Creating macroscopic quantum superpositions with Bose-Einstein condensates

D. Gordon* and C. M. Savage
Department of Physics and Theoretical Physics, The Australian National University, Australian Capital Territory 0200, Australia
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We use a simple two-mode model to investigate the quantum state dynamics of a two-species Bose-Einsten condensate, such as that produced in recent experiments, undergoing weak Josephson coupling. We find that in certain parameter regimes the quantum state of the system evolves into a macroscopic superposition of two states which differ in the atom number difference between the two species. The ''size'' of the macroscopic superposition created by such a method can be varied by adjusting the Josephson coupling coefficient, and is found to be near maximal for a certain critical value of this coefficient. [S1050-2947(99)01906-X]

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I. INTRODUCTION

The question of the compatibility of quantum mechanics with macroscopic realism is one of the important, as yet unanswered, philosophical questions posed by quantum mechanics. Macroscopic realism asserts that a system with several macroscopically distinguishable states available to it will always be in one of these states. This is incompatible with quantum mechanics, which permits superpositions of different states [1]. If we are to believe in macroscopic realism, then we must modify quantum mechanics in such a way that superpositions will exist only on a microscopic level. For example, Penrose [2] and others have suggested that such a modification might involve a quantum theory of gravity. Such theories should be testable, in the sense that the production of macroscopic or mesoscopic superposition states could put limits on the regime of state vector reduction.

Recently there have been several experiments performed in which two-species Bose-Einstein condensates (BECs) are created and manipulated [3–6]. These experiments have involved two-species BECs in which the two species consist of two hyperfine sublevels of ^{87}Rb—the $|F,m_f\rangle=|1,-1\rangle$ and $|2,2\rangle$ sublevels in the case of [3] and the $|1,-1\rangle$ and $|2,1\rangle$ sublevels in the case of [4–6]. Such a configuration has the advantage that the two species can be coupled to one another via a Josephson-type coupling realized by a multiphoton transition, allowing a rich variety of experiments to be performed.

At the time of writing, there are at least two proposals for producing macroscopic superpositions or Schrödinger cat states using such systems. Cirac *et al.* [7] have shown that if the two species are Josephson coupled, then in certain parameter regimes the ground state of the Hamiltonian is a superposition of two states involving a particle number imbalance between the two species. Such a state represents a superposition of two states which are macroscopically (or mesoscopically) distinguishable, and hence can be called a Schrödinger cat state. Using the scheme described in [7], the production of such a state would involve the adiabatic transfer of the double condensate to the ground state of the Josephson coupling Hamiltonian. Ruostekoski *et al.* [8] have also shown that such states can be created by a mechanism involving the coherent scattering of far-detuned light fields. Their model neglects the collisional interactions between particles. One might also put forward a scheme analogous to the Yurke-Stoler scheme of quantum optics [9]. The major drawback of such a scheme is that the time needed to evolve to a cat state can be shown to be rather long, and thus problems due to decoherence would be greatly increased.

In this paper, we use a two-mode model to investigate the quantum state dynamics of such a system, including both interspecies and intraspecies two-body collisions between atoms. We show that the interplay between the atom-atom collisions and the Josephson coupling can lead to evolution which results in macroscopic superposition states of the type discussed in Cirac *et al.* [7] and Ruostekoski *et al.* [8]. The ''size'' of the Schrödinger cat can be adjusted by changing the strength of the Josephson coupling. Our scheme for producing cat states differs from [7] in that the macroscopic superposition is produced by the normal dynamic evolution of the system rather than by adiabatic transfer to the ground state of the Josephson coupling Hamiltonian. Indeed, in the parameter regimes we investigate, the ground state shows squeezing in the relative particle number and is thus certainly not a Schrödinger cat state [10].

Our scheme may be compared to a typical experiment in quantum optics, where a coherent light beam is passed through a nonlinear crystal. During the time it takes for one photon to traverse the length of a crystal, the quantum state of the beam is modified by the nonlinearity—for example, squeezing or second-harmonic generation could take place. In the situation which we describe here, the nonlinearity is provided by the various collisional interactions and the Josephson coupling. We envisage preparing the initial state and then turning on the Josephson coupling for some amount of time. The period during which the Josephson coupling is active is analogous to the time spent by the light field in the crystal; after the Josephson coupling is turned off, the quantum state of the system will have been modified. In the example here, the end result will be a Schrödinger cat state.

Although the production of such states would no doubt involve considerable experimental difficulty, we believe that it would be worthwhile, since the investigation of the boundary between the quantum (microscopic) world and the every-

*Electronic address: Dan.Gordon@anu.edu.au

day macroscopic world is sure to provide fertile ground for new discoveries.

