Magnetic-field-stimulated transitions of excited states in fast muonic helium ions

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It is shown that one can stimulate, by using present day laboratory magnetic fields, transitions between the lm sublevels of fast μHe^+ ions formed in muon-catalyzed fusion. Strong fields also cause the self-ionization from highly excited states of such muonic ions. Both effects are the consequence of the interaction of the bound muon with the oscillating field of the Stark term coupling the center-of-mass and muon motions of the μHe^+ ion due to the nonseparability of the collective and internal variables in this system. The performed calculations show a possibility to drive the population of the lm sublevels by applying a field of a few tesla, which affects the reactivation rate and is especially important to the $K\alpha$ x-ray production in muon catalyzed fusion. It is also shown that the $2s$-$2p$ splitting in μHe^+ due to the vacuum polarization slightly decreases the stimulated transition rates. [S1050-2947(99)02306-9]

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I. INTRODUCTION

We show that using present day laboratory magnetic fields may change essentially the population of the lm sublevels of the excited states ($n \geqslant 2$) of the fast muonic ions μHe^+ produced in muon-catalyzed fusion (μCF) [1–4]. This may affect the muon stripping and, especially, the intensities of the x rays from the ions. Stronger fields also cause the self-ionization [5,6] of the highly excited states of the μHe^+ ions.

Both effects are consequences of the nonseparability of the collective and internal degrees of freedom of the charged two-body system (μHe^+) in the presence of an external magnetic field [7]. The coupling between the center-of-mass (c.m.) and light particle (muon) motions is described by an oscillating Stark term which is proportional to the mass ratio m_μ/M (m_μ and M are the muon and the μHe^+ mass, respectively), the c.m. momentum (see below) and the magnetic-field strength B [7]. A rather strong coupling of the c.m. and muon motions due to the essential nonadiabaticity ($m_\mu/M \simeq 1/40$) and the high c.m. velocity ($v \simeq 6v_0 = 6\alpha c, \alpha \simeq 1/137$) makes the system μHe^+, formed in μCF, sufficiently sensitive to the external magnetic field. Thus, as it will be shown below, one can drive the sublevel populations P_{nlm} of the fast μHe^+ in excited states $n \geqslant 2$ by applying magnetic fields of a few tesla. However, from the first glance the obtained result looks quite unexpected, because the muonic atomic unit of the magnetic field, $B_0 = (e/\hbar)^3 m_\mu^2 c \simeq 1.01\times 10^{14}$ G, is extremely large, exceeding the existing present day laboratory fields by nine orders of magnitude.

In Sec. II we describe our time-dependent approach for treating the evolution of the two-body charged system in an external magnetic field. The calculations of the transition rates between the sublevels lm of the fast μHe^+ ion in the presence of a magnetic field are discussed in Sec. III. Section IV is devoted to a discussion of the self-ionization process of highly excited states of the fast μHe^+ ion in a strong field. Finally we provide some conclusions in Sec. V.

II. THEORETICAL APPROACH TO THE FAST MUONIC HELIUM ION IN A MAGNETIC FIELD

We start our analysis with the transformed Hamiltonian [7,8] of the moving μHe^+ ion in a magnetic field. The Hamiltonian of the system $H = H_0 + h + \Delta U$ consists of two terms describing the c.m. motion

$$H_0(\boldsymbol{P},\boldsymbol{R}) = \frac{1}{2M}\left(\boldsymbol{P} - \frac{Q}{2}\boldsymbol{B}\times\boldsymbol{R}\right)^2 \tag{1}$$

and internal degrees of freedom

$$h(\boldsymbol{p},\boldsymbol{r}) = \frac{1}{2m_\mu}\left(\boldsymbol{p} - \frac{e}{2}\boldsymbol{B}\times\boldsymbol{r} + \frac{Q}{2}\frac{m_\mu^2}{M^2}\boldsymbol{B}\times\boldsymbol{r}\right)^2 + \frac{1}{2M_0}\left(\boldsymbol{p} + \left[\frac{e}{2} - \frac{Q}{2M}\frac{m_\mu}{M}(M+M_0)\right]\boldsymbol{B}\times\boldsymbol{r}\right)^2 - \frac{ze^2}{r}, \tag{2}$$

