Quantum-state transformation by dispersive and absorbing four-port devices

L. Knöll, S. Scheel, E. Schmidt, and D.-G. Welsch
Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, D-07743 Jena, Germany

A. V. Chizhov
Joint Institute for Nuclear Research, Laboratory of Theoretical Physics, 141980 Dubna, Moscow Region, Russia

(Received 20 August 1998)

The recently derived input-output relations for radiation at a dispersive and absorbing four-port device [T. Gruner and D.-G. Welsch, Phys. Rev. A **54**, 1661 (1996)] are used to derive the unitary transformation that relates the output quantum state to the input quantum state, including radiation and matter and without placing frequency restrictions. It is shown that for each frequency component the transformation can be regarded as a U(4) group transformation, which can directly be calculated from the underlying complex refractive-index profile of the device without additional postulates. If for narrow-bandwidth radiation far from the medium resonances the absorption matrix of the four-port device can be disregarded, the well-known U(2) group transformation for a lossless device is recognized. Explicit formulas for the transformation of Fock states, coherent states, and phase-space functions are given. [S1050-2947(99)03406-X]

PACS number(s): 42.50.Ct, 42.25.Bs, 42.79.−e

I. INTRODUCTION

Four-port devices such as beam splitters are indispensable to optical investigation, and a number of fundamental experiments in quantum optics necessarily require the use of them. The quantum theory of dispersionless and nonabsorbing beam splitters has been well established [1–7]. A beam splitter can be realized by a multislab dielectric plate, which is a dispersive and absorbing device in general. Even if the effects of dispersion and absorption (in a chosen frequency interval) are small, their influence on nonclassical radiation should be considered carefully. On the other hand, in practice multislab dielectric configurations with strongly varying dispersive and absorptive properties, e.g., near optical band gaps, have been of increasing interest, and a description of their action in the quantum domain is desired.

To give a quantum theory of dispersive and absorbing four-port devices, a Kramers-Kronig consistent quantization scheme of the electromagnetic field in dispersive and absorbing inhomogeneous media is required [8–12]. Recently, quantization of the electromagnetic field within the framework of the phenomenological Maxwell theory (with given complex permittivity in the frequency domain) has been performed, using an expansion of the electromagnetic field operators in terms of the Green function of the classical problem and an appropriately chosen infinite set of bosonic basic fields [8,11,12]. This quantization scheme applies to any linear inhomogeneous, dispersive, and absorbing matter—cases for which familiar concepts of mode expansion fail—and is fully consistent with both the Kramers-Kronig relations and the canonical (equal-time) field commutation relations in QED [11,12].

The formalism has been used in order to derive input-output relations for radiation at a dispersive and absorbing (multilayer) dielectric plate described in terms of a complex refractive-index profile $n(x,\omega)$ (x, space coordinate; ω, frequency) and to express the (low-order) moments and correlations of the outgoing fields in terms of those of the incoming fields and the (initial) dielectric-matter excitations [9,10,13,15], without any additional assumptions and approximations. Such a complex refractive-index profile may serve as a model for a number of four-port devices, such as beam splitters, mirrors, thin films, interferometers, and optical fibers. The results have been used for studying low-order correlations in two-photon interference effects [13,14,16].

In this paper we study the transformation of the quantum state as a whole and present closed formulas that enable us to calculate for a given complex refractive-index profile $n(x,\omega)$ the output quantum state from the input quantum state. It is worth noting that the theory applies to optical fields at arbitrary frequencies and bandwidths. Since the action of the device is fully determined by its complex refractive-index profile, there is no need to heuristically introduce into the theory device parameters such as transmission and reflection coefficients and postulate relations between them. All these quantities and relations including their specific dependence on frequency are natural consequences of the basic theoretical QED concept.

In particular, for narrow-bandwidth light whose frequencies are far from medium resonances so that dispersion and absorption may be disregarded and a frequency-independent real refractive-index profile may be assumed, the well-known results of mode expansion and U(2) group transformation are observed. In the general case of nonvanishing absorption it turns out that for each frequency component a U(4) group transformation must be performed. Each of these U(4) group transformations can be decomposed into U(2) group transformations, which correspond to a network of lossless four-port devices for radiation and matter. This decomposition also follows from the basic theory and need not be postulated. In particular, for each frequency component the U(4) matrix and the U(2) matrices can be exactly calculated from the underlying complex refractive-index profile.

The paper is organized as follows. In Sec. II the underlying theory is outlined and the basic input-output relations are given. The problem of quantum-state transformation is studied in Sec. III and closed solutions are presented. To illustrate the theory, in Sec. IV explicit transformation rules for Fock states and coherent states as well as for phase-space functions are presented. A summary and some conclusions are given in Sec. V.

II. BASIC EQUATIONS

Let us consider two light beams (of fixed polarization) that propagate along the (positive) x_1 and x_2 axes and impinge on a dispersive and absorbing four-port device that gives rise to two outgoing beams propagating along the (positive) y_1 and y_2 axes. Following [13], the operator of the vector potential in each of the four channels of the device can be given by

$$\hat{A}_j(z_j)=\int_0^\infty \mathrm{d}\omega \left[\sqrt{\frac{\hbar \beta_j(\omega)}{4\pi c\omega\epsilon_0 n_j^2(\omega)\mathcal{A}}} \times e^{i\beta_j(\omega)\omega z_j/c}\hat{c}(z_j,\omega)+\mathrm{H.c.}\right] \tag{1}$$

$(j=1,2)$, where

$$n_j(\omega)=\sqrt{\epsilon_j(\omega)}=\beta_j(\omega)+i\,\gamma_j(\omega) \tag{2}$$

is the complex refractive index of the adjacent medium on the jth side of the device ($\mathcal{A}$, plan area of the beam). In Eq. (1), $\hat{c}_j(z_j,\omega)$ stands for the amplitude operators $\hat{a}_j(x_j,\omega)$ and $\hat{b}_j(y_j,\omega)$, respectively, of the incoming and outgoing damped waves at frequency ω. The input-output relations for the amplitude operators can be derived to be

$$\hat{b}_j(\overline{y_j},\omega)=\sum_{j'=1}^{2} T_{jj'}(\omega)\hat{a}_{j'}(\overline{x_{j'}},\omega)+\sum_{j'=1}^{2} A_{jj'}(\omega)\hat{g}_{j'}(\omega), \tag{3}$$

where it is assumed that the incoming beams enter the device at $x_j=\overline{x_j}$ and the outgoing beams leave the device at $y_j=\overline{y_j}$. The bosonic operators $\hat{g}_j(\omega)$ play the role of operator noise sources and describe device excitations. The 2×2 matrices $T_{jj'}(\omega)$ and $A_{jj'}(\omega)$, respectively, are the characteristic transformation and absorption matrices of the device and are given in terms of its complex refractive-index profile $n(x,\omega)$ [for the calculation of $T_{jj'}(\omega)$ and $A_{jj'}(\omega)$, see [13]]. Whereas the matrix $T_{jj'}(\omega)$ describes the effects of reflection and transmission, the matrix $A_{jj'}(\omega)$ results from the losses inside the device.

For notational reasons it is convenient to introduce the definitions

$$\hat{\mathbf{a}}(\omega)=\begin{pmatrix}\hat{a}_1(\omega)\\ \hat{a}_2(\omega)\end{pmatrix}, \tag{4}$$

$$\hat{\mathbf{g}}(\omega)=\begin{pmatrix}\hat{g}_1(\omega)\\ \hat{g}_2(\omega)\end{pmatrix}, \tag{5}$$

$$\hat{\mathbf{b}}(\omega)=\begin{pmatrix}\hat{b}_1(\omega)\\ \hat{b}_2(\omega)\end{pmatrix} \tag{6}$$

$[\hat{a}_j(\omega)\equiv\hat{a}_j(\overline{x_j},\omega), \hat{b}_j(\omega)\equiv\hat{b}_j(\overline{y_j},\omega)]$, and

$$\mathbf{T}(\omega)=\begin{pmatrix}T_{11}(\omega) & T_{12}(\omega)\\ T_{21}(\omega) & T_{22}(\omega)\end{pmatrix}, \tag{7}$$

$$\mathbf{A}(\omega)=\begin{pmatrix}A_{11}(\omega) & A_{12}(\omega)\\ A_{21}(\omega) & A_{22}(\omega)\end{pmatrix}. \tag{8}$$

The input-output relations for radiation at a four-port device can then be given in the compact form of

$$\hat{\mathbf{b}}(\omega)=\mathbf{T}(\omega)\hat{\mathbf{a}}(\omega)+\mathbf{A}(\omega)\hat{\mathbf{g}}(\omega). \tag{9}$$

When the device is embedded in vacuum, then the matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ can be shown to satisfy the relation, see [13],

$$\mathbf{T}(\omega)\mathbf{T}^{+}(\omega)+\mathbf{A}(\omega)\mathbf{A}^{+}(\omega)=\mathbf{I}, \tag{10}$$

and the amplitude operators of both the incoming and outgoing waves are bosonic operators. When the device is embedded in a medium, then the photonic amplitude operators are not bosonic operators in general. In this case a unitary transformation can be introduced such that the transformed operators are bosonic operators at least at one position, so that the corresponding (scaled and transformed) transformation and absorption matrices satisfy the condition (10).