II. THE MODEL

The two-mode model is derived from the multimode Hamiltonian as follows: We label the two species of atom as *A* and *B*. The condensate modes for species *A* and *B* are those wave functions which satisfy the coupled two-species Gross-Pitaevskii equations or Hartree-Fock equations, i.e., they are those spatial modes for species *A* and *B* which are macroscopically occupied. We also include a set of noncondensate modes such that we have a complete orthonormal basis $|\phi_i\rangle$ (such modes could be determined by the Hartree-Fock equations [11]).

In such a basis, the second quantized Hamiltonian is

$$H=\sum_{i,j} H^A_{ij}\hat{a}^\dagger_i\hat{a}_j+H^B_{ij}\hat{b}^\dagger_i\hat{b}_j+\frac{1}{2}\lambda_{ij}(\hat{a}^\dagger_i\hat{b}_j+\hat{b}^\dagger_j\hat{a}_i)+\sum_{i,j,k,l}\frac{1}{2}W^{AA}_{ijkl}\hat{a}^\dagger_i\hat{a}^\dagger_j\hat{a}_k\hat{a}_l+\frac{1}{2}W^{BB}_{ijkl}\hat{b}^\dagger_i\hat{b}^\dagger_j\hat{b}_k\hat{b}_l+W^{AB}_{ijkl}\hat{a}^\dagger_i\hat{b}^\dagger_j\hat{a}_k\hat{b}_l. \tag{1}$$

The $H^{A(B)}_{ij}$ are the matrix elements of the single-particle Hamiltonian in our basis:

$$H^{A(B)}_{ij}=\langle\phi_i|\hat{\mathbf{p}}^2/(2m)+V^{A(B)}(\mathbf{r})|\phi_j\rangle. \tag{2}$$

The λ_{ij} are the matrix elements describing the Josephson coupling between the two species, which for a position-independent coupling is defined by

$$\lambda_{ij}=\Lambda\int d^3\mathbf{r}\,\phi^A_i(\mathbf{r})\phi^B_j(\mathbf{r}), \tag{3}$$

where Λ describes the strength of the coupling and the ϕ's are taken to be real. The W^{pq}_{ijkl} are the matrix elements of the two-body potentials describing collisions between an atom of species *p* and an atom of species *q*, where *p* and *q* stand for *A* or *B*:

$$W^{pq}_{ijkl}=U^{pq}_0\int d^3\mathbf{r}\,\phi^p_i(\mathbf{r})\phi^q_j(\mathbf{r})\phi^p_k(\mathbf{r})\phi^q_l(\mathbf{r}). \tag{4}$$

The U^{pq}_0 are the two-body interaction parameters:

$$U^{pq}_0=4\pi a_{pq}\hbar^2/m, \tag{5}$$

where a_{pq} is the scattering length for a two-body collision between an atom of species *p* and an atom of species *q*, and *m* is the mass of the atom.

The two-mode approximation consists in neglecting all modes except the condensate modes. At zero temperature, this amounts to ignoring the atoms which have left the condensate mode due to the two-body potentials. The validity of this approximation is discussed at the end of Sec. IV.

Under this approximation the Hamiltonian becomes

$$\hat{H}=E_A\hat{a}^\dagger\hat{a}+E_B\hat{b}^\dagger\hat{b}+\frac{1}{2}\lambda(\hat{a}^\dagger\hat{b}+\hat{b}^\dagger\hat{a})+\frac{W_{AA}}{2}\hat{a}^{\dagger 2}\hat{a}^2+\frac{W_{BB}}{2}\hat{b}^{\dagger 2}\hat{b}^2+W_{AB}\hat{a}^\dagger\hat{b}^\dagger\hat{a}\hat{b}, \tag{6}$$

where $E_{A(B)}=H^{A(B)}_{00}$, $W_{pq}=W^{pq}_{0000}$, $\lambda=\lambda_{00}$, and the condensate modes have the index 0. This is the same as the two-mode system considered by Cirac *et al.* [7], who then went on to investigate some of the effects of including the full multimode structure; in the dynamic regime considered here we shall content ourselves with this simpler form of the Hamiltonian.

For example, if we consider the case where there are an equal number of particles in each species, equal cylindrically symmetric harmonic trapping potentials for each species, and scattering lengths satisfying $a_{AA}=a_{BB}$ and $a_{AB}\leqslant a_{AA},a_{BB}$, then we find that the ground-state solution to these equations has $\phi^A_0(\mathbf{r})=\phi^B_0(\mathbf{r})$, and thus we can easily solve the coupled Gross-Pitaevskii equations for the condensate wave function in the Thomas-Fermi limit. Doing so yields

$$W_{pq}=\frac{2\times 15^{2/5}}{7}\left(\frac{m\omega_\perp^6\lambda_a^2\hbar^4}{N^3(a+a_{AB})^3}\right)^{1/5}a_{pq}, \tag{7}$$

where we have set $a_{AA}=a_{BB}=a$, $\omega_\perp$ is the trap angular frequency of the trap perpendicular to the axis of cylindrical symmetry, and λ_a is the trap anisotropy, i.e., the ratio $\omega_\parallel/\omega_\perp$, where $\omega_\parallel$ is the angular frequency parallel to the axis of cylindrical symmetry.