as well as the coupling term

$$\Delta U(\boldsymbol{P},\boldsymbol{R},\boldsymbol{r}) = \gamma\frac{e}{M}\left[\boldsymbol{B}\times\left(\boldsymbol{P} - \frac{Q}{2}\boldsymbol{B}\times\boldsymbol{R}\right)\right]\boldsymbol{r} \tag{3}$$

between the c.m. and internal motions. Here ($\boldsymbol{R},\boldsymbol{P}$) and ($\boldsymbol{r},\boldsymbol{p}$) are the canonical coordinate-momentum pairs for the c.m. and internal motions, M_0 is the helium nuclear mass. The magnetic-field vector is denoted as $\boldsymbol{B}$ and oriented along the z axis, the charges of the nucleus, the ion, and the muon are $-ze$, Q, and e, respectively and $\gamma = (M_0 + zm_\mu)/M$.

In Refs. [9,10] it was shown that for the one-electron ion He^+ the corrections to the c.m. motion due to the coupling term (3) are negligible in not too strong fields and at low c.m. velocities. Therefore, the c.m. can be treated as a pseudoparticle with mass M and unit charge $(Q=-e)$ [9,10], performing the cyclic motion

$$\dot{\boldsymbol{R}}=v_{\perp}(\cos\omega t\boldsymbol{n}_x-\sin\omega t\boldsymbol{n}_y) \tag{4}$$

with the frequency $\omega=QB/M$ in the xy plane orthogonal to the vector of the magnetic field $\boldsymbol{B}$. $v_{\perp}$ is the projection of the initial c.m. velocity $\boldsymbol{v}(t=0)=\dot{\boldsymbol{R}}(t=0)$ onto the xy plane. By using the classical Hamiltonian equations connecting the c.m. momentum $\boldsymbol{P}$ and the velocity $\dot{\boldsymbol{R}}$ of the center of mass of the He^{+} ion in the field B [6,7], one can transform the mixing term (3) to the form

$$\Delta U(\boldsymbol{P},\boldsymbol{R},\boldsymbol{r})=\gamma e\left[\boldsymbol{B}\times\left(\dot{\boldsymbol{R}}+\frac{e\gamma}{M}\boldsymbol{B}\times\boldsymbol{r}\right)\right]\boldsymbol{r}\simeq\gamma e[\boldsymbol{B}\times\dot{\boldsymbol{R}}]\boldsymbol{r}, \tag{5}$$

representing an oscillating Stark field [see Eq. (4)]. Thus we get the simplified effective Hamiltonian of the electron motion $H(\boldsymbol{r},\dot{\boldsymbol{R}}(t))$, with $\dot{\boldsymbol{R}}(t)$ defined by Eq. (4). Such an approach has, however, many limitations and in particular fails to describe correctly a number of interesting phenomena occurring in ion physics. In particular for the muonic ion μHe^{+} in a magnetic field the coupling (3) between the internal and c.m. variables is $m_e/m_\mu\simeq 200$ times stronger compared to the electronic counterpart He^{+}, which demands an accurate treatment of the nonseparability of the system.

For analyzing the time evolution of the fast μHe^{+} ions in a magnetic field we suggest an approach including the nonadiabatic effects in the problem [all the terms of the order m_μ/M and m_μ/M_0 in Eqs. (1)–(3)]. It is a mixed treatment by a coupled system of equations describing quantum mechanically the muon degrees of freedom and treating classically the c.m. motion. It is formulated as the initial-value problem

$$i\frac{\partial}{\partial t}\psi(\boldsymbol{r},t)=H(\boldsymbol{P}(t),\boldsymbol{R}(t),\boldsymbol{r})\psi(\boldsymbol{r},t), \tag{6}$$