III. QUANTUM-STATE TRANSFORMATION

The operator input-output relation (9) enables one to calculate arbitrary correlations of the outgoing beams from the correlations of the incoming beams and the device excitations [13]. To obtain the quantum state of the outgoing beams as a whole, the question arises of which quantum-state transformation in the "Schrödinger picture" corresponds to the operator input-output relation (9) in the "Heisenberg picture." To answer the question, let us assume that for any frequency the input-output relation (9) is rewritten as a unitary operator transformation

$$\hat{\mathbf{b}}(\omega)=\hat{U}^{\dagger}\hat{\mathbf{a}}(\omega)\hat{U}, \quad \hat{U}^{\dagger}=\hat{U}^{-1}. \tag{11}$$

Further, let $\hat{\varrho}_{\mathrm{in}}$ be the density operator of the quantum state the incoming fields and the device are prepared in. The effect of the device can then equivalently be described by leaving the photonic operators $\hat{a}_j(\omega)$ unchanged but transforming the input-state density operator $\hat{\varrho}_{\mathrm{in}}$ to obtain the output-state density operator $\hat{\varrho}_{\mathrm{out}}$ as

$$\hat{\varrho}_{\mathrm{out}}=\hat{U}\hat{\varrho}_{\mathrm{in}}\hat{U}^{\dagger}. \tag{12}$$

A. Lossless device

Let us first restrict attention to fields in a sufficiently small frequency interval of width $\Delta\omega$ in which absorption may be disregarded. For this frequency window the four-port device can be regarded as being lossless, and Eqs. (9) and (10) approximately reduce to

$$\hat{\mathbf{b}}(\omega)=\mathbf{T}(\omega)\hat{\mathbf{a}}(\omega), \tag{13}$$

$$\mathbf{T}(\omega)\mathbf{T}^{+}(\omega)=\mathbf{I}. \tag{14}$$

Equation (13) [together with Eq. (14)] is an example of a U(N) [or, if the determinant of the transformation matrix is equal to unity, SU(N)] group transformation for $N=2$, which can be handled using standard Lie algebra techniques (see, e.g., [17]). Following [1,5], the unitary exponential operator $\hat{U}$ that corresponds to the U(2) [or, if $\det\mathbf{T}(\omega)=1$, SU(2)] group transformation matrices $\mathbf{T}(\omega)$ can then be given by

$$\hat{U}=\exp\left[-i\int_{\Delta\omega}d\omega\,[\hat{\mathbf{a}}^{\dagger}(\omega)]^{T}\mathbf{V}(\omega)\hat{\mathbf{a}}(\omega)\right], \tag{15}$$

where for chosen ω the 2×2 Hermitian matrix $\mathbf{V}(\omega)$ is related to the matrix $\mathbf{T}(\omega)$ as

$$\exp[-i\mathbf{V}(\omega)]=\mathbf{T}(\omega) \tag{16}$$

(for possible factorizations of $\hat{U}$, see, e.g., [5]). Note that in Eq. (15) the superscript T introduces transposition of the vector operator $\hat{\mathbf{a}}^{\dagger}(\omega)$, so that

$$[\hat{\mathbf{a}}^{\dagger}(\omega)]^{T}\mathbf{V}(\omega)\hat{\mathbf{a}}(\omega)=\sum_{j,j'=1}^{2}\hat{a}_{j}^{\dagger}(\omega)V_{jj'}(\omega)\hat{a}_{j'}(\omega). \tag{17}$$

B. Dispersive and absorbing device

1. Transformation law

In order to extend the formalism to arbitrary devices, without restriction to frequencies far from absorption transitions, we first express the input-output relation (9) [together with Eq. (10)] in terms of a U(4) group transformation. For this purpose we combine the two-dimensional vector operators $\hat{\mathbf{a}}(\omega)$ and $\hat{\mathbf{g}}(\omega)$ to obtain a four-dimensional input vector operator

$$\hat{\boldsymbol{\alpha}}(\omega)=\begin{pmatrix}\hat{\mathbf{a}}(\omega)\\ \hat{\mathbf{g}}(\omega)\end{pmatrix}=\begin{pmatrix}\hat{a}_1(\omega)\\ \hat{a}_2(\omega)\\ \hat{g}_1(\omega)\\ \hat{g}_2(\omega)\end{pmatrix} \tag{18}$$

and supply the two-dimensional vector operator $\hat{\mathbf{b}}(\omega)$ with some other two-dimensional vector operators $\hat{\mathbf{h}}(\omega)$ to obtain a four-dimensional output vector operator

$$\hat{\boldsymbol{\beta}}(\omega)=\begin{pmatrix}\hat{\mathbf{b}}(\omega)\\ \hat{\mathbf{h}}(\omega)\end{pmatrix}=\begin{pmatrix}\hat{b}_1(\omega)\\ \hat{b}_2(\omega)\\ \hat{h}_1(\omega)\\ \hat{h}_2(\omega)\end{pmatrix}. \tag{19}$$

Now we relate the four-dimensional vectors $\hat{\boldsymbol{\beta}}(\omega)$ and $\hat{\boldsymbol{\alpha}}(\omega)$ to each other as

$$\hat{\boldsymbol{\beta}}(\omega)=\boldsymbol{\Lambda}(\omega)\hat{\boldsymbol{\alpha}}(\omega), \tag{20}$$

$$\boldsymbol{\Lambda}(\omega)\boldsymbol{\Lambda}^{+}(\omega)=\boldsymbol{I}, \tag{21}$$

where the U(4) group transformation matrix $\boldsymbol{\Lambda}(\omega)$ is chosen such that the input-output relation (9) between $\hat{\mathbf{b}}(\omega)$ and $\hat{\mathbf{a}}(\omega)$ is preserved. As we have shown in Appendix A, the matrix $\boldsymbol{\Lambda}(\omega)$ can be expressed in terms of the 2×2 matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ as

$$\boldsymbol{\Lambda}(\omega)=\begin{pmatrix}\mathbf{T}(\omega) & \mathbf{A}(\omega)\\ -\mathbf{S}(\omega)\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega) & \mathbf{C}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega)\end{pmatrix}, \tag{22}$$

where

$$\mathbf{C}(\omega)=\sqrt{\mathbf{T}(\omega)\mathbf{T}^{+}(\omega)} \tag{23}$$

and

$$\mathbf{S}(\omega)=\sqrt{\mathbf{A}(\omega)\mathbf{A}^{+}(\omega)} \tag{24}$$

are commuting positive Hermitian matrices, and

$$\mathbf{C}^{2}(\omega)+\mathbf{S}^{2}(\omega)=\mathbf{I}. \tag{25}$$

The unitary matrix $\mathbf{D}(\omega)$ that appears in Eq. (A11) in Appendix A has been omitted in Eq. (22), since it corresponds to an irrelevant change of the device variables $\hat{\mathbf{h}}(\omega)$, as can be seen from the second line in the large parentheses in Eq. (22). Note that after separation of phase factors $e^{i\varphi(\omega)}$ and $e^{i\psi(\omega)}$, respectively, from the matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ and inclusion of them in the operators $\hat{\mathbf{a}}(\omega)$ and $\hat{\mathbf{g}}(\omega)$ the matrix $\boldsymbol{\Lambda}(\omega)$ can be regarded as an SU(4) matrix.

The U(4) [or SU(4)] group transformation (20) implies the unitary operator transformation

$$\hat{\boldsymbol{\beta}}(\omega)=\hat{U}^{\dagger}\hat{\boldsymbol{\alpha}}(\omega)\hat{U}, \tag{26}$$

where the unitary operator $\hat{U}$ that corresponds to the 4×4 unitary matrices $\boldsymbol{\Lambda}(\omega)$ can be given by

$$\hat{U}=\exp\left[-i\int_{0}^{\infty}d\omega\,[\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)]^{T}\boldsymbol{\Phi}(\omega)\hat{\boldsymbol{\alpha}}(\omega)\right]. \tag{27}$$

Here, $\boldsymbol{\Phi}(\omega)$ is a 4×4 Hermitian matrix which is related to the matrix $\boldsymbol{\Lambda}(\omega)$ by

$$\exp[-i\boldsymbol{\Phi}(\omega)]=\boldsymbol{\Lambda}(\omega). \tag{28}$$

Note that the integrand in Eq. (27) explicitly reads as

$$[\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)]^{T}\boldsymbol{\Phi}(\omega)\hat{\boldsymbol{\alpha}}(\omega)=\sum_{\nu,\nu'=1}^{4}\hat{\alpha}_{\nu}^{\dagger}(\omega)\Phi_{\nu\nu'}(\omega)\hat{\alpha}_{\nu'}(\omega). \tag{29}$$

For narrow-bandwidth radiation far from medium resonances the ω integral in Eq. (27) can be restricted to a small interval in which absorption may be disregarded, $\mathbf{A}(\omega)\approx\mathbf{0}$, and hence

$$\boldsymbol{\Lambda}(\omega)\approx\begin{pmatrix}\mathbf{T}(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{pmatrix}, \tag{30}$$

$$\boldsymbol{\Phi}(\omega)\approx\begin{pmatrix}\mathbf{V}(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}. \tag{31}$$

In this case Eq. (27) reduces to Eq. (15) and the U(2) [or SU(2)] group transformation for a lossless device is recognized.