Angular-momentum basis

We find it convenient to use operators satisfying the usual angular-momentum commutation relations:

$$\hat{J}_x=\frac{1}{2}(\hat{b}^\dagger\hat{a}+\hat{a}^\dagger\hat{b}),\quad \hat{J}_y=\frac{i}{2}(\hat{b}^\dagger\hat{a}-\hat{a}^\dagger\hat{b}),\quad \hat{J}_z=\frac{1}{2}(\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b}), \tag{8}$$

for which the Casimir invariant is $\hat{J}^2=\hat{N}(\hat{N}+1/2)$, where $\hat{N}=(1/2)(\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b})$. Note that for the Hamiltonian (6), $\hat{N}$ is a constant of motion. We shall limit ourselves to work with eigenstates of this operator. Hence we make the substitution $\hat{N}\to N$ in what follows. We use as a basis for the state of the system the eigenstates of the operator $\hat{J}_z$. Such a restricted basis contains $2N+1$ basis vectors defined by

$$\hat{J}_z|m\rangle=m|m\rangle, \tag{9}$$

where *m* runs from $-N$ to *N*.

In terms of the operators (8) and neglecting terms which simply describe a shift in the zero point of the energy, the Hamiltonian (6) is

$$H=[E_A-E_B+(N-\tfrac{1}{2})(W_{AA}-W_{BB})]\hat{J}_z +(W-W_{AB})\hat{J}_z^2+\lambda\hat{J}_x, \quad (10)$$

where we have defined $W=(W_{AA}+W_{BB})/2$. We can see from the structure of this Hamiltonian that we lose no generality by considering the case $W_{AA}=W_{BB}=W$, since the term proportional to $W_{AA}-W_{BB}$ can be compensated for by changing the values of E_A and E_B. In the case where $E_A-E_B+(N-1/2)(W_{AA}-W_{BB})=0$, which holds if both traps are identical and the two species have equal intraspecies scattering lengths, the first term in the Hamiltonian is zero and the expression is formally the same as the Hamiltonian considered by Milburn *et al.* [12] in their analysis of the double-well system. Note, however, the parameter dependence of the second term, which can be close to zero for realistic experimental parameters [6]. In such a case the decay time of the relative phase between the two species approaches infinity, an effect noted by Law *et al.* [13] with a macroscopic model of collapse and revival and by Villain *et al.* [14] with a fully quantum-field model. The system will then exhibit purely sinusoidal Rabi-type Josephson oscillations.

In our chosen basis we have the following:

$$\hat{J}_x|m\rangle=\frac{1}{2}[\sqrt{(N-m)(N+m+1)}|m+1\rangle + \sqrt{(N+m)(N-m+1)}|m-1\rangle],$$
$$\hat{J}_y|m\rangle=\frac{i}{2}[\sqrt{(N+m)(N-m+1)}|m-1\rangle - \sqrt{(N-m)(N+m+1)}|m+1\rangle], \quad (11)$$
$$\hat{J}_z|m\rangle=m|m\rangle,$$

so that it can be seen that the matrix representing H is tridiagonal. In the limit of zero Josephson coupling, H is diagonal so that the unitary evolution matrix can immediately be written down, and the dynamics solved. For nonzero Josephson coupling we need to first diagonalize a real tridiagonal matrix, which is in general the simplest kind of diagonalization to perform numerically. The time needed to perform such a diagonalization scales as N^3, and we have solved for values of N up to a few thousand.

III. QUANTUM STATE DYNAMICS

Before considering the creation of Schrödinger cat states, it is interesting to consider the evolution of some basic expectation values. We shall consider in turn the limits of zero Josephson coupling, small Josephson coupling, and large Josephson coupling. For simplicity we shall also assume that the two condensates are in the same trap and that the intraspecies scattering lengths are equal, so that we have $E_A=E_B$ and $W_{AA}=W_{BB}=W$.

A. Initial state

In current experiments, the two-species BEC is created by coupling two hyperfine sublevels with electromagnetic fields [3–6]. If we start with all $2N$ atoms in state A and apply a strong $\pi/2$ pulse (strong in the sense that the pulse duration is smaller than the time scale characterizing the dynamics of the system), then we will end up with the state

$$|\psi\rangle=2^{-N}\sum_{m=-N}^{N}\exp[i(N+m)\phi]\sqrt{\binom{2N}{N-m}}|m\rangle, \quad (12)$$

where $\phi=\pi/2$ for the situation described here [15].

