Scaling of hydrogenic atoms and ions interacting with laser fields: Positronium in a laser field

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Scaling laws are derived for hydrogenlike atoms and ions interacting with laser fields. In particular, the scaling of (appearance) intensities is derived. This scaling is independent of the physical mechanism responsible for the ionization process, be it tunneling or multiphoton ionization. It provides a firm basis and explanation for the validity of earlier models for the estimation of appearance intensities and of the extrapolation into the tunneling regime of scaling laws obtained through lowest-order perturbation theory. As an example of the applicability of the scaling laws, we calculate two-, three-, and six-photon generalized cross sections and ionization rates for positronium at laser frequencies of current experimental interest. [S1050-2947(99)04806-4]

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I. INTRODUCTION

Recent experimental advances at the Aarhus negative ion [1] and positronium beam-lines suggest that studies of the photobreakup, excitation, and even strong field driving of positronium (Ps) by intense laser radiation will be feasible shortly. Initial test experiments have in fact already been carried out [2]. This may well be the beginning of the exploration of the interaction of lasers with exotic atoms, although for the moment given the wavelength range of available lasers, the only exotic atom amenable to photobreakup studies by pulsed electromagnetic radiation is Ps, and of course the muonium atom which is for all practical purposes identical to hydrogen. It is, however, conceivable that future developments in coherent short wavelength and x-ray sources may render feasible similar studies in other exotic atoms. The size and energy scales of some of these exotic atoms differ significantly from those of ordinary atoms owing to the different relation between the masses of the bound particles, which alters one of the basic features of atoms, namely the heavy nucleus to which the electron is bound. Positronium, for example, made up of an electron and a positron bound to each other, represents the lightest leptonic hydrogenlike atom. Thus in a photobreakup process, both particles fly away, which implies kinematical behavior much different from the usual process of atomic photon ionization where the light electron flies away from a practically stationary [in the center-of-mass (CM) system] nucleus.

These are only the obvious differences. Others, more profound, originate from the change of the reduced mass. With m_e being the mass of the electron, the reduced mass of Ps is $\mu=m_e/2$ instead of almost m_e as for H or any other ordinary atom. Since nonrelativistic theory will be sufficient for our purpose here, the energy levels of Ps are given by $E_n(\mu,Z)=(\mu/m_e)Z^2E_n^{(\mathrm{H})}$, where $E_n^{(\mathrm{H})}$ denotes the corresponding level of H. For $\mu=m_e/2$ and $Z=1$ this means that the energy differences are half those of H. But μ also affects the radius a_μ of the ground and all other states because $a_\mu=\hbar^2/(\mu e^2)=2a_0$ with a_0 being the Bohr radius. Thus the energy levels are lowered by a factor of 2 while the size is increased by the same factor. An even more profound effect of the reduced mass will be the magnitude of the matrix elements coupling the system to radiation. An immediate consequence is easily recognized in the doubling of the quiver radius and the ponderomotive potential with respect to those of hydrogen, for fixed frequency and intensity of the laser. But what about the cross sections, the yield of nonlinear processes, and in particular their dependence on the order of nonlinearity?

This led us to the consideration of scaling laws which took us well beyond our initial aim, namely Ps. In short, we have found scaling laws for strong-field ionization—perturbative or otherwise—in any hydrogenlike system. We have shown, moreover, that scaling in the perturbative (multiphoton) regime carries over into the nonperturbative regime where tunneling takes over as the principal mechanism. For hydrogenlike systems, this is a rigorous result emerging from the scaling properties. By extension, it follows that scaling laws in the perturbative regime for any atom should be valid in the tunneling regime as well, which sheds light on and confirms the validity and usefulness of semiempirical scaling laws for any atom obtained some time ago [3].