Combining Eqs. (12) and (27), we obtain the output quantum state $\hat{\varrho}_{\rm out}$, from which the quantum state of the outgoing fields, $\hat{\varrho}_{\rm out}^{(F)}$, can be derived,

$$\hat{\varrho}_{\rm out}^{(F)}=\mathrm{Tr}^{(D)}\{\hat{\varrho}_{\rm out}\}=\mathrm{Tr}^{(D)}\{\hat{U}\hat{\varrho}_{\rm in}\hat{U}^{\dagger}\}, \tag{32}$$

where $\mathrm{Tr}^{(D)}$ means the trace with respect to the device. The input density operator $\hat{\varrho}_{\rm in}$ is an operator functional of $\hat{\boldsymbol{\alpha}}(\omega)$ and $\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)$,

$$\hat{\varrho}_{\rm in}=\hat{\varrho}_{\rm in}[\hat{\boldsymbol{\alpha}}(\omega),\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)], \tag{33}$$

and hence the transformed density operator $\hat{\varrho}_{\rm out}$ can be given by

$$\hat{\varrho}_{\rm out}=\hat{\varrho}_{\rm in}[\hat{U}\hat{\boldsymbol{\alpha}}(\omega)\hat{U}^{\dagger},\hat{U}\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)\hat{U}^{\dagger}]. \tag{34}$$

Recalling Eqs. (20) and (26), we see that

$$\hat{U}\hat{\boldsymbol{\alpha}}(\omega)\hat{U}^{\dagger}=\boldsymbol{\Lambda}^{+}(\omega)\hat{\boldsymbol{\alpha}}(\omega), \tag{35}$$

$$\hat{U}\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)\hat{U}^{\dagger}=\boldsymbol{\Lambda}^{T}(\omega)\hat{\boldsymbol{\alpha}}^{\dagger}(\omega), \tag{36}$$

and hence

$$\hat{\varrho}_{\rm out}=\hat{\varrho}_{\rm in}[\boldsymbol{\Lambda}^{+}(\omega)\hat{\boldsymbol{\alpha}}(\omega),\boldsymbol{\Lambda}^{T}(\omega)\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)]. \tag{37}$$

Combining Eqs. (32) and (37) yields

$$\hat{\varrho}_{\rm out}^{(F)}=\mathrm{Tr}^{(D)}\{\hat{\varrho}_{\rm in}[\boldsymbol{\Lambda}^{+}(\omega)\hat{\boldsymbol{\alpha}}(\omega),\boldsymbol{\Lambda}^{T}(\omega)\hat{\boldsymbol{\alpha}}^{\dagger}(\omega)]\}. \tag{38}$$

Using the formulas given in [13], both the matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ can directly be calculated from the underlying complex-valued refractive-index profile $n(x,\omega)$ of the matter for any frequency. The unitary operator $\hat{U}$ in Eq. (27) together with Eq. (28) and the matrices $\boldsymbol{\Lambda}(\omega)$ expressed in terms of $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$, Eq. (22), then enables us to derive, for chosen input quantum states, closed solutions for the output quantum states in a straightforward way, Eqs. (32) and (34) and Eqs. (37) and (38), without placing any frequency restrictions. It is worth noting that the formulas are also suited for studying the behavior of quantum states in the vicinity of absorption lines (Sec. IV) where commonly used mode expansion fails. Since the presence of matter is fully described by the complex refractive-index profile, there is no need for introducing phenomenological replacement schemes.

Radiation fields are frequently described in terms of discrete modes. Let us restrict our attention to (quasi) monochromatic discrete modes. We subdivide the frequency axis into sufficiently small intervals $\Delta\omega_m$ with midfrequencies ω_m and define the bosonic input operators

$$\hat{\boldsymbol{\alpha}}_m=\frac{1}{\sqrt{\Delta\omega_m}}\int_{\Delta\omega_m}d\omega\;\hat{\boldsymbol{\alpha}}(\omega), \tag{39}$$

and the bosonic output operators $\hat{\boldsymbol{\beta}}_m$ accordingly. The operator input-output relation (20) then reads as

$$\hat{\boldsymbol{\beta}}_m=\boldsymbol{\Lambda}_m\hat{\boldsymbol{\alpha}}_m \tag{40}$$

$[\boldsymbol{\Lambda}_m\equiv\boldsymbol{\Lambda}(\omega_m)]$, which can be rewritten as, according to Eq. (26),

$$\hat{\boldsymbol{\beta}}_m=\hat{U}^{\dagger}\hat{\boldsymbol{\alpha}}_m\hat{U}=\hat{U}_m^{\dagger}\hat{\boldsymbol{\alpha}}_m\hat{U}_m, \tag{41}$$

where [in place of Eq. (27)]

$$\hat{U}=\prod_m\hat{U}_m, \tag{42}$$

with

$$\hat{U}_m=\exp(-i[\hat{\boldsymbol{\alpha}}_m^{\dagger}]^T\boldsymbol{\Phi}_m\hat{\boldsymbol{\alpha}}_m), \tag{43}$$

$[\boldsymbol{\Phi}_m\equiv\boldsymbol{\Phi}(\omega_m)]$. The matrices $\boldsymbol{\Phi}_m$ and $\boldsymbol{\Lambda}_m$ are related to each other according to Eq. (28), and Eqs. (30)–(38) apply accordingly.

2. Relation to U(2) and SU(2) group transformations

The U(4) group transformation defined by the matrix $\boldsymbol{\Lambda}$ in Eq. (A11) [or Eq. (22)] can be decomposed in different ways. As we have shown in Appendix B, it can be given in terms of five U(2) group transformations. That is to say, for chosen frequency component the action of an absorbing four-port device formally corresponds, e.g., to the combined action of five lossless four-port devices [for possible factorizations of a U(N) matrix into U(2) matrices, see also [18]]. Each of the lossless devices contributes a unitary operator of the type given in Eq. (15),

$$\hat{U}[\mathbf{M};\hat{\mathbf{q}}]\equiv\exp\left[-i\int_0^{\infty}d\omega\,[\hat{\mathbf{q}}^{\dagger}(\omega)]^T\mathbf{W}(\omega)\hat{\mathbf{q}}(\omega)\right], \tag{44}$$

to the overall (product) unitary operator. In Eq. (44), $\mathbf{W}(\omega)$ is a 2×2 Hermitian matrix that is related to a U(2) group transformation matrix $\mathbf{M}(\omega)$ as

$$\exp[-i\mathbf{W}(\omega)]=\mathbf{M}(\omega), \tag{45}$$

and $\hat{\mathbf{q}}(\omega)$ is a vector whose two components are bosonic operators. Note that for narrow-bandwidth radiation far from medium resonances Eq. (44) [together with Eq. (45)] corresponds to Eq. (15) [together with Eq. (16)], with $\mathbf{M}(\omega)=\mathbf{T}(\omega)$, $\mathbf{W}(\omega)=\mathbf{V}(\omega)$, and $\hat{\mathbf{q}}(\omega)=\hat{\mathbf{a}}(\omega)$.

When the irrelevant matrix $\mathbf{D}(\omega)$ in Eq. (A11) is set equal to the unit matrix $\mathbf{I}$, then Eq. (A11) reduces to Eq. (22). As shown in Appendix B, the unitary operator $\hat{U}\equiv\hat{U}[\boldsymbol{\Lambda};\hat{\boldsymbol{\alpha}}]$ given in Eq. (27) can be decomposed into a product of eight operators $U[\mathbf{M};\hat{\mathbf{q}}]$ as follows:

$$\hat{U}[\boldsymbol{\Lambda};\hat{\boldsymbol{\alpha}}]=\hat{U}[\mathbf{C}+i\mathbf{S};(i\hat{\mathbf{a}}+\hat{\mathbf{g}})/\sqrt{2}]\hat{U}[\mathbf{C}-i\mathbf{S};(\hat{\mathbf{a}}+i\hat{\mathbf{g}})/\sqrt{2}]\times\hat{U}[\mathbf{S}^{-1}\mathbf{A};\hat{\mathbf{g}}]\,\hat{U}[\mathbf{C}^{-1}\mathbf{T};\hat{\mathbf{a}}] \tag{46}$$

[cf. Eqs. (B13), (B18), and (B24)], and decomposition of $\hat{U}[\mathbf{C}-i\mathbf{S};(\hat{\mathbf{a}}+i\hat{\mathbf{g}})/\sqrt{2}]$ and $\hat{U}[\mathbf{C}+i\mathbf{S};(i\hat{\mathbf{a}}+\hat{\mathbf{g}})/\sqrt{2}]$ eventually yields

$$\hat{U}[\mathbf{\Lambda};\hat{\boldsymbol{\alpha}}]=\hat{U}^{\dagger}[\mathbf{P};\hat{\mathbf{d}}_2]\,\hat{U}^{\dagger}[\mathbf{P};\hat{\mathbf{d}}_1]\hat{U}[\mathbf{C}+i\mathbf{S};\hat{\mathbf{g}}]\,\hat{U}[\mathbf{C}-i\mathbf{S};\hat{\mathbf{a}}]\times\hat{U}[\mathbf{P};\hat{\mathbf{d}}_2]\,\hat{U}[\mathbf{P};\hat{\mathbf{d}}_1]\hat{U}[\mathbf{S}^{-1}\mathbf{A};\hat{\mathbf{g}}]\,\hat{U}[\mathbf{C}^{-1}\mathbf{T};\hat{\mathbf{a}}], \tag{47}$$

where

$$\hat{\mathbf{d}}_j(\omega)=\begin{pmatrix}\hat{a}_j(\omega)\\ \hat{g}_j(\omega)\end{pmatrix} \tag{48}$$

($j=1,2$) and

$$\mathbf{P}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix} \tag{49}$$

[cf. Eqs. (B26) and (B27)]. It should be pointed out that when $\mathbf{\Lambda}(\omega)$ is an SU(4) group transformation, then the matrices $\mathbf{P}$, $\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega)$, and $\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega)$ correspond to SU(2) group transformations. The matrices $\mathbf{C}(\omega)+i\mathbf{S}(\omega)$ and $\mathbf{C}(\omega)-i\mathbf{S}(\omega)$ correspond to U(2) group transformations in general, i.e., SU(2) transformations and additional phase shifts. Further it is worth noting that all the matrices can be exactly calculated from the underlying complex refractive-index profile $n(x,\omega)$, because they follow from the basic theoretical concepts of QED and need not be introduced phenomenologically within ''beam splitter'' replacement schemes. Needless to say, each of the operators $\hat{U}[\mathbf{M};\hat{\mathbf{q}}]$ on the right-hand side in Eq. (47) can be further factored [5].