This state is a particular case of an *atomic coherent state*, or *Bloch state* [12,16]. It describes a state with a well defined relative phase ϕ between the two species.

In what follows, we will need to consider a range of values of ϕ—in particular, the $\phi=0$ state will be shown to evolve into a Schrödinger cat state. One way of varying ϕ would be to apply a strong pulse (such as a highly detuned intense light field) to one or both of the species, in order to shift the zero point of their energy. Providing that the pulse interacts with each species with different coupling strengths, this will result in fast phase evolution which will change the relative phase between the two species.

Another scheme for changing the relative phase between the two components involves shifting the phase of the light fields providing the Josephson coupling. We imagine applying the $\pi/2$ pulse to create the initial state, and then switching to Josephson coupling beams with a different phase than that of the original $\pi/2$ pulse. If we remain in our original basis, then this change of phase will show up as complex matrix elements for the Josephson coupling term:

$$H_{\text{JOS}}=\frac{1}{2}[\lambda\exp(i\delta\phi)\hat{b}^\dagger\hat{a}+\lambda\exp(-i\delta\phi)\hat{a}^\dagger\hat{b}]=\lambda\hat{J}_x', \quad (13)$$

where $\hat{J}_x'=(1/2)[\exp(i\delta\phi)\hat{b}^\dagger\hat{a}+\exp(-i\delta\phi)\hat{a}^\dagger\hat{b}]$. This operator can be obtained from J_x by a unitary transformation corresponding to a phase rotation of $\delta\phi$. Under this same unitary transformation, J_z remains unchanged. Thus, if we apply this unitary transformation to the new Hamiltonian which has complex Josephson coupling matrix elements, we will end up with a Hamiltonian which again looks like the original Hamiltonian (10). However, in making our change of basis we must also apply this unitary transformation to the state vector; doing so is found to rotate the relative phase between the two species by an amount $\delta\phi$. Thus, in summary, if we change the phase of our Josephson coupling beams immediately following the initial $\pi/2$ pulse, and then make a certain unitary change of basis, then the system will be unchanged except that the relative phase between the two species will have been rotated.

B. Zero Josephson coupling

In this case, as has been remarked, the Hamiltonian is diagonal and is given by

$$H=(W-W_{AB})\hat{J}_z^2. \quad (14)$$

The dynamics can be immediately solved and is given by

$$\langle m|\psi\rangle = \exp\left(-\frac{i}{\hbar}(W-W_{AB})m^2 t\right). \tag{15}$$

For the initial state (12) with $\phi=0$, the expectation value of the operator $\hat{a}^\dagger\hat{b}=\hat{J}_x+i\hat{J}_y$ is given by

$$\langle(\hat{a}^\dagger\hat{b})(t)\rangle = 2^{-2N}\sum_{m=-N}^{N-1}(N-m)\binom{2N}{N-m} \times\exp\left(-\frac{2i}{\hbar}(W-W_{AB})m\ \ t\right). \tag{16}$$

The real part of this equation gives the expectation value of $\hat{J}_x$ and the imaginary part gives the expectation value of $\hat{J}_y$. The operator $\hat{a}^\dagger\hat{b}$ can be considered to describe the relative phase between the two condensates: for large N, the state (12) is approximately an eigenstate of this operator with eigenvalue $N\exp(-iN\phi)$, and for a product of two coherent states in species A and B, the expectation value of $\hat{a}^\dagger\hat{b}$ is precisely this value. We can see from the expression (16) that its expectation value for the state (12) evolving under the Hamiltonian (14) consists of a sum of sinusoids with periods $T_m=\hbar\pi/(m[W-W_{AB}])$. Such a system exhibits collapse and revival of the relative phase between the two components. As in the single condensate calculation of [17], the collapse time can be calculated by considering the spread in frequency over the spread in relative particle number of the state, and for the case of Eq. (16) is given by

$$T_C \approx \frac{2\pi\hbar}{|W-W_{AB}|\sqrt{2N}}. \tag{17}$$

It is apparent that the collapse time will go to infinity in the case $W=W_{AB}$, and indeed the system will exhibit no dynamics other than the normal rotation of the phase of the entire wave function. This effect has been noted by Law *et al.* [13] and Villain *et al.* [14].

C. The effect of Josephson coupling

The atom number dynamics of the system including a term describing Josephson coupling are identical to those described by Milburn *et al.* [12], since under the conditions $E_A=E_B$ and $W_{AA}=W_{AB}=W$, the Hamiltonian (10) is formally the same as that considered by these authors. Milburn *et al.* [12] consider the evolution of $\langle J_z\rangle$ (i.e., the particle number difference between the species) from an initial state in which all the atoms are of the same species. In terms of the notation and physical system used in this paper, they find that there exists a critical value of the Josephson coupling parameter λ given by

$$\lambda_C = \frac{N}{2}(W-W_{AB}). \tag{18}$$

For $\lambda \gg \lambda_C$, the system oscillates in a Rabi-type manner between all particles being species A and all particles being species B. These oscillations are eventually ''damped'' by the phase diffusion in the system, and also exhibit revivals due to the finite particle number.