Returning to Ps, we show that its behavior can be described in terms of parameters obtained from the appropriate scaling of those for H. There are of course fundamental differences between H and Ps since the latter is metastable against annihilation—a feature of all particle-antiparticle bound systems. There is a difference in lifetime between the singlet and the triplet ground states. The triplet state annihilating predominantly by three-photon emission lives longer than the singlet state whose annihilation is dominated by two-photon emission, with respective lifetimes $\tau_{\mathrm{trip}}=1.4\times10^{-7}$ s and $\tau_{\mathrm{sing}}=1.25\times10^{-10}$ s [4]. Thus the lifetime of the triplet state is orders of magnitude longer than the pulse duration of essentially any laser. A laser pulse of nanosecond duration (such as the one employed in the Aarhus experiment) will then for all practical purposes interact only with the triplet. In any case, since the electric dipole interaction does not mix the singlet and triplet manifolds, calculations for either of the two carry over to the other.

To the best of our knowledge, this is the first theoretical treatment of the photobreakup and related multiphoton pro-

cesses of Ps in a strong pulsed laser of optical frequency. Previous work [5–7] that we are aware of has been mainly concerned with the possibility of the delay of annihilation through laser excitation. As we shall see below, nonperturbative behavior sets in at an intensity 16 times smaller than in H at the appropriately scaled frequency. By contrast, nonperturbative behavior in a Ze ion sets in at an intensity Z^6 times larger than in H, again at the appropriately scaled frequency.

The paper is organized as follows. In the next section we outline the kinematical differences in ionization from ordinary as opposed to exotic atoms, e.g., positronium. In Sec. III we derive general scaling relations directly from the time-dependent Schrödinger equation, while in Sec. IV we first summarize formulas describing multiphoton ionization in perturbation theory and then we discuss scaling in the perturbative regime. Finally, in Sec. V the results are discussed.

II. KINEMATICAL CONSIDERATIONS

A hydrogenic system is made up of two charged particles interacting through the Coulomb potential. The problem is in general solved by introducing two fictitious particles: the center-of-mass (CM) particle of mass $M=m_1+m_2$ and the reduced mass particle $\mu=m_1m_2/(m_1+m_2)$. The charged particles may have very different masses as in hydrogen or they may have equal masses as in positronium. It is not *a priori* obvious that the approximations made for hydrogen also apply for the much lighter system of Ps. Do we, for example, have to take into account recoil and Doppler energies when considering ionization? Does the angular distribution of ionization products look different in the laboratory and CM frames? For an N-photon process it is easily verified for typical optical wave numbers k and total mass $M=m_1+m_2$ that the recoil energy $\hbar^2N^2k^2/(2M)$ can be neglected even for positronium, $M=2m_e$. Similarly, it can be shown that the Doppler energy is negligible. Therefore, the energy relation $E_f=E_i+N\hbar\omega$, for the reduced mass particle, is fulfilled to an excellent approximation.

The momentum is absorbed by the CM particle. To obtain the (angular, partial) rate of, for example, ionization of the reduced mass particle, we have to sum over the final states of this unresolved CM momentum. One can thus use standard methods to obtain cross sections and rates of the reduced mass particle. How these quantities relate to those of the true particles depends on the mass relation between them. As an example, we may consider the photoelectron spectrum, including above threshold ionization (ATI). In hydrogen, the proton is much heavier than the electron, which as a result carries all of the photon energy, and the photoelectron peaks are, accordingly, separated by $\hbar\omega$. For positronium, on the other hand, the photon energy is equally shared between the electron and the positron. This gives rise to photoelectron peaks separated by $\hbar\omega/2$. The spectrum of the true particle is thus obtained from that of the reduced particle at half the energy.

Finally, we note that CM angular differential ionization rates and cross sections at (θ,ϕ) have to be multiplied by a standard kinematical factor to transform into the corresponding quantities in the laboratory system at angles (θ_L,ϕ_L). Assuming light linearly polarized and letting the z axis be along the laser polarization and the atomic beam direction (crossed beam experiment), this factor reads [8] $(1+2q\cos\theta+q^2)^{3/2}/|1+q\cos\theta|$, where $q=V_L/v$ is the ratio between the velocity of the atomic beam particle with respect to the CM system and the velocity of the electron in the CM frame. For ordinary atomic targets of thermal velocities, this factor is very close to 1 (since $q\ll1$) due to the large mass of the atoms and is therefore not considered in discussions of photoelectron angular distributions. For thermal positronium, however, this factor may deviate substantially from unity, due to the equal masses of the electron and the positron, and it is thus essential to include when comparing theory to experimental data. Note that the kinematical factor enhances the fragment emission in the direction of the polarization of the laser by $(1+q^2)$, which would make the angular distribution in Ps more peaked than, for example, in hydrogen. If one is interested only in angle-integrated quantities, there is of course no difference between results in the laboratory and CM system when first the transformation from the reduced mass particle to the physical particle has been made.

