area of small dimensions compared to the wavelength of the field or inside several areas separated by multiples of the wavelength. In every case, the existence of correlations comes from the indistinguishability of atom-plus-field systems which enable the photons to be indifferently emitted by any atom. Accordingly, the theory is space independent and accounts for probability conservation. Since

the sum of the probabilities that each atom emits a photon is unity, each individual probability is strongly dependent on the other ones. In particular, for remote atoms the probability fluctuations experienced by any one of them are felt by the remaining ones and provide a way for transmitting information (STOC). This effect, which was previously studied in two-atom systems, is also observed in the case of three atoms. From unpublished results obtained in four-atom systems, it appears that the number of atoms has little influence upon the entanglement of photon states. The sensitivity of correlations with respect to intensity could be improved with multiphoton transitions. As in the case of multiple atoms, the calculation is to be done by resorting to a nonperturbative theory, which implies making resummations of perturbation series of more than two noncommuting operators.

Our main concern is to prove the existence of such correlations via an indisputable experiment. To this end we resort to an experimental device that has given beautiful results concerning locality violation in quantum theory. We expect that the question one asks about the nature of the photon states will get an answer via the results supplied by this fourth-order interference experiment.

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