IV. APPLICATIONS

A. Fock-state and coherent-state bases

To illustrate the theory, let us first consider the transformation of Fock states as fundamental basis states for quantum-state representation. Here and in the following we restrict our attention to single-mode radiation of chosen frequency (in each channel), so that the mode subscript can be omitted. The results can easily be extended to multimode fields by taking the direct product of single-mode states. Let

$$\hat{\varrho}_{\rm in}=|\psi_{\rm in}\rangle\langle\psi_{\rm in}|, \tag{50}$$

$$|\psi_{\rm in}\rangle=|n_1,n_2,n_3,n_4\rangle=\prod_{\nu=1}^{4}\frac{\hat{\alpha}_\nu^{\dagger n_\nu}}{\sqrt{n_\nu!}}|0\rangle, \tag{51}$$

be the density operator of the system in the case when n_1 and n_2 photons impinge on the device that is excited in Fock states with n_3 and n_4 quanta. From application of Eq. (37) we then obtain

$$\hat{\varrho}_{\rm out}=|\psi_{\rm out}\rangle\langle\psi_{\rm out}|, \tag{52}$$

with

$$|\psi_{\rm out}\rangle=\prod_{\nu=1}^{4}\frac{1}{\sqrt{n_\nu!}}\left(\sum_{\mu=1}^{4}\Lambda_{\mu\nu}\hat{\alpha}_\mu^{\dagger}\right)^{n_\nu}|0\rangle. \tag{53}$$

We use the decomposition

$$\left(\sum_{\mu=1}^{4}\Lambda_{\mu\nu}\hat{\alpha}_\mu^{\dagger}\right)^{n_\nu}=\sum_{\{k_{\nu\mu}\}}{}'\prod_{\mu=1}^{4}\frac{n_\nu!}{k_{\nu\mu}!}(\Lambda_{\mu\nu}\hat{\alpha}_\mu^{\dagger})^{k_{\nu\mu}}, \tag{54}$$

where the notation Σ' is used to indicate that the (nonnegative) integers $k_{\nu\mu}$ satisfy the condition

$$\sum_{\mu=1}^{4}k_{\nu\mu}=n_\nu, \tag{55}$$

and rewrite Eq. (53) as

$$|\psi_{\rm out}\rangle=\sum_{\{k_\mu\}}C_{k_1,k_2,k_3,k_4}|k_1,k_2,k_3,k_4\rangle. \tag{56}$$

Here the coefficients C_{k_1,k_2,k_3,k_4} are given by

$$C_{k_1,k_2,k_3,k_4}=\left(\prod_{\nu=1}^{4}\sqrt{n_\nu!}\right)\left(\prod_{\mu=1}^{4}\sqrt{k_\mu!}\right)\sum_{\{k_{\nu\mu}\}}{}''\prod_{\mu,\nu=1}^{4}\frac{\Lambda_{\mu\nu}^{k_{\nu\mu}}}{k_{\nu\mu}!}, \tag{57}$$

the $k_{\nu\mu}$ satisfying the conditions

$$\sum_{\mu=1}^{4}k_{\nu\mu}=n_\nu,\quad\sum_{\nu=1}^{4}k_{\nu\mu}=k_\mu \tag{58}$$

(notation Σ''). Obviously, when the input state is a superposition of Fock states (51), then the output state is the corresponding superposition of Fock states (56). Application of Eq. (38) eventually yields the density operator of the outgoing fields,

$$\hat{\varrho}_{\rm out}^{(F)}={\rm Tr}^{(D)}\{|\psi_{\rm out}\rangle\langle\psi_{\rm out}|\}. \tag{59}$$

In particular, for a single Fock state we obtain, on using Eqs. (56) and (57),

$$\hat{\varrho}_{\rm out}^{(F)}=\sum_{k_1,k_2}\sum_{k_1',k_2'}D_{k_1,k_2,k_1',k_2'}|k_1,k_2\rangle\langle k_1',k_2'|, \tag{60}$$

$$D_{k_1,k_2,k_1',k_2'}=\sum_{k_3,k_4}C_{k_1,k_2,k_3,k_4}C^{*}_{k_1',k_2',k_3,k_4}. \tag{61}$$

Alternatively, the density operator $\hat{\varrho}_{\rm out}^{(F)}$ can be represented as follows. Let us define linear combinations

$$\hat{x}_\nu^{\dagger}=\sum_{i=1}^{2}\Lambda_{i\nu}\,\hat{a}_i^{\dagger} \tag{62}$$

and

$$\hat{y}_\nu^{\dagger}=\sum_{i=1}^{2}\Lambda_{2+i\,\nu}\,\hat{g}_i^{\dagger} \tag{63}$$

of photonic and device operators, respectively. Writing

$$\sum_{\mu=1}^{4} \Lambda_{\mu\nu}\hat{\alpha}_{\mu}^{\dagger} = \hat{x}_{\nu}^{\dagger} + \hat{y}_{\nu}^{\dagger}, \tag{64}$$

and combining Eqs. (53) and (59) yields

$$\varrho_{\text{out}}^{(F)} = \sum_{\{p_\mu\}\{q_\nu\}} Y_{\{p_\mu\}\{q_\nu\}} \prod_{\mu=1}^{4} \hat{x}_{\mu}^{\dagger p_\mu} |0^{(F)}\rangle\langle 0^{(F)}| \prod_{\nu=1}^{4} \hat{x}_{\nu}^{q_\nu} \tag{65}$$

$(p_\mu, q_\nu = 0,1,\ldots,n_\nu)$, where

$$Y_{\{p_\mu\}\{q_\nu\}} = \left[\prod_{\nu=1}^{4} \frac{1}{\sqrt{n_\nu!}} \binom{n_\nu}{q_\nu}\right]\left[\prod_{\mu=1}^{4} \frac{1}{\sqrt{n_\mu!}} \binom{n_\mu}{p_\mu}\right] \times \left\langle 0^{(D)} \left| \left(\prod_{\nu=1}^{4} \hat{y}_{\nu}^{n_\nu - q_\nu}\right)\left(\prod_{\mu=1}^{4} \hat{y}_{\mu}^{\dagger n_\mu - p_\mu}\right) \right| 0^{(D)} \right\rangle, \tag{66}$$

$|0^{(F)}\rangle$ and $|0^{(D)}\rangle$, respectively, being the field and device ground states. The device ground-state expectation value in Eq. (66) can be calculated by moving the operators $\hat{y}_\nu$ from the left to the right, employing the commutation relations between $\hat{y}_\nu$ and $\hat{y}_{\mu}^{\dagger}$. In this way $Y_{\{p_\mu\}\{q_\nu\}}$ can be expressed in terms of the complex numbers $\upsilon_{\nu\mu}$,

$$\upsilon_{\nu\mu} = [\hat{y}_\nu, \hat{y}_{\mu}^{\dagger}], \tag{67}$$

which form a 4×4 matrix $\boldsymbol{\upsilon}$ that is related to the matrices $\mathbf{T}$ and $\mathbf{A}$ as

$$\boldsymbol{\upsilon} = \begin{pmatrix} \mathbf{I} - \mathbf{T}^{+}\mathbf{T} & -\mathbf{T}^{+}\mathbf{A} \\ -\mathbf{A}^{+}\mathbf{T} & \mathbf{I} - \mathbf{A}^{+}\mathbf{A} \end{pmatrix}. \tag{68}$$

Next let us consider the transformation of coherent states

$$|\psi_{\text{in}}\rangle = |\boldsymbol{\gamma}\rangle = \exp[\boldsymbol{\gamma}^{T}\hat{\boldsymbol{\alpha}}^{\dagger} - \boldsymbol{\gamma}^{*T}\hat{\boldsymbol{\alpha}}]|0\rangle, \tag{69}$$

where

$$\boldsymbol{\gamma} = \begin{pmatrix} \mathbf{c} \\ \mathbf{d} \end{pmatrix}, \tag{70}$$

with c_i and d_i $(i=1,2)$, respectively, being the coherent amplitudes of the input fields and the device. Application of Eq. (37) yields Eq. (52), where $|\psi_{\text{out}}\rangle$ is the coherent state

$$|\psi_{\text{out}}\rangle = |\boldsymbol{\gamma}'\rangle, \tag{71}$$

with

$$\boldsymbol{\gamma}' = \boldsymbol{\Lambda}\boldsymbol{\gamma} \tag{72}$$

in place of $\boldsymbol{\gamma}$ in Eq. (69). The extension to a superposition of coherent states is straightforward.

From Eqs. (59), (71), and (72) it follows that when the incoming modes and the device are prepared in coherent states, then the outgoing modes are prepared in coherent states, i.e.,

$$\hat{\varrho}_{\text{out}}^{(F)} = |\mathbf{c}'\rangle\langle\mathbf{c}'|. \tag{73}$$