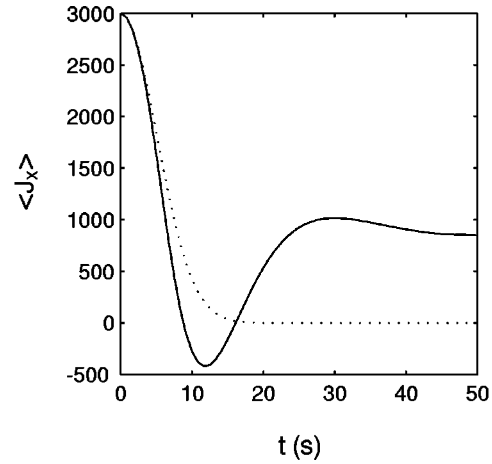

FIG. 1. Expectation value of J_x for an initial state with $\phi=0$ and weak Josephson coupling. Parameters are $N=3000$ (6000 atoms total), $W-W_{AB}=2.6\times10^{-3}\hbar$ m^{-1}, giving $\lambda_C=3.9\hbar$ s^{-1}. The parameter $W-W_{AB}$ was chosen to be consistent with experimental values for ^{87}Rb, except that we have chosen to scale the trap frequencies such that the collapse time for the relative phase, which could be considered to give a time scale for the phase dynamics, is equivalent to that of a total of 5×10^5 atoms in the trap as used in recent experiments. The solid line shows the case of $\lambda=2\times10^{-3}\hbar$ s^{-1} and the dotted line shows the zero Josephson coupling case $(\lambda=0)$. It can be seen that, in the former case, the relative phase initially follows the standard (approximately Gaussian) collapse, but instead of staying at zero until some long time as would be the case for no Josephson coupling, it is immediately partially revived.

For $\lambda<\lambda_C$, the particle number difference oscillates, but incompletely, so that for λ appreciably less than λ_C most of the particles remain in the initial species for all times. This effect describes a kind of self-trapping, and is due to collisional terms in the Hamiltonian. Smerzi *et al.* [18] have found the same effect in their mean-field calculations, which neglect the quantum statistical effects in the system, but take into account the spatial variation of the wave function.

It is interesting to note that the presence of even weak Josephson coupling can have a large effect on the phase dynamics of the system, even when the expectation value of J_z remains constant or nearly constant. This is illustrated for an initial state (16) with $\phi=0$ in Fig. 1. Such a state is not expected to exhibit Josephson oscillations due to the zero phase difference between the two species, however the state vector is affected by the Josephson coupling, greatly modifying the picture of collapse and revival.

IV. MACROSCOPIC SUPERPOSITION STATES

In the following section we shall show that in certain parameter regimes the system considered here will dynamically evolve into states which are a superposition of two macroscopically (or mesoscopically) distinguishable states — so-called ''Schrödinger's cat states.'' As in the cases discussed by Cirac *et al.* [7] and Ruostekoski *et al.* [8], these states consist of superpositions of two states which differ in average relative particle number. Cirac *et al.* [7] have shown that for $W_{AB}>W$ and certain strengths of the Josephson coupling parameter, such states can arise as the ground state of the Hamiltonian. Steel and Collett [10] demonstrate this result from a slightly different viewpoint. Ruostekoski *et al.* [8] have considered the dynamic production of Schrödinger's cat states in a pair of free condensates by a mechanism involving the stimulated scattering of light between the two components, where the atom-atom collisions are ignored. In this paper, we show that similar states can arise from the unitary evolution under the Hamiltonian (10) due to the interplay between the Josephson coupling and the atom-atom collisions. The production of these states does not require the condition that $W_{AB}>W$ as in [7]. Properties of the states, such as the ''size'' of the Schrödinger's cat or the degree of number squeezing, can be controlled by varying the strength of the Josephson coupling parameter. Once the system has evolved to such a state, the Josephson coupling can be switched off, effectively ''freezing'' the evolution of the number distribution.

Up until now we have looked at expectation values of relevant quantities. In what follows, we shall instead concentrate on the dynamics of the state vector evolving under the Hamiltonian (10) and for an initial state (12). Figure 2(a) shows the result for $\lambda \gg \lambda_C$ and for a phase difference of $\pi/2$, such as would be expected if we started with one species of atom and applied a $\pi/2$ pulse. We see that the Rabi-like Josephson dynamics are due to a wave-packet-like motion of the state backwards and forwards. If we now consider the case where $\lambda<\lambda_C$, we can see from Fig. 2(b) that the Josephson oscillations are of smaller amplitude and are centered around $m=0$—an effect similar to the self-trapping effect discussed previously.