$$\psi(\boldsymbol{r},t=0)=\phi_{n'l'm'}(\boldsymbol{r})$$

with the effective Hamiltonian

$$H(\boldsymbol{P}(t),\boldsymbol{R}(t),\boldsymbol{r})=-\frac{1}{2m}\Delta_{\boldsymbol{r}}-\frac{2}{r}+\frac{\gamma'}{2m}\boldsymbol{B}\cdot\boldsymbol{L}+\frac{1}{8m}\left(\gamma'^2+\frac{4m}{M}\gamma^2\right)[\boldsymbol{B}\times\boldsymbol{r}]^2-\frac{\gamma}{M}\left[\boldsymbol{B}\times\left(\boldsymbol{P}(t)-\frac{Q}{2}\boldsymbol{B}\times\boldsymbol{R}(t)\right)\right]\boldsymbol{r}, \tag{7}$$

depending on the parameters $\boldsymbol{P}(t)$ and $\boldsymbol{R}(t)$ defined by the classical Hamiltonian equations of motion

$$\frac{d}{dt}\boldsymbol{P}_j(t)=-\frac{\partial}{\partial\boldsymbol{R}_j}H_{\rm cl}(\boldsymbol{P}(t),\boldsymbol{R}(t)),$$

$$\frac{d}{dt}\boldsymbol{R}_j(t)=\frac{\partial}{\partial\boldsymbol{P}_j}H_{\rm cl}(\boldsymbol{P}(t),\boldsymbol{R}(t)), \tag{8}$$

[here $\phi_{n'l'm'}(\boldsymbol{r})$ are the Coulomb wave functions of the bound muon in μHe^{+} without external field, $\boldsymbol{L}$ is the muon angular momentum, $\gamma'=(M_0^2-zm_\mu^2)/M^2$ and $m=m_\mu/(1+m_\mu/M_0)$] where the Hamiltonian $H_{\rm cl}$ is determined as

$$H_{\rm cl}(\boldsymbol{P},\boldsymbol{R})=H_0(\boldsymbol{P},\boldsymbol{R})+\langle\psi(\boldsymbol{r},t)|h(\boldsymbol{p},\boldsymbol{r})+\Delta U(\boldsymbol{P},\boldsymbol{R},\boldsymbol{r})|\psi(\boldsymbol{r},t)\rangle \tag{9}$$

by averaging the initial Hamiltonian (1)–(3) over the internal variables $\boldsymbol{r}$ at every time moment and simultaneously integration of the coupled Eqs. (6)–(9). Here and below we use $-e=\hbar=1$.

The c.m. coordinate Z is separated and two pairs of the classical Hamiltonian equations of motion

$$\begin{aligned}\frac{d}{dt}P_x&=\frac{\omega}{2}\left(P_y-\frac{QB}{2}X+\gamma B\langle x\rangle\right),\\ \frac{d}{dt}P_y&=\frac{\omega}{2}\left(-P_x-\frac{QB}{2}Y+\gamma B\langle y\rangle\right),\\ \frac{d}{dt}X&=\frac{1}{M}P_x+\frac{\omega}{2}Y-\frac{\gamma\omega}{Q}\langle y\rangle,\\ \frac{d}{dt}Y&=\frac{1}{M}P_y-\frac{\omega}{2}X+\frac{\gamma\omega}{Q}\langle x\rangle,\end{aligned} \tag{10}$$

coupled also with the Schrödinger equation (6) via the terms $\langle x\rangle=\langle\psi(\boldsymbol{r},t)|x|\psi(\boldsymbol{r},t)\rangle$ and $\langle y\rangle=\langle\psi(\boldsymbol{r},t)|y|\psi(\boldsymbol{r},t)\rangle$, need to be integrated.

Such an approach is analogous to the ones given in Refs. [11–13] suggested for semiclassically treating the dynamics of molecular processes [12,13]. It has the property of conserving the total energy of the system μHe^{+} and includes the coupling between the c.m. and internal variables, which is important for the problem of a fast muonic ion in a magnetic field due to the essential nonadiabaticity of the system $(m_\mu/M, m_\mu/M_0\sim 1/40)$ and high c.m. velocity $(v_\perp\simeq 6\alpha c)$.