Note that the coherent amplitudes c_1' and c_2' of the outgoing modes are not only determined by the characteristic transformation matrix $\mathbf{T}$ but also by the absorption matrix $\mathbf{A}$ via the coherent-state amplitudes of the device, as can be seen from Eq. (72),

$$\mathbf{c}' = \mathbf{T}\mathbf{c} + \mathbf{A}\mathbf{d}. \tag{74}$$

B. Phase-space functions

It is often useful to describe quantum states in terms of phase-space functions (see, e.g., [19]). For notational convenience we will again restrict our attention to single-mode fields. Let $P_{\text{in}}(\boldsymbol{\alpha};s)$ be the s-parametrized phase-space function of an input state $\hat{\varrho}_{\text{in}}$. Equation (37) implies that the phase-space function $P_{\text{out}}(\boldsymbol{\alpha};s)$ that corresponds to $\hat{\varrho}_{\text{out}}$ is simply given by

$$P_{\text{out}}(\boldsymbol{\alpha};s) = P_{\text{in}}(\boldsymbol{\Lambda}^{+}\boldsymbol{\alpha};s), \tag{75}$$

so that the phase-space function $P_{\text{out}}^{(F)}(\mathbf{a};s)$ of the outgoing radiation reads as

$$P_{\text{out}}^{(F)}(\mathbf{a};s) = \int d^2\mathbf{g}\, P_{\text{out}}(\boldsymbol{\alpha};s) = \int d^2\mathbf{g}\, P_{\text{in}}(\boldsymbol{\Lambda}^{+}\boldsymbol{\alpha};s), \tag{76}$$

where the notation

$$\boldsymbol{\alpha} = \begin{pmatrix} \mathbf{a} \\ \mathbf{g} \end{pmatrix} \tag{77}$$

has been introduced, with a_i and g_i $(i=1,2)$, respectively, being the complex phase-space variables of the fields and the device. We change the variables,

$$\mathbf{g} = \mathbf{S}^{-1}(\mathbf{C}\mathbf{a} - \mathbf{C}^{-1}\mathbf{g}'), \tag{78}$$

$$d^2\mathbf{g} = |\det(\mathbf{CS})^{-1}|^2\, d^2\mathbf{g}', \tag{79}$$

and rewrite Eq. (76) as, on recalling Eqs. (22)–(24),

$$P_{\text{out}}^{(F)}(\mathbf{a};s) = \int \left\{ \frac{d^2\mathbf{g}'}{|\det\mathbf{T}\det\mathbf{A}|^2} P_{\text{in}}\left[\begin{pmatrix} \mathbf{T}^{-1}\mathbf{g}' \\ \mathbf{A}^{-1}(\mathbf{a}-\mathbf{g}') \end{pmatrix};s\right]\right\}. \tag{80}$$

Equations (75) and (76) [or Eq. (80)] are the most general relations for s-parametrized phase-space functions at an arbitrary, linear four-port device.

When the states of the input field and the device are not entangled, then the phase-space function $P_{\text{in}}(\boldsymbol{\alpha};s)$ is a product of the phase-space functions of the incoming fields and the device,

$$P_{\text{in}}(\boldsymbol{\alpha};s) = P_{\text{in}}^{(F)}(\mathbf{a};s) P_{\text{in}}^{(D)}(\mathbf{g};s). \tag{81}$$

In this case Eq. (80) reads as

$$P_{\text{out}}^{(F)}(\mathbf{a};s) = \int \left\{ \frac{d^2\mathbf{g}'}{|\det\mathbf{T}\det\mathbf{A}|^2} P_{\text{in}}^{(F)}(\mathbf{T}^{-1}\mathbf{g}';s) \times P_{\text{in}}^{(D)}[\mathbf{A}^{-1}(\mathbf{a}-\mathbf{g}');s]\right\}. \tag{82}$$

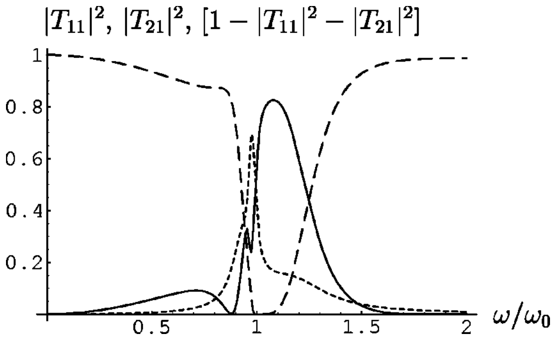

FIG. 1. The reflection coefficient $|T_{11}|^2$ (solid line), the transmission coefficient $|T_{21}|^2$ (dashed line), and the absorption coefficient $(1-|T_{11}|^2-|T_{21}|^2)$ (dotted line) of a dielectric plate are shown as functions of frequency ω for $\epsilon_s=1.5$ and $\gamma/\omega_0=0.01$ in Eq. (84), and the plate thickness $2c/\omega_0$.

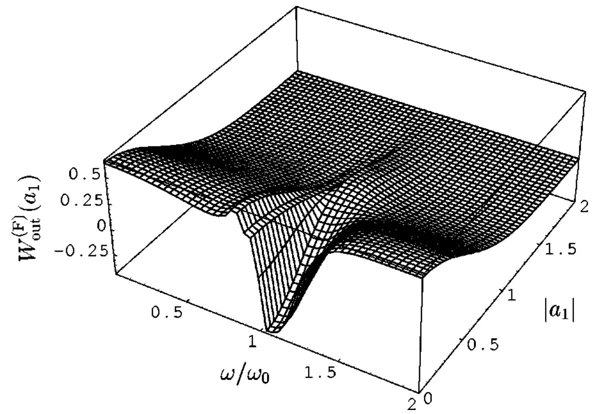

FIG. 2. The (radial-symmetric) Wigner function $W_{\rm out}^{(F)}(a_1)$ of the quantum state of the field reflected at a dielectric plate is shown as a function of frequency ω for a single-photon Fock input state and the plate data given in Fig. 1.

The phase-space function of the outgoing field is a convolution of the phase-space functions of the incoming fields and the device, the arguments being transformed by the inverse transformation matrix and the inverse absorption matrix, respectively. Note that when absorption is disregarded, i.e., in Eqs. (80) and (82) the absorption matrix approaches zero, then the well-known relation for nonabsorbing four-port devices is recognized,

$$P_{\rm out}^{(F)}(\mathbf{a};s)=P_{\rm in}^{(F)}(\mathbf{T}^{-1}\mathbf{a};s). \tag{83}$$

C. Simple examples

To illustrate the new possibilities offered, let us consider a single dielectric plate in the ground state and assume a single-resonance medium of Lorentz type whose complex permittivity can be given by

$$\epsilon=1+\frac{\epsilon_s-1}{1-(\omega/\omega_0)^2-2i\gamma\omega/\omega_0^2}. \tag{84}$$

Using the formulas given in [13], we have calculated the reflection coefficient $|T_{11}|^2$, the transmission coefficient $|T_{21}|^2$, and the absorption coefficient $(1-|T_{11}|^2-|T_{21}|^2)$. In Fig. 1 they are shown as functions of ω for $\epsilon_s=1.5$, $\gamma/\omega_0=0.01$, and plate thickness $2c/\omega_0$.

1. $|1,0,0,0\rangle$ Fock state

In the simplest case when one of the quasimonochromatic input modes is prepared in a single-photon Fock state and the other input mode and the device are prepared in the vacuum state, Eq. (51) reads as

$$|\psi_{\rm in}\rangle=|1,0,0,0\rangle. \tag{85}$$

The input Wigner function $W_{\rm in}(\boldsymbol{\alpha})\equiv P_{\rm in}(\boldsymbol{\alpha};0)$ is then given by

$$W_{\rm in}(\boldsymbol{\alpha})=-\left(\frac{2}{\pi}\right)^4 L_1(4|a_1|^2)\exp(-2|\boldsymbol{\alpha}|^2) \tag{86}$$

[$L_k(z)$, Laguerre polynomial; for the Wigner function of Fock states, see, e.g., [20]]. Applying Eq. (75) yields

$$W_{\rm out}(\boldsymbol{\alpha})=-\left(\frac{2}{\pi}\right)^4 L_1\left(4\left|\sum_{\nu=1}^{4}\Lambda_{\nu1}^{*}\alpha_\nu\right|^2\right)\exp(-2|\boldsymbol{\Lambda}^{+}\boldsymbol{\alpha}|^2), \tag{87}$$

from which the Wigner function of the outgoing fields, $W_{\rm out}^{(F)}(\mathbf{a})$, is easily obtained by integration over the device variables $\mathbf{g}$ according to Eq. (76). Further integration over a_2 (a_1) yields the Wigner function of the field in the first (second) output channel. After some algebra we obtain, on recalling the definition (22) of the unitary matrix $\boldsymbol{\Lambda}$,

$$W_{\rm out}^{(F)}(a_i)=\frac{4}{\pi}[|T_{i1}|^2(2|a_i|^2-1)+\tfrac{1}{2}]\exp(-2|a_i|^2) \tag{88}$$

$(i=1,2)$. The matrix elements T_{i1} are to be taken at the midfrequency ω of the quasi-monochromatic radiation mode considered. The states of the reflected and transmitted modes are mixtures of zero- and one-photon Fock states in general, which can be easily seen, rewriting Eq. (88) as

$$W_{\rm out}^{(F)}(a_i)=(1-|T_{i1}|^2)W_0(a_i)+|T_{i1}|^2W_1(a_i), \tag{89}$$

where $W_k(a_i)$ is the Wigner functions of the k-photon Fock state,

$$W_k(a_i)=\frac{2(-1)^k}{\pi}L_k(4|a_i|^2)\exp(-2|a_i|^2). \tag{90}$$

The dependence on frequency of the Wigner functions $W_{\rm out}^{(F)}(a_1)$ and $W_{\rm out}^{(F)}(a_2)$ of the reflected and transmitted fields are shown in Figs. 2 and 3, respectively. Note that in the case under consideration $W_{\rm out}^{(F)}(a_1)$ and $W_{\rm out}^{(F)}(a_2)$ are radial symmetric, because they only depend on the absolute values $|a_1|$ and $|a_2|$ [see Eq. (88)]. Sufficiently far from the medium resonance $(\omega/\omega_0\ll 1)$ the relation $|T_{11}|^2+|T_{21}|^2\approx 1$ holds, and the plate acts like a lossless beam splitter, with $|T_{11}|^2\approx 0$ and $|T_{21}|^2\approx 1$. In this case the reflected mode is prepared in a state close to the vacuum state and the transmitted mode is prepared in a state close to a single-photon Fock state. Near resonance $(\omega/\omega_0\approx 1)$ the transmission coefficient approaches zero, $|T_{21}|^2\approx 0$, and the transmitted mode is prepared in a state close to the vacuum. Since the

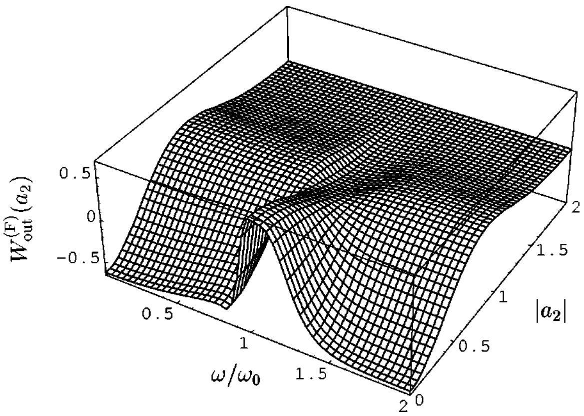

FIG. 3. The (radial-symmetric) Wigner function $W_{\rm out}^{(F)}(a_2)$ of the quantum state of the field transmitted through a dielectric plate is shown as a function of frequency ω for a single-photon Fock input state and the plate data given in Fig. 1.

reflection coefficient is less than unity owing to absorption, the reflected mode is prepared in a state that deviates from a single-photon Fock state.