As has been discussed, a $\pi/2$ pulse will produce an initial state with relative phase of $\phi=\pi/2$ between the two components but can be varied by either applying a strong light field to one or both species or by changing the phase of the Josephson coupling relative to the initial $\pi/2$ pulse. In what follows, we will concentrate on the case $\phi=0$. In this case, the distribution of relative particle number must remain symmetric around $m=0$, since both the state vector and the Hamiltonian will be invariant under the interchange $A\leftrightarrow B$. For $W_{AA}-W_{AB}=0$, the initial state is an eigenstate of the Hamiltonian, and thus the number distribution remains constant in time. For $W_{AA}-W_{AB}\neq 0$, the dynamics will be affected by the diffusive collisional terms in the Hamiltonian. Figure 3 shows the evolution of the state vector for $\lambda<\lambda_C$. We see that the interplay of collisional effects and Josephson coupling leads to the creation of a state which is doubly peaked about $m=0$, similar to the case of [7,8]. One peak describes a situation in which more of the atoms are to be found in species *A*, and the other peak describes the converse.

We find that by varying the strength of the Josephson coupling parameter λ, the ''size'' of the cat can be varied. For $\lambda=\lambda_C$ the distribution has peaks at $m=\pm N$, so that the size of the cat is maximal in this case: the state is close to a superposition of $2N$ atoms in species *A* and $2N$ atoms in species *B*.

We can get a better idea about the quantum state by looking at the *Q* function on the Bloch sphere [10,16]. This function is defined in terms of the so-called Bloch states or atomic coherent states, of which the states (12) are a particular case. They are defined in our basis as

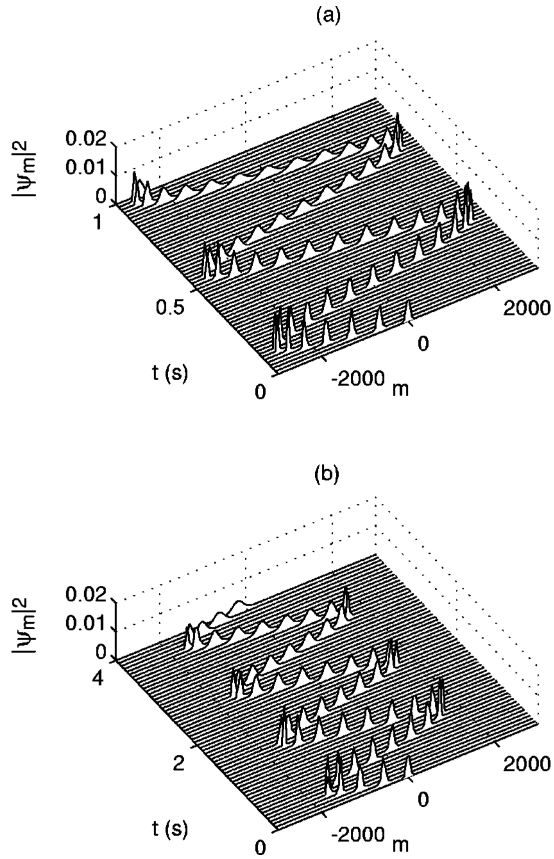

FIG. 2. The evolution of the state vector under strong Josephson coupling. The initial state is given by Eq. (12) with $\phi=\pi/2$. All parameters except for λ are the same as those given in Fig. 1; as before we have $\lambda_C=3.9\hbar\ \mathrm{s}^{-1}$. ψ_m denotes the *m* component of the state vector. (a) $\lambda=12\hbar\ \mathrm{s}^{-1}>\lambda_C$. The Rabi-like Josephson oscillations can be seen to be due to a highly coherent wave-packet motion back and forth. Some wave-packet spreading is evident; this is due to the presence of collisional terms in the Hamiltonian and will eventually lead to a collapse of the oscillations. (b) $\lambda=2.5\hbar\ \mathrm{s}^{-1}<\lambda_C$. The oscillations are seen to be incomplete; this is due to the same self-trapping mechanism discussed in Milburn *et al.* [11].

$$|\theta,\phi\rangle=\sum_m \sqrt{\binom{2N}{N-m}}\sin^{N+m}(\tfrac{1}{2}\theta)\times\cos^{N-m}(\tfrac{1}{2}\theta)\exp[-i(N+m)\phi]|m\rangle. \quad (19)$$

The parameter θ fixes the number of particles in each species, with $\theta=\pi/2$ giving equal numbers of particles in each species. The parameter ϕ describes the relative phase between the two species. The Bloch *Q* function is defined as

$$Q(\theta,\phi)=|\langle\psi|\theta,\phi\rangle|^2. \quad (20)$$

Figure 4 shows the *Q* function for the state shown in Fig. 3(b). It can be seen that the two peaks in number difference are correlated with two separate phases. This means that a phase measurement would be able to distinguish between the two macroscopic states. The dissipative decay of such states would thus be sensitive to processes which mimicked phase measurements as well as those which detected atoms of a given species.