The time-dependent three-dimensional Schrödinger equation (6) is integrated by the method developed in Refs. [14–16] simultaneously with the system of coupled Hamiltonian equations of motion (10).

In principle, one can think also of a time-dependent perturbation theoretical approach in order to describe the sublevel mixing induced by the oscillating motional electric field. However, the large center-of-mass velocity together with the strong nonadiabaticity in the case of our muonic helium would certainly require a very careful estimate of the possible range of validity of such an approach. For small center-of-mass velocities of the ‘‘electronic’’ He^{+} ion a perturbation theoretical approach for the classical dynamics of the ion has been developed in Ref. [17].

III. TRANSITIONS BETWEEN SUBLEVELS *lm* OF FAST MUONIC HELIUM IONS IN A MAGNETIC FIELD

The fast ions μHe^{+} are formed in μCF due to the muon ‘‘sticking’’ process to helium

$$dt\mu \rightarrow \mu^4\text{He} + n \tag{11}$$

partly in excited states $n \geqslant 2$ with the kinetic energy $E_{\text{cm}} = Mv^2/2 \simeq 3.5$ MeV [1–4,14]. At the present time the effect is rather well investigated both experimentally and theoretically due to its importance for the efficiency of μCF in the deuterium-tritium mixture [3]. Particularly an essential dependence of the muon ''stripping'' from the μHe^+ ions (characterized by the reactivation rate R [18,19]) on the population P_{nlm} of its excited states ($nl \neq 1s$) was found [18–21]. Here we analyze this parameter in the presence of an external magnetic field.

We have calculated the time evolution of the population

$$P_{nl}(t) = \sum_{m=-l}^{l} |\langle \phi_{nlm}(\boldsymbol{r}) | \psi(\boldsymbol{r},t) \rangle|^2 \tag{12}$$

of the sublevels l for states $n=2$ and 3 of μHe^+ for the present day laboratory fields B with strengths of a few tesla. The maximal value $v_\perp = 6v_0 = 6\alpha c$ of the μHe^+ ion c.m. velocity projection onto the xy plane, perpendicular to the direction of the magnetic field $\boldsymbol{B}$, is chosen to coincide with the initial velocity of the ion, $v \simeq 6v_0$ ($E_{\text{cm}} = Mv^2/2 \simeq 3.5$ MeV), formed in reaction (11).

The coupled Schrödinger equation (6) and classical equations (10) with the initial conditions

$$\psi(\boldsymbol{r},t=0) = \phi_{n'l'm'}(\boldsymbol{r}),$$
$$P_x(t=0) = Mv_\perp, \quad P_y(t=0) = 0, \tag{13}$$
$$X(t=0) = Y(t=0) = 0,$$

were integrated simultaneously with the same step of integration Δt over time t. Details of the computational scheme applied for the integration of the Schrödinger equation (6) can be found in Refs. [15,16]. The grids over $r \in [0, r_m = 400a_0]$ (250 grid points) and the angular variables $\{\theta, \phi\}$ (25 grid points) were constructed according to Ref. [15]. The step of integration over t was chosen as $\Delta t \leqslant 2\times 10^3 t_0$, which permitted us to keep the accuracy of the evaluated quantities (12) about a few percent after $10^5 - 10^6$ steps of integration. Here and below some values are given in muonic atomic units of $a_0 = \hbar^2/(m_\mu e^2) = 2.56\times 10^{-11}$ cm and $t_0 = \hbar^3/(m_\mu e^4) = 1.17\times 10^{-19}$ s.