2. $|n,0,0,0\rangle$ Fock state

The above given results can easily be extended to an n-photon input state such that

$$|\psi_{\rm in}\rangle=|n,0,0,0\rangle, \tag{91}$$

with the Wigner function

$$W_{\rm in}(\boldsymbol{\alpha})=(-1)^n\left(\frac{2}{\pi}\right)^4 L_n(4|a_1|^2)\exp(-2|\boldsymbol{\alpha}|^2). \tag{92}$$

The Wigner function of the mode in the ith output channel reads then

$$W_{\rm out}^{(F)}(a_i)=\sum_{k=0}^{n}\binom{n}{k}|T_{i1}|^{2k}(1-|T_{i1}|^2)^{n-k}W_k(a_i). \tag{93}$$

Needless to say, Eq. (93) holds for any phase-space function and reveals that the density operator of the mode in the ith output channel, $\hat{\varrho}_{{\rm out}\,i}^{(F)}$, can be expanded in the Fock basis as

$$\hat{\varrho}_{{\rm out}\,i}^{(F)}=\sum_{k=0}^{n}\binom{n}{k}|T_{i1}|^{2k}(1-|T_{i1}|^2)^{n-k}|k\rangle\langle k|, \tag{94}$$

which also can be obtained by applying Eqs. (65) and (66) in Sec. IV A which give

$$\hat{\varrho}_{\rm out}^{(F)}=\sum_{k=0}^{n}\binom{n}{k}v^{n-k}\frac{\hat{x}_1^{\dagger\,k}}{\sqrt{k!}}|0^{(F)}\rangle\langle 0^{(F)}|\frac{\hat{x}_1^{k}}{\sqrt{k!}}, \tag{95}$$

with

$$\hat{x}_1^{\dagger}=T_{11}\hat{a}_1^{\dagger}+T_{21}\hat{a}_2^{\dagger} \tag{96}$$

and

$$v=v_{11}=1-|T_{11}|^2-|T_{21}|^2, \tag{97}$$

or equivalently,

$$\hat{\varrho}_{\rm out}^{(F)}=\sum_{k=0}^{n}\binom{n}{k}v^{n-k}\sum_{p,q=0}^{k}\sqrt{\binom{k}{p}\binom{k}{q}}\times T_{11}^{p}T_{11}^{*\,q}T_{21}^{k-p}T_{21}^{*\,k-q}|p,k-p\rangle\langle q,k-q|. \tag{98}$$

Tracing over one output channel yields Eq. (94).

3. $|1,1,0,0\rangle$ Fock state

Finally let us consider the case when both input field modes are prepared in single-photon Fock states. In this case we have

$$|\psi_{\rm in}\rangle=|1,1,0,0\rangle. \tag{99}$$

Following the line given above, after some algebra we obtain

$$\begin{aligned}\hat{\varrho}_{\rm out}^{(F)}=&\hat{x}_1^{\dagger}\hat{x}_2^{\dagger}|0^{(F)}\rangle\langle 0^{(F)}|\hat{x}_1\hat{x}_2+v_{11}\hat{x}_2^{\dagger}|0^{(F)}\rangle\langle 0^{(F)}|\hat{x}_2\\&+v_{12}\hat{x}_1^{\dagger}|0^{(F)}\rangle\langle 0^{(F)}|\hat{x}_2+v_{21}\hat{x}_2^{\dagger}|0^{(F)}\rangle\langle 0^{(F)}|\hat{x}_1\\&+v_{22}\hat{x}_1^{\dagger}|0^{(F)}\rangle\langle 0^{(F)}|\hat{x}_1+[v_{11}v_{22}+v_{12}v_{21}]|0^{(F)}\rangle\langle 0^{(F)}|,\end{aligned} \tag{100}$$

where

$$\hat{x}_i^{\dagger}=T_{1i}\hat{a}_1^{\dagger}+T_{2i}\hat{a}_2^{\dagger}, \tag{101}$$

and v_{ij} are the elements of the matrix $\boldsymbol{v}$ of Eq. (68). After tracing over one output channel we get

$$\begin{aligned}\hat{\varrho}_{{\rm out}\,i}^{(F)}=&[1-|T_{i1}|^2(1-|T_{i2}|^2)-|T_{i2}|^2(1-|T_{i1}|^2)]|0\rangle\langle 0|\\&+(|T_{i1}|^2+|T_{i2}|^2-4|T_{i1}|^2|T_{i2}|^2)|1\rangle\langle 1|\\&+2|T_{i1}|^2|T_{i2}|^2|2\rangle\langle 2|.\end{aligned} \tag{102}$$

It should be pointed out that for a lossless 50%:50% beam splitter Eq. (102) reduces to

$$\hat{\varrho}_{{\rm out}\,i}^{(F)}=\tfrac{1}{2}(|0\rangle\langle 0|+|2\rangle\langle 2|), \tag{103}$$

and the well-known effect of photon correlation is observed. The dependence on frequency of the Wigner function $W_{\rm out}^{(F)}(a_1)$ is shown in Fig. 4. Note that $W_{\rm out}^{(F)}(a_2)=W_{\rm out}^{(F)}(a_1)$, because of the equal input states and the properties of the plate that $|T_{11}|^2=|T_{22}|^2$ and $|T_{12}|^2=|T_{21}|^2$.

Let us briefly address the degree of entanglement of the two output states. A quantitative measure of the degree of entanglement can be defined [21], employing the von Neumann entropy. The index of correlation I_c is defined by

$$I_c=S_1+S_2-S_{12}, \tag{104}$$

where S_i is the von Neumann entropy of the ith single-channel output state $\hat{\varrho}_{{\rm out}\,i}^{(F)}$ [Eq. (102)],

$$S_i=-\,{\rm Tr}[\hat{\varrho}_{{\rm out}\,i}^{(F)}\ln\hat{\varrho}_{{\rm out}\,i}^{(F)}], \tag{105}$$

and S_{12} is the entropy of the (entangled) density matrix of the two-channel output state $\hat{\varrho}_{\rm out}^{(F)}$,

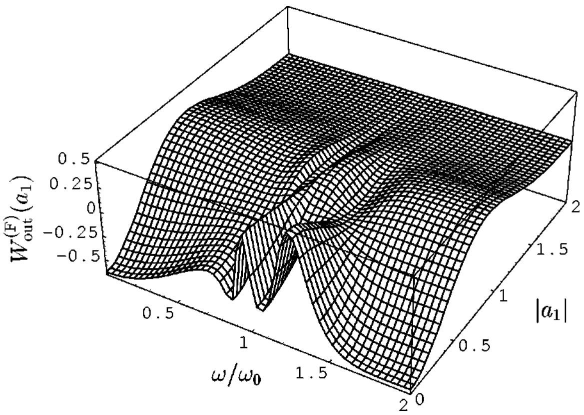

FIG. 4. The (radial-symmetric) Wigner function $W^{(F)}_{\rm out}(a_1)$ of the quantum state of the field in the first output channel of a dielectric plate is shown as a function of frequency ω for the two-photon Fock input state $|1,1,0,0\rangle$ and the plate data given in Fig. 1.

$$S_{12}=-\,{\rm Tr}[\hat{\varrho}^{(F)}_{\rm out}\ln\hat{\varrho}^{(F)}_{\rm out}]. \tag{106}$$

The index of correlation I_c is bounded from below by zero and from above by S_1+S_2. As already mentioned, in the case under consideration the relation $\hat{\varrho}^{(F)}_{\rm out\,1}=\hat{\varrho}^{(F)}_{\rm out\,2}$ is valid, so that $S_1=S_2$, and the possible maximum value of the index of correlation (for every frequency ω) is given by

$$I_c^{\rm max}=S_1+S_2=2S_1=2S_2. \tag{107}$$

Figure 5 presents the index of correlation I_c as a function of frequency. The output states are maximally entangled at about $0.68\omega_0$ and $1.35\omega_0$, whereas at the resonance frequency ω_0 they are not correlated at all because the input photons are reflected or absorbed and there is no mixing of their states by the device. Far below the resonance when absorption is small, the index of correlation approximately equals its theoretically possible maximum value (107).

V. SUMMARY

Starting from the canonical quantization of the electromagnetic field in arbitrary linear Kramers-Kronig media, we have developed the quantum theory of the action of dispersive and absorbing optical four-port devices. In particular, we have presented formulas for calculating the complete quantum state of the outgoing fields from the quantum states of the incoming fields and the device for given complex frequency-dependent refractive-index profile of the device, without any frequency restriction. The theory is a natural extension of the standard theory of lossless four-port devices where mode expansion applies.

Given the complex frequency-dependent refractive-index profile of a device, for each frequency component a U(4) group transformation matrix can be calculated that includes transmission, reflection, and absorption. The U(4) matrices can be decomposed in different ways. In particular, each matrix can be decomposed into five U(2) matrices. That is to say, for a chosen frequency the action of an absorbing four-port device formally corresponds, e.g., to the combined action of five lossless four-port devices for radiation and matter. From a more fundamental point of view, the theory can be regarded, in a sense, as justification for the concepts of replacement schemes. From the point of view of practical computations the theory enables one to exactly calculate all relevant device parameters from the complex refractive-index profile. Obviously, the introduction of replacement schemes for broadband radiation makes little sense.