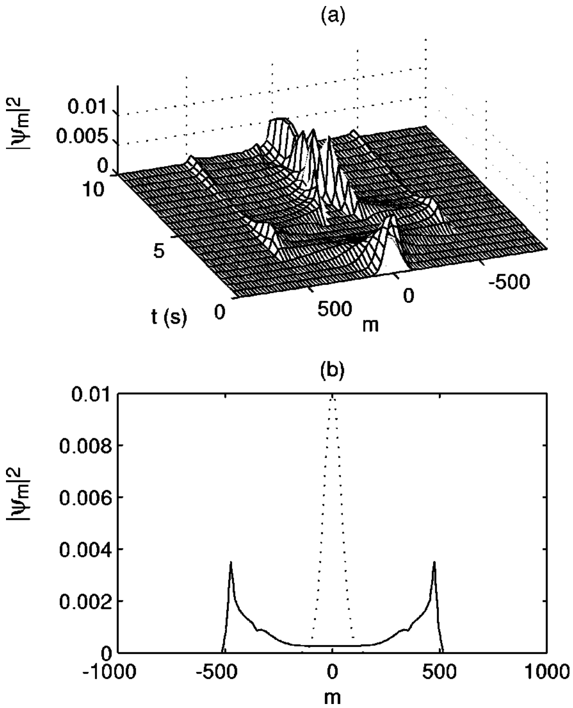

FIG. 3. (a) Evolution of the relative number distribution for $\lambda = 0.1\hbar\ \mathrm{s}^{-1} < \lambda_C$ and the other parameters as in Fig. 1. At $t \approx 4$ s it can be seen that the number distribution has become doubly peaked; this state represents a macroscopic quantum superposition state. In order to give a better idea of the gross features of the probability distribution, the state vector has been "smoothed" to eliminate fine structure. (b) shows the smoothed probability distribution at $t=0$ s (dotted line) and $t=4$ s (solid line).

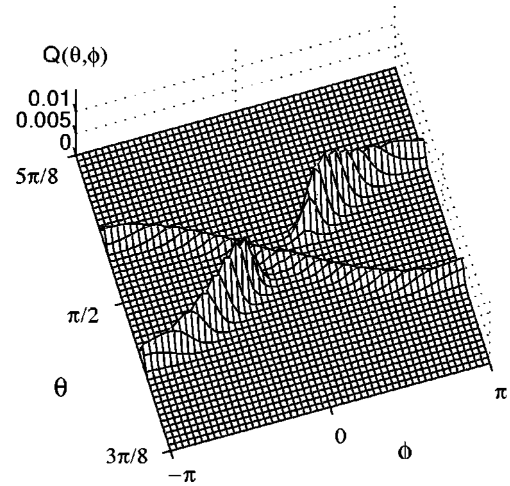

FIG. 4. Q-function plot on the Bloch sphere for the state shown in Fig. 3(b) for $t=4$ s. The coordinates θ and ϕ label the spherical polar coordinates on the Bloch sphere. ϕ can be equated with the relative phase between the two components, and θ describes the particle number difference: $\theta=0$ and $\theta=\pi$ describe the extreme situations in which all atoms are in one species, and $\theta=\pi/2$ describes equal numbers of atoms in each species ($m=0$). It can be seen that the state is a superposition between two macroscopic states of differing phase as well as differing particle number. As time progresses, the "ends" of the Q-function distribution shown above curl around towards $\phi=0,2\pi$ and $\theta=\pi/2$ and also become much more highly peaked, leading to a better defined but "smaller" Schrödinger cat state. The resulting relocalization of the phase shows up as a partial revival of $\langle \hat{J}_x \rangle$ such as is shown in Fig. 1.

This evolution into Schrödinger cat states can be partially understood by considering the effect of the phase diffusion on the Josephson dynamics while ignoring the effect of the Josephson coupling on the phase dynamics. This approximation will be valid only for some short time. At $t=0$, the condensate starts in a state with well defined relative phase and equal particle numbers in each species. The phase then diffuses due to the energy spread which is present in the initial state of the condensate and which is caused by a spread in relative particle number and the atom-atom collisions. The Josephson coupling then acts on these different phases present in the state vector, causing the negative phase half of the wave function to move in the direction of increasing m and the positive phase half to move in the direction of decreasing m, eventually causing the wave function to "split" into two wave packets evolving in opposite directions. These wave packets are eventually stopped in their motion by the self-trapping mechanism mentioned earlier, and become highly peaked.