Results of the calculation are presented in Figs. 1 and 2. We have analyzed two cases: the muon is initially in the $2s$ or $3s$ states of μHe^+, i.e., the quantum numbers $n'l'm'$ were fixed as 200 or 300 in Eqs. (13). The obtained data demonstrate that applying the magnetic fields of the order of a few tesla stimulates fast transitions between the lm sublevels in excited states ($n \geqslant 2$) of the μHe^+ ions moving with the velocities defined by the energy output in the fusion reaction (11). These transitions occur through energy transfer from the c.m. to the internal muon motion and become faster with increasing field strength B or the $v_\perp$ component of the initial c.m. velocity v (see Figs. 2) in agreement with the classical equations [7]

$$\frac{d}{dt}E_{\text{c.m.}} = -\frac{d}{dt}E_{\text{int}} = e\gamma(\boldsymbol{B}\times\dot{\boldsymbol{R}})\dot{\boldsymbol{r}}, \tag{14}$$

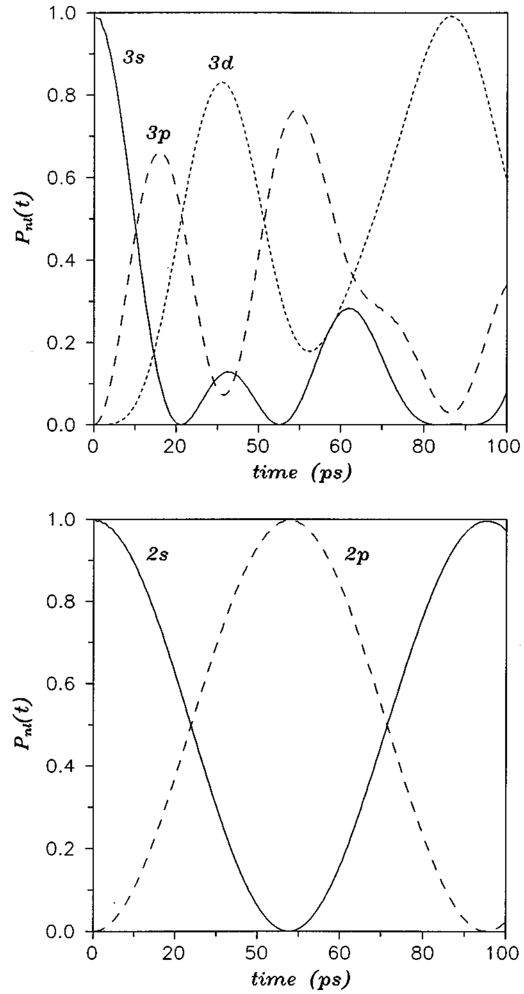

FIG. 1. Magnetic field stimulating the transitions between the lm sublevels of excited states $n=3$ and 2 in the fast $(\mu\text{He}^+)_{nlm}$ ion. The calculation of the populations $P_{nl}(t)$ was performed for the fixed projection $v_\perp = 6v_0$ of the μHe^+ initial c.m. velocity on the plane perpendicular to the magnetic field $\boldsymbol{B}$ for $B=4$ T. The initial populations have been chosen as $P_{3s}(t=0)=1$ and $P_{2s}(t=0)=1$.

where $E_{\text{c.m.}}$ and E_{int} are the energy of the center of mass and internal motion, respectively. This equation is the consequence of the classical Hamiltonian equations of motion of the two-body charged system in a magnetic field [5–7] and shows a permanent exchange of energy between the c.m. and muonic degrees of freedom.

The above results were obtained in the nonrelativistic limit for which the splitting between different lm states is given exclusively by the interaction of the muonic atom with the magnetic field. However due to its compactness the muonic helium has considerable relativistic corrections which could, in principle, suppress the above-calculated transition rates. The measured $2s_{1/2}$-$2p_{3/2}$ splitting is $\Delta E_{2S\text{-}2P} = 1.527$ eV [22]. The main contribution in the $\Delta E_{2S\text{-}2P}$ splitting is given by the vacuum polarization (VP) effect $\Delta E^{\text{VP}}_{2S\text{-}2P} = 1.667$ eV [23], which is dominant among other relativistic and short-range corrections in the muonic helium [24]. Muonic ions (atoms) are much more sensitive than electronic ones to the VP alteration