In consequence of absorption the quantum state of the outgoing radiation depends on the quantum state the device is prepared in when the incoming fields impinge on the device. In combination with conditional measurement this offers novel possibilities of quantum-state manipulation. In particular, the theory enables one to study (in the linear regime) the effect of resonance frequencies on quantum-state transformation and the action on quantum fields of dielectric structures whose complex refractive-index profile shows strongly varying dispersion and absorption.

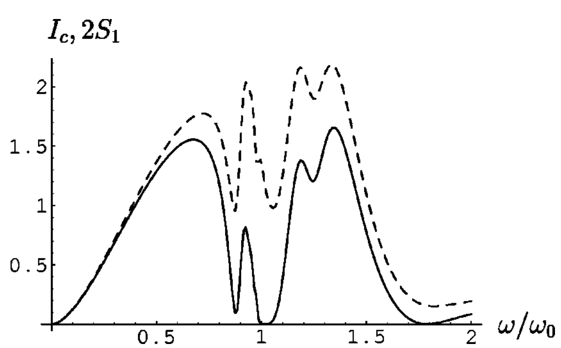

FIG. 5. Index of correlation I_c (full line) and maximal index of correlation $I_c^{\rm max}=2S_1$ (dashed line) as a function of frequency for the plate data given in Fig. 1.

ACKNOWLEDGMENTS

We acknowledge discussions with S.M. Barnett, J. Jeffers, J. Clausen, and J. Rataj. This work was supported by the Deutsche Forschungsgemeinschaft.

APPENDIX A: DERIVATION OF THE U(4) GROUP MATRIX

Let us write the sought U(4) matrix $\mathbf{\Lambda}(\omega)$ as

$$\mathbf{\Lambda}(\omega)=\begin{pmatrix}\mathbf{T}(\omega) & \mathbf{A}(\omega)\\ \mathbf{F}(\omega) & \mathbf{G}(\omega)\end{pmatrix}, \tag{A1}$$

where $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ are defined in Eqs. (7) and (8) and satisfy the relation (10). The 2×2 matrices $\mathbf{F}(\omega)$ and $\mathbf{G}(\omega)$ are to be determined such that $\mathbf{\Lambda}(\omega)$ is unitary, i.e.,

$$\mathbf{F}(\omega)\mathbf{F}^{+}(\omega)+\mathbf{G}(\omega)\mathbf{G}^{+}(\omega)=\mathbf{I}, \tag{A2}$$

$$\mathbf{F}(\omega)\mathbf{T}^{+}(\omega)+\mathbf{G}(\omega)\mathbf{A}^{+}(\omega)=\mathbf{0}. \tag{A3}$$

From Eq. (A3) we find that

$$\mathbf{F}(\omega)=-\mathbf{G}(\omega)\mathbf{A}^{+}(\omega)[\mathbf{T}^{+}(\omega)]^{-1}. \tag{A4}$$

We substitute in Eq. (A2) for $\mathbf{F}(\omega)$ the result of Eq. (A4) and derive

$$\mathbf{G}(\omega)\{\mathbf{I}+\mathbf{A}^{+}(\omega)[\mathbf{T}(\omega)\mathbf{T}^{+}(\omega)]^{-1}\mathbf{A}(\omega)\}\mathbf{G}^{+}(\omega)=\mathbf{I}, \tag{A5}$$

and hence

$$\mathbf{I}+\mathbf{A}^{+}(\omega)[\mathbf{T}(\omega)\mathbf{T}^{+}(\omega)]^{-1}\mathbf{A}(\omega)=[\mathbf{G}^{+}(\omega)\mathbf{G}(\omega)]^{-1}. \quad \text{(A6)}$$

Recalling Eq. (10), from Eq. (A6) we find that

$$\mathbf{G}^{+}(\omega)\mathbf{G}(\omega)=\mathbf{I}-\mathbf{A}^{+}(\omega)\mathbf{A}(\omega). \quad \text{(A7)}$$

A particular solution of Eq. (A7) is

$$\mathbf{G}(\omega)=\mathbf{C}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega), \quad \text{(A8)}$$

where $\mathbf{C}(\omega)$ and $\mathbf{S}(\omega)$ are defined in Eqs. (23) and (24), respectively. Obviously, the general solution reads as

$$\mathbf{G}(\omega)=\mathbf{D}(\omega)\mathbf{C}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega), \quad \text{(A9)}$$

where $\mathbf{D}$ is an arbitrary unitary 2×2 matrix. From Eq. (A4) it then follows that $\mathbf{F}(\omega)$ is given by

$$\mathbf{F}(\omega)=-\mathbf{D}(\omega)\mathbf{S}(\omega)\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega). \quad \text{(A10)}$$

Combining Eqs. (A1), (A9), and (A10), we obtain

$$\mathbf{\Lambda}(\omega)=\begin{pmatrix}\mathbf{T}(\omega) & \mathbf{A}(\omega)\\ -\mathbf{D}(\omega)\mathbf{S}(\omega)\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega) & \mathbf{D}(\omega)\mathbf{C}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega)\end{pmatrix}, \quad \text{(A11)}$$

which reveals that for given matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ the U(4) matrix $\mathbf{\Lambda}(\omega)$ is only determined up to a U(2) matrix $\mathbf{D}(\omega)$.

APPENDIX B: FACTORIZATION OF THE U(4) GROUP TRANSFORMATION

The U(4) matrix $\mathbf{\Lambda}(\omega)$ in Eq. (A11) can be rewritten as a product of three U(4) matrices as follows:

$$\mathbf{\Lambda}(\omega)=\mathbf{\Lambda}_3(\omega)\,\mathbf{\Lambda}_2(\omega)\,\mathbf{\Lambda}_1(\omega), \quad \text{(B1)}$$

where

$$\mathbf{\Lambda}_1(\omega)=\begin{pmatrix}\mathbf{D}(\omega)\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{D}(\omega)\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega)\end{pmatrix}, \quad \text{(B2)}$$

$$\mathbf{\Lambda}_2(\omega)=\begin{pmatrix}\mathbf{D}(\omega)\mathbf{C}(\omega)\mathbf{D}^{+}(\omega) & \mathbf{D}(\omega)\mathbf{S}(\omega)\mathbf{D}^{+}(\omega)\\ -\mathbf{D}(\omega)\mathbf{S}(\omega)\mathbf{D}^{+}(\omega) & \mathbf{D}(\omega)\mathbf{C}(\omega)\mathbf{D}^{+}(\omega)\end{pmatrix}, \quad \text{(B3)}$$

$$\mathbf{\Lambda}_3(\omega)=\begin{pmatrix}\mathbf{D}^{+}(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{pmatrix}. \quad \text{(B4)}$$

When we choose the matrix $\mathbf{D}(\omega)$ such that the matrices $\mathbf{D}(\omega)\mathbf{C}(\omega)\mathbf{D}^{+}(\omega)$ and $\mathbf{D}(\omega)\mathbf{S}(\omega)\mathbf{D}^{+}(\omega)$ become diagonal matrices [note that $\mathbf{C}(\omega)$ and $\mathbf{S}(\omega)$ defined in Eqs. (23) and (24), respectively, can be diagonalized by the same unitary matrix], then the U(4) group transformation corresponds to five U(2) group transformations.

Let $\mathbf{D}(\omega)$ be the unit matrix, $\mathbf{D}(\omega)=\mathbf{I}$. In this case Eq. (B1) reduces to

$$\mathbf{\Lambda}(\omega)=\mathbf{\Lambda}_2(\omega)\,\mathbf{\Lambda}_1(\omega), \quad \text{(B5)}$$

where now

$$\mathbf{\Lambda}_1(\omega)=\begin{pmatrix}\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{S}^{-1}(\omega)\mathbf{A}(\omega)\end{pmatrix} \quad \text{(B6)}$$

and

$$\mathbf{\Lambda}_2(\omega)=\begin{pmatrix}\mathbf{C}(\omega) & \mathbf{S}(\omega)\\ -\mathbf{S}(\omega) & \mathbf{C}(\omega)\end{pmatrix}. \quad \text{(B7)}$$

The matrix $\mathbf{\Lambda}_2(\omega)$ can be given by the unitary transform of a quasidiagonal matrix $\mathbf{\Lambda}_2'(\omega)$,

$$\mathbf{\Lambda}_2(\omega)=\mathbf{Y}^{+}\mathbf{\Lambda}_2'(\omega)\mathbf{Y}, \quad \text{(B8)}$$

where

$$\mathbf{\Lambda}_2'(\omega)=\begin{pmatrix}\mathbf{C}(\omega)-i\mathbf{S}(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{C}(\omega)+i\mathbf{S}(\omega)\end{pmatrix} \quad \text{(B9)}$$

and

$$\mathbf{Y}=\frac{1}{\sqrt{2}}\begin{pmatrix}\mathbf{I} & i\mathbf{I}\\ i\mathbf{I} & \mathbf{I}\end{pmatrix}. \quad \text{(B10)}$$

Combining Eqs. (B5) and (B8), we obtain

$$\mathbf{\Lambda}(\omega)=\mathbf{Y}^{+}\mathbf{\Lambda}_2'(\omega)\mathbf{Y}\mathbf{\Lambda}_1(\omega), \quad \text{(B11)}$$

which corresponds to a decomposition of the U(4) group transformation into eight U(2) group transformations.