V. VALIDITY AND FEASIBILITY

In making the calculations above, we have relied on the two-mode approximation. In order for this approximation to provide a reasonably accurate picture, we must assume that the parameters $E_{A,B}$, $W_{AA,BB,AB}$, and λ are reasonably constant for the cases investigated. These parameters all depend on the self-consistently defined condensate modes of the system, which in turn depend on the particle numbers N_A and N_B. Thus the two-mode approximation will be most accurate when the state vector has a highly localized particle number distribution. This introduces a possible problem for our approach: we want to investigate the production of Schrödinger cat states consisting of *superpositions* of states with different relative particle number, and thus, to some degree at least, we wish to move away from regimes in which the particle number distribution is highly localized.

However, we can always find regimes in which the dependence of the density profile on the particle number distribution is weak. Some examples are as follows. (i) Low density of particles: in this case, the density profiles for the two species approach the single particle eigenfunctions for the ground state of the trap and thus do not vary greatly with particle number. (ii) The regime in which $W_{AA}=W_{BB}$ and W_{AB} is only slightly less than $W_{AA/BB}$. In this case, although we do not show it here, the dependence of the density profiles on the relative particle number between the two species is weak and can be made to approach zero. However, we find that in such regimes, the characteristic evolution time is slow and the production of Schrödinger cat states could take a long time, thus exacerbating problems due to decoherence. (iii) If the density of atoms is high enough such that the healing lengths of the two-species condensate are small compared to the size of the trap, then "hard" traps such as square well traps also show little dependence of the density profiles on particle numbers, providing we are working in the regime in which the two species form a homogeneous mixture, e.g., $W_{AA}=W_{BB}$ and $W_{AB}<W_{AA/BB}$.

Furthermore, even in regimes in which the two-mode approximation is not accurate, Schrödinger cat states might be produced under that same conditions as are predicted by the two-mode approximation; this was found to be the case in the calculations of Cirac *et al.* [7].

Such states would be hard to produce in practice. One serious experimental problem would be decoherence; in the extreme example of a maximal Schrödinger cat state with a superposition between $m=-N$ and $m=N$, the ''detection,'' or loss, of one atom would be enough to destroy the superposition, since such a detection would tell us which species was populated and hence allow us to distinguish between the two macroscopic states. For less extreme cases, this condition would relax somewhat, but certainly a macroscopic (mesoscopic) loss from the system would always be enough to destroy a macroscopic (mesoscopic) superposition state.

The paper of Cirac *et al.* [7] also lists as a condition for the production of such states cooling close to the collective ground state, which is far more restrictive than simply demanding Bose-Einstein condensation, i.e., off-diagonal long-range order. In terms of the single-particle states, the condition they give is that less than one atom can be out of the single-particle ground state. This condition is equivalent to demanding that the many-body wave function has a significant population in the collective ground state (for exactly one particle not in the single-particle ground state, it turns out that the fraction of the many-body population in the *collective* ground state is $1/e$, which is of the order of 50%). The rest of the many-body wave functions will be thermally distributed among the other eigenstates of the many-body Hamiltonian, which will have very different number and/or relative phase distributions from the Schrödinger cat state. In summary, the scheme of Cirac *et al.* [7] relies on cooling to a *particular* eigenstate of the many-body Hamiltonian.

The present scheme is somewhat different, since it relies on unitary evolution rather than cooling in order to arrive at a cat state. In the present case, the effect of having nonzero temperature would be to produce an incoherent ensemble of initial states, each containing a slightly different total number and/or relative number of atoms. Thus in order to be able to observe a Schrödinger cat state, we would want the final state for each member of the initial ensemble to have a similar number distribution and relative phase. We would thus demand that the final state not be too sensitive to changes in the initial state. In the worst case scenario, in which varying the particle number by one atom would be enough to completely destroy the characteristics of the Schrödinger cat state, then we would recover the condition of Cirac *et al.*, since we would then *need* a significant population in a particular many-body state in order to observe the effects of a Schrödinger cat state.

We have found that the most critical factor here appears to be that the peak of the atom distribution must be accurately centered about $m=0$ compared to the spread in the relative number distribution. Since this latter quantity is of the order $\sqrt{N}$, we require that the variation in the average relative particle number be significantly less than $\sqrt{N}$. If this condition is not satisfied, then the cat will be lopsided, i.e., ''more alive than dead'' or *vice versa*. Recall the condition of Cirac *et al*. [7] that no more than one particle be out of the single-particle ground state. In the present case, varying the particle number by one will cause a variation in the relative particle number of one, which is much less than the spread in relative particle number ($\sqrt{N}$). Thus we believe that the present scheme might exhibit Schrödinger cats at higher temperatures than that of Cirac *et al.* [7].

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