The quantities presented in the following refer solely to the reduced particle. Accordingly, the appropriate kinematical factors described above have to be considered for a direct comparison with angle-resolved experimental data.

III. SCALING OF THE SCHRÖDINGER EQUATION

Submitting a particle with reduced mass μ and charge $(-e)$ moving in the field of a nucleus of charge Ze to an electric field

$$\boldsymbol{E}(\omega,t)=\frac{1}{2}[\mathcal{E}(t)e^{-i\omega t}\boldsymbol{\epsilon}+\mathcal{E}^{\star}(t)e^{i\omega t}\boldsymbol{\epsilon}^{\star}] \tag{1}$$

of amplitude $\mathcal{E}(t)$, angular frequency ω, and polarization $\boldsymbol{\epsilon}$ introduces the coupling $e\boldsymbol{r}\cdot\boldsymbol{E}(\omega,t)$ in the Schrödinger equation

$$i\hbar\partial_t\Psi(\boldsymbol{r},t)=\left(-\frac{\hbar^2}{2\mu}\nabla_r^2-\frac{Ze^2}{r}+e\boldsymbol{r}\cdot\boldsymbol{E}(\omega,t)\right)\Psi(\boldsymbol{r},t). \tag{2}$$

Equation (2) is based upon a few approximations. First, we have made the dipole approximations. Second, we have assumed that the ionic case $(Z\neq1)$ in practice only occurs in systems where the nucleus is much heavier than the electron, in which case $\mu\sim m_e$ irrespective of corrections due to the charge and the interaction of the CM motion with the laser field $(\propto1/M)$ can be neglected. With scaling length and time according to

$$\boldsymbol{x}=\left(\frac{\mu}{m_e}\right)Z\boldsymbol{r},\quad \tau=\left(\frac{\mu}{m_e}\right)Z^2t, \tag{3}$$

we are led to the following equation:

$$i\hbar\partial_\tau\Psi(\boldsymbol{x},\tau)=\left(-\frac{\hbar^2}{2m_e}\nabla_x^2-\frac{e^2}{x}+e\boldsymbol{x}\cdot\boldsymbol{E}'(\omega',\tau)\right)\Psi(\boldsymbol{x},\tau), \tag{4}$$

describing hydrogen subject to the scaled field

$$\boldsymbol{E}'(\omega',\tau)=\left(\frac{\mu}{m_e}\right)^{-2}Z^{-3}\boldsymbol{E}(\omega',\tau) \tag{5}$$

with the scaled frequency

$$\omega'=\left(\frac{\mu}{m_e}\right)^{-1}Z^{-2}\omega. \tag{6}$$

In deriving Eqs. (4) and (5), we have used the fact that typical forms of pulse evolution, $\mathcal{E}(t)$, depend on time through t/T, where T is a parameter related to the temporal pulse width and slope, and we have accordingly scaled T in Eqs. (4) and (5) as

$$T(\mu,Z)=\left(\frac{\mu}{m_e}\right)^{-1}Z^{-2}T(m_e,Z=1), \tag{7}$$

which ensures that the number of laser field cycles is the same in Eqs. (2) and (4).

We have thus shown that the Schrödinger equation for hydrogen in a laser field is regained by using the scaling relations in Eq. (3) for length and time, scaling the field amplitude as

$$\mathcal{E}_0(\mu,Z)=\left(\frac{\mu}{m_e}\right)^{2}Z^{3}\mathcal{E}_0(m_e,Z=1), \tag{8}$$

and the frequency and the temporal width as in Eqs. (6) and (7), respectively.

If the Floquet ansatz is valid, we can easily show that

$$\Psi(\mu,Z,\mathcal{E}_0,\omega,\boldsymbol{r},t)=\mathrm{const}\Psi\left(m_e,Z=1,\mathcal{E}_0\left(\frac{\mu}{m_e}\right)^{-2}Z^{-3},\omega\left(\frac{\mu}{m_e}\right)^{-1}Z^{-2},\boldsymbol{x},\tau\right). \tag{9}$$

If we introduce the complex quasienergy E, we have that $E\rightarrow(\mu/m_e)^{-1}Z^{-2}E$ from the Floquet-Fourier ansatz so that the ionization rate should be scaled by this factor. Thus, if we wish to know the ionization rate of the $1s$ ground state of the (μ,Z) system at a frequency ω and peak intensity $\mathcal{E}_0$, we can find it from

$$\Gamma(\mu,Z)=-2\times\left(\frac{\mu}{m_e}\right)Z^2\ \mathrm{Im}(E), \tag{10}$$

where E is the quasienergy of the ground state of hydrogen subjected to light of frequency ω' and peak field strength $\mathcal{E}_0(\mu/m_e)^{-2}Z^{-3}$. The ionization rate in Eq. (10) is the total rate summed over all channels and integrated over all angles. If the rate of a (μ,Z) system is known, Eq. (10) may be inverted to obtain the hydrogenic rate. Equation (10) is of course only valid when the concept of a rate is meaningful, i.e., away from avoided crossings where several Floquet states mix and only when the pulse is sufficiently long and adiabatic to allow for the single Floquet state approach.

The scaling in Eq. (10) is obvious if one considers the scaling of time in Eq. (3) and remember that the rate is measured in $1/s$. Using this argument for the partial rate corresponding to N-photon ionization, we get

$$\Gamma^{(\mathrm{N})}(\mu,Z,\mathcal{E}_0,\omega)=\left(\frac{\mu}{m_e}\right)Z^2\times\Gamma^{(\mathrm{N})}\left(m_e,Z=1,\frac{\mathcal{E}_0}{(\mu/m_e)^2Z^3},\frac{\omega}{(\mu/m_e)Z^2}\right), \tag{11}$$

which is consistent with Eq. (10), as the partial rates add up to the total rate. The scaling relations for the total and partial rates do of course also apply for the corresponding *angular* quantities. Additional factors of kinematical origin only come into play when one wants to transform to the laboratory frame as discussed in Sec. II.

In Sec. IV we derive results in lowest-order perturbation theory which are consistent with the above results.

IV. SCALING IN THE PERTURBATIVE REGIME

In this section we first summarize relations of relevance to a discussion of scaling in the perturbative regime. In lowest-order perturbation theory, the N-photon transition amplitude has the form

$$M_{fg}^{(N)}=\sum_s\cdots\sum_n\sum_m\frac{\langle f|\boldsymbol{\epsilon}\boldsymbol{r}|s\rangle}{[E_s-E_g-(N-1)\hbar\omega]}\cdots\times\frac{\langle n|\boldsymbol{\epsilon}\boldsymbol{r}|m\rangle}{(E_n-E_g-2\hbar\omega)}\frac{\langle m|\boldsymbol{\epsilon}\boldsymbol{r}|g\rangle}{(E_m-E_g-\hbar\omega)}, \tag{12}$$

where the $(N-1)$-fold sum runs over complete sets of atomic states and atomic energy levels, and where $|g\rangle$ and $|f\rangle$ are the initial and final atomic state, respectively.

The corresponding generalized N-photon cross section is [9]

$$\hat{\sigma}_N=\frac{(2\pi\alpha)^N}{4\pi^2}\frac{\mu K}{\hbar}\omega^N\int d\Omega_K|M_{fg}^{(N)}|^2, \tag{13}$$

where it is assumed that the continuum wave functions are normalized so that $\rho(\boldsymbol{K})=(2\pi)^{-3}$. The generalized cross section is measured in units of $\mathrm{cm}^{2N}\mathrm{s}^{N-1}$. The transition probability per unit time (the ionization rate) is given by

$$\Gamma^{(\mathrm{N})}=\hat{\sigma}_N F^N, \tag{14}$$

where F is the photon flux, i.e., the number of photons per square centimeter per second which is again related to the intensity measured in W/cm^2 by

$$F=0.624\times 10^{19} I/\hbar\omega \ \ (\mathrm{eV}). \tag{15}$$

The probability of ionization P_N (ion yield at the end of the pulse) is given by the integral

$$P_N=\int_0^\infty \hat{\sigma}_N F^N(t)\,dt, \tag{16}$$

where $F(t)=F_0 f(t)$ describes the photon flux having a peak value of F_0, and $f(t)$ depends on the particular pulse shape. Although ultimately we must calculate integrals such as those in Eq. (16), we may gain insight into the basic features by considering

$$P_N \simeq \hat{\sigma}_N F_0^N \tau_L, \tag{17}$$

as an approximation which would be exact for a square pulse of duration τ_L. One can in fact always write an exact relation of the form (16) if τ_L is replaced by an effective duration which is equal to $\int_0^\infty f^N(t)\,dt=\hat{\tau}^{(N)}$, where the superscript N is intended to indicate the dependence of the effective duration on the order of the process. In crossed beam experiments, it is often the time of flight of the target atoms through the laser focus, τ_f, which determines the interaction time. The saturation flux (and therefore the saturation intensity) is defined by $P_N=1$.

In the perturbative regime, it is possible to follow the scaling in detail as a function of photon flux or intensity. From the definition of the generalized cross section (13) and from the scaling of length and time (3) it follows immediately that the generalized cross section scales according to

$$\hat{\sigma}_N(\mu,Z,\omega)=\left(\frac{\mu}{m_e}\right)^{-3N+1} Z^{-4N+2} \times \hat{\sigma}_N\left(m_e, Z=1, \frac{\omega}{(\mu/m_e)Z^2}\right). \tag{18}$$

This result may also be obtained by scaling the individual quantities of Eq. (13). Such a procedure was followed in [3] in an investigation of the Z scaling. There the correct scaling of the final electron wave number was not considered. Furthermore, the difference in the scaling of bound-bound and bound-free matrix elements was not taken into account. The final result is, however, very similar, $\hat{\sigma}_N \propto 1/Z^{4N-4}$. In the limit $N \gg 1$ this Z scaling coincides with the above result.

From Eq. (14) we obtain the relation between the N-photon transition rates in terms of photon flux

$$\Gamma^{(N)}(\mu,Z,\omega)=\left(\frac{\mu}{m_e}\right)^{-3N+1} Z^{-4N+2}\left(\frac{F}{F^{(\mathrm{H})}}\right)^N \times \Gamma^{(N)}\left(m_e, Z=1, \frac{\omega}{(\mu/m_e)Z^2}\right) \tag{19}$$

or in terms of intensity

$$\Gamma^{(N)}(\mu,Z,\omega)=\left(\frac{\mu}{m_e}\right)^{-4N+1} Z^{-6N+2}\left(\frac{I}{I^{(\mathrm{H})}}\right)^N \times \Gamma^{(N)}\left(m_e, Z=1, \frac{\omega}{(\mu/m_e)Z^2}\right). \tag{20}$$

In these equations the superscript ''H'' corresponds to hydrogen. For $N=2$, equal intensities, and nonexotic atoms, the result in Eq. (20) reduces to a formula which was given by Zernick many years ago [10] and which was recently used by Lambrecht *et al.* [11]. We also note that for intensities scaled as in Eqs. (10) and (11) we regain Eq. (11) for the relation between partial rates at appropriately scaled laser and energy parameters. Finally, we mention that Eqs. (19) and (20) also apply, with the same scaling factors, for the angular differential rates. Similarly, Eq. (18) translates simply to the differential generalized cross section.

V. RESULTS AND DISCUSSION

A. The scaling of intensities

One immediate consequence of the scaling of the field strengths discussed in Sec. III is the scaling of the intensities

$$I=\left(\frac{\mu}{m_e}\right)^4 Z^6 I^{(\mathrm{H})}, \tag{21}$$

which implies that the behavior of a hydrogenlike system of charge Z and reduced mass μ under intensity I is exactly the same as that of hydrogen placed in an electromagnetic field of intensity $I^{(\mathrm{H})}$, at appropriately scaled frequency (6) and pulse duration (7). This relation also holds, of course, for the relation between appearance intensities. It is based only on scaling and does therefore apply irrespective of the physical mechanisms responsible for the considered process, i.e., it holds across tunneling, intermediate, and multiphoton regimes. This explains why the correct scaling for the appearance intensity can be obtained from the scaling of generalized cross sections in lowest-order perturbation theory. And since for large N, which corresponds to tunneling, the exact scaling obtained above coincides with the approximate one obtained some time ago [3] for arbitrary atoms and ions, it also explains why that scaling gave correct predictions for high-order cross sections and appearance intensities whenever it was tested [12,13]. It finally provides a firm proof for the validity of a simple model [4,14,15] often used for the prediction of appearance intensities on the basis of over the barrier ionization (tunneling). In this tunneling model the atomic system ionizes at the appearance intensity, which

TABLE I. Generalized cross sections for the six-, three-, and two-photon ionization for hydrogen and positronium.

N	$\lambda^{(\mathrm{H})}$ (nm)	$\lambda^{(\mathrm{Ps})}$ (nm)	$\hat{\sigma}_N^{(\mathrm{H})}$ (cm^{2N} s^{N-1})	$\hat{\sigma}_N^{(\mathrm{Ps})}$ (cm^{2N} s^{N-1})
6	532	1064	3×10^{-180}	4×10^{-175}
3	266	532	5×10^{-83}	1×10^{-80}
2	177.5	355	1×10^{-50}	3×10^{-49}

forces the combined potential of the Coulomb and electron-field interaction to match the ionization potential, E_p,

$$I = cE_p^4/128\pi e^6 Z^2, \tag{22}$$

and for hydrogenic systems we therefore have the following scaling of the appearance intensities:

$$I(\mu, Z) = \left(\frac{\mu}{m_e}\right)^4 Z^6 I^{(\mathrm{H})}, \tag{23}$$

which is identical to the scaling obtained in Eq. (21) by considering the Schrödinger equation. (Indeed, a quality check of any theoretical model may be to see whether it satisfies the above intensity scaling or not.)

B. The special case of positronium

We now turn to the specific case of Ps where the scaling relations of Eqs. (18) and (20) simply read

$$\hat{\sigma}_N(m_e/2, \omega) = 2^{3N-1}\hat{\sigma}_N^{(\mathrm{H})}(m_e, 2\omega), \tag{24}$$

$$\Gamma^{(N)}(m_e/2, \omega) = 2^{4N-1}\left(\frac{I}{I^{(\mathrm{H})}}\right)^N \Gamma_{(\mathrm{H})}^{(N)}(m_e, 2\omega). \tag{25}$$

The six-photon ionization cross section, for example, is thus $\sim 2^{17}$ times larger for positronium than for hydrogen. This implies by Eq. (25) that equal rates for H and Ps are obtained with an intensity $\sim 2^4$ times smaller in the case of Ps.

The relevant wavelengths for the ongoing experimental work are $\lambda = 1064$, 532, and 355 nm corresponding to six-, three-, and two-photon ionization, respectively [2]. In Table I we show the generalized six-, three-, and two- photon cross sections at these wavelengths. The values for hydrogen are taken from the existing literature [16,17]. The generalized cross sections for positronium have been obtained by the scaling law of Eq. (24). Even though the generalized cross sections for Ps are significantly larger than the hydrogenic ones of the corresponding order — especially for higher-order processes — this effect does not change the fact that the yield decreases rapidly as a function of the order of the process.

We can estimate the saturation intensities for hydrogen and positronium. The atomic beam and laser beam will cross at right angles. A typical velocity of the thermal positronium atoms is 10^5 m/s [2,18]. Taking the diameter of the laser focus to be 20 μm, we get an interaction time $\tau_f = 0.2$ ns. We use the same value for hydrogen. Equation (17) with τ_f instead of τ_L gives the estimates of the saturation intensities for, e.g., six-photon ionization. We find $I_S \simeq 1.3\times10^{13}$ and $I_S \simeq 9\times10^{11}$ W/cm^2 for hydrogen and positronium, respectively. The corresponding rates are $\sim 5\times10^9\,\mathrm{s}^{-1}$ in both cases, leading to a lifetime against breakup much shorter than the lifetime of the triplet against annihilation, which is important for the comfortable observability of the process. It is interesting to calculate the value of the ponderomotive potential, $U_p = I/(4\mu\omega^2)$ (a.u.), to see if one may expect any channel closing at the saturation intensities. We find $U_p(\mathrm{H}) = 0.34$ eV and $U_p(\mathrm{Ps}) = 0.19$ eV which are both very close to the energy of the six-photon ionized electron 0.38 and 0.19 eV for hydrogen and positronium, respectively. This estimate, accordingly, shows that the ionization will most likely be influenced by channel closing at the saturation intensities at this wavelength. The values of U_p and the excess energies are, on the other hand, so close that this conclusion depends critically on the experimental uncertainty of the laser intensity and the value of the generalized cross section. Note that in the case of Ps, the ponderomotive energy is the sum of the quiver energy of the electron and the positron. Each particle, however, carries only half of the excess energy (0.09 eV) but also has to overcome only half of the ponderomotive energy.

As an example for calibration by comparison with available experience we consider the experiment of Wolff *et al.* [19], who studied multiphoton ionization of hydrogen with a 10 ns pulsed laser of a wavelength of 532 nm operated at an intensity of $\sim 1.6\times10^{13}$ W/cm^2. From the above estimate we see that this intensity is above the saturation intensity. For hydrogen we obtain the rate $\Gamma_N(H) \simeq 2\times10^{10}\,\mathrm{s}^{-1}$, which means that a large fraction of the hydrogen atoms will ionize during their way through the laser focus. Accordingly, only low-order ATI peaks are expected to be measured. This is in accordance with the experimental findings [19]. Similar experimental findings should be possible in positronium if one scales the quantities as discussed in Secs. III and IV: doubling the wavelength leads to the six-photon ionization regime of positronium. Furthermore, the laser intensity should be decreased by a factor of 16 to $\sim 1.0\times10^{12}$ W/cm^2. A pulse of much shorter duration, say subpicosecond, would produce a more extended photoelectron energy spectrum (ATI). Notice that the quiver radiis of Ps and H are equal at scaled field and frequency, while the ponderomotive shift of Ps is half that of H in accordance with the overall energy scaling.

VI. CONCLUSIONS AND OUTLOOK

We have presented a detailed study of the scaling of any hydrogenic system. The scaling laws make it possible to calculate ionization rates and generalized cross sections for any hydrogenic system using ionization rates of hydrogen at appropriately scaled variables. Since the theoretical literature on hydrogen in laser fields covers a large span of intensities and frequencies, much of that information can be carried over to ionic or exotic systems after proper scaling.

As an example of the applicability of the scaling laws, we

have explicitly calculated generalized two-, three-, and six-photon cross sections for Ps, based on accurate values for hydrogen. These cross sections are of interest in ongoing experiments [2]. As discussed above, the scaling laws indicate that at scaled frequencies, Ps breaks up at lower intensity than H does. That information in itself, however, does not reveal the expected behavior of nonlinear processes. If, for example, one examines the rate of decrease of N-photon processes with increasing N in Ps, it turns out that their values fall off much more slowly than in H. Thus, in an experiment with properly scaled frequency but fixed intensity, one should expect an early plateau in the ATI spectrum as compared to that of H. This would be consistent with the onset of saturation for Ps at lower intensity as noted above, and appears to have some resemblance to recent data by DiMauro *et al.* [20] on potassium under strong infrared radiation. The two systems are rather different and it would be interesting to eventually explore the underlying reasons for that similarity.

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