Using Eqs. (20) and (B5) and recalling Eqs. (26)–(28), we may write

$$\begin{aligned}\hat{\boldsymbol{\beta}}(\omega)&=\mathbf{\Lambda}_2(\omega)\,\mathbf{\Lambda}_1(\omega)\,\hat{\boldsymbol{\alpha}}(\omega)=\mathbf{\Lambda}_2(\omega)\,\hat{U}_1^{\dagger}\hat{\boldsymbol{\alpha}}(\omega)\hat{U}_1\\ &=\hat{U}_1^{\dagger}\,\mathbf{\Lambda}_2(\omega)\,\hat{\boldsymbol{\alpha}}(\omega)\,\hat{U}_1=\hat{U}_1^{\dagger}\,\hat{U}_2^{\dagger}\,\hat{\boldsymbol{\alpha}}(\omega)\,\hat{U}_2\hat{U}_1\\ &=\hat{U}^{\dagger}\,\hat{\boldsymbol{\alpha}}(\omega)\,\hat{U}, \end{aligned}\quad \text{(B12)}$$

with

$$\hat{U}\equiv\hat{U}[\mathbf{\Lambda};\hat{\boldsymbol{\alpha}}]=\hat{U}[\mathbf{\Lambda}_2;\hat{\boldsymbol{\alpha}}]\hat{U}[\mathbf{\Lambda}_1;\hat{\boldsymbol{\alpha}}]. \quad \text{(B13)}$$

Here, $\hat{U}_i\equiv\hat{U}[\mathbf{\Lambda}_i;\hat{\boldsymbol{\alpha}}]$ $(i=1,2)$ is given by Eq. (27), with $\mathbf{\Phi}_i(\omega)$ in place of $\mathbf{\Phi}(\omega)$, and

$$\exp[-i\mathbf{\Phi}_i(\omega)]=\mathbf{\Lambda}_i(\omega). \quad \text{(B14)}$$

From the quasidiagonal structure of $\mathbf{\Lambda}_1(\omega)$, Eq. (B6), it then follows that

$$\mathbf{\Phi}_1(\omega)=\begin{pmatrix}\mathbf{W}_1(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}+\begin{pmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{W}_2(\omega)\end{pmatrix}, \tag{B15}$$

where

$$\exp[-i\mathbf{W}_1(\omega)]=\mathbf{C}^{-1}(\omega)\mathbf{T}(\omega) \tag{B16}$$

and

$$\exp[-i\mathbf{W}_2(\omega)]=\mathbf{S}^{-1}(\omega)\mathbf{A}(\omega). \tag{B17}$$

Thus, $\hat{U}[\mathbf{\Lambda}_1;\hat{\boldsymbol{\alpha}}]$ can be expressed in terms of two unitary operators of the type given in Eq. (44) [together with Eq. (45)],

$$\hat{U}[\mathbf{\Lambda}_1;\hat{\boldsymbol{\alpha}}]=\hat{U}[\mathbf{S}^{-1}\mathbf{A};\hat{\mathbf{g}}]\,\hat{U}[\mathbf{C}^{-1}\mathbf{T};\hat{\mathbf{a}}]. \tag{B18}$$

To decompose $\hat{U}[\mathbf{\Lambda}_2;\hat{\boldsymbol{\alpha}}]$, we note that Eq. (B8) implies that

$$\hat{U}[\mathbf{\Lambda}_2;\hat{\boldsymbol{\alpha}}]=\hat{U}[\mathbf{\Lambda}_2';\mathbf{Y}(\omega)\hat{\boldsymbol{\alpha}}], \tag{B19}$$

where

$$\exp[-i\mathbf{\Phi}_2'(\omega)]=\mathbf{\Lambda}_2'(\omega) \tag{B20}$$

and

$$\mathbf{Y}(\omega)\hat{\boldsymbol{\alpha}}(\omega)=\frac{1}{\sqrt{2}}\begin{pmatrix}\hat{\mathbf{a}}(\omega)+i\hat{\mathbf{g}}(\omega)\\ i\hat{\mathbf{a}}(\omega)+\hat{\mathbf{g}}(\omega)\end{pmatrix}. \tag{B21}$$

The quasidiagonal structure of $\mathbf{\Lambda}_2'(\omega)$, Eq. (B9), enables us to write

$$\mathbf{\Phi}_2'(\omega)=\begin{pmatrix}\mathbf{W}_3(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}+\begin{pmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{W}_3(\omega)\end{pmatrix}, \tag{B22}$$

where

$$\exp[-i\mathbf{W}_3(\omega)]=\mathbf{C}(\omega)-i\mathbf{S}(\omega), \tag{B23}$$

so that $\hat{U}[\mathbf{\Lambda}_2;\hat{\boldsymbol{\alpha}}]$ can also be expressed in terms of two unitary operators of the type given in Eq. (44) [together with Eq. (45)],

$$\hat{U}[\mathbf{\Lambda}_2;\hat{\boldsymbol{\alpha}}]=\hat{U}[\mathbf{C}+i\mathbf{S};(i\hat{\mathbf{a}}+\hat{\mathbf{g}})/\sqrt{2}]\hat{U}[\mathbf{C}-i\mathbf{S};(\hat{\mathbf{a}}+i\hat{\mathbf{g}})/\sqrt{2}]. \tag{B24}$$

Recalling the definitions of $\hat{\mathbf{d}}_j(\omega)$, Eq. (48), and $\mathbf{P}$, Eq. (49), it is seen that $(\hat{\mathbf{a}}+i\hat{\mathbf{g}})/\sqrt{2}$ and $(i\hat{\mathbf{a}}+\hat{\mathbf{g}})/\sqrt{2}$ can be given by

$$\frac{1}{\sqrt{2}}\begin{pmatrix}\hat{a}_j(\omega)+i\hat{g}_j(\omega)\\ i\hat{a}_j(\omega)+\hat{g}_j(\omega)\end{pmatrix}=\mathbf{P}\,\hat{\mathbf{d}}_j(\omega)=\hat{U}^{\dagger}[\mathbf{P};\hat{\mathbf{d}}_j]\,\hat{\mathbf{d}}_j(\omega)\,\hat{U}[\mathbf{P};\hat{\mathbf{d}}_j] \tag{B25}$$

$(j=1,2)$, and hence

$$\hat{U}[\mathbf{C}-i\mathbf{S};(\hat{\mathbf{a}}+i\hat{\mathbf{g}})/\sqrt{2}]=\hat{U}^{\dagger}[\mathbf{P};\hat{\mathbf{d}}_1]\,\hat{U}^{\dagger}[\mathbf{P};\hat{\mathbf{d}}_2]\,\hat{U}[\mathbf{C}-i\mathbf{S};\hat{\mathbf{a}}]\,\hat{U}[\mathbf{P};\hat{\mathbf{d}}_2]\,\hat{U}[\mathbf{P};\hat{\mathbf{d}}_1], \tag{B26}$$

$$\hat{U}[\mathbf{C}+i\mathbf{S};(i\hat{\mathbf{a}}+\hat{\mathbf{g}})/\sqrt{2}]=\hat{U}^{\dagger}[\mathbf{P};\hat{\mathbf{d}}_1]\,\hat{U}^{\dagger}[\mathbf{P};\hat{\mathbf{d}}_2]\,\hat{U}[\mathbf{C}+i\mathbf{S};\hat{\mathbf{g}}]\,\hat{U}[\mathbf{P};\hat{\mathbf{d}}_2]\,\hat{U}[\mathbf{P};\hat{\mathbf{d}}_1]. \tag{B27}$$

---

[1] B. Yurke, S.L. McCall, and J.R. Klauder, Phys. Rev. A **33**, 4033 (1986).

[2] S. Prasad, M.O. Scully, and W. Martienssen, Opt. Commun. **62**, 139 (1987).

[3] Z.Y. Ou, C.K. Hong, and L. Mandel, Opt. Commun. **63**, 118 (1987).

[4] H. Fearn and R. Loudon, Opt. Commun. **64**, 485 (1987).

[5] R.A. Campos, B.E.A. Saleh, and M.C. Teich, Phys. Rev. A **40**, 1371 (1989).

[6] U. Leonhardt, Phys. Rev. A **48**, 3265 (1993).

[7] A. Luis and L.L. Sánchez-Soto, Quantum Semiclassic. Opt. **7**, 153 (1995).

[8] T. Gruner and D.-G. Welsch, in *Quantum Field Theory under the Influence of External Conditions*, edited by M. Bordag (Teubner Verlagsgesellschaft, Leipzig, 1996); Phys. Rev. A **53**, 1818 (1996).

[9] R. Matloob, R. Loudon, S.M. Barnett, and J. Jeffers, Phys. Rev. A **52**, 4823 (1995).

[10] R. Matloob and R. Loudon, Phys. Rev. A **53**, 4567 (1996).

[11] H.T. Dung, L. Knöll, and D.-G. Welsch, Phys. Rev. A **57**, 3931 (1998).

[12] S. Scheel, L. Knöll, and D.-G. Welsch, Phys. Rev. A **58**, 700 (1998).

[13] T. Gruner and D.-G. Welsch, Phys. Rev. A **54**, 1661 (1996).

[14] T. Gruner and D.-G. Welsch, Opt. Commun. **134**, 447 (1997).

[15] S.M. Barnett, C.R. Gilson, B. Huttner, and N. Imoto, Phys. Rev. Lett. **77**, 1739 (1996).

[16] S.M. Barnett, J. Jeffers, A. Gatti, and R. Loudon, Phys. Rev. A **57**, 2134 (1998).

[17] J.F. Cornwell, *Group Theory in Physics* (Academic, London, 1984).

[18] M. Reck, A. Zeilinger, H.J. Bernstein, and P. Bertani, Phys. Rev. Lett. **73**, 58 (1994).

[19] W. Vogel and D.-G. Welsch, *Lectures on Quantum Optics* (Akademie Verlag, Berlin, 1994).

[20] C.W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991).

[21] S.M. Barnett and S.J.D. Phoenix, Phys. Rev. A **40**, 2404 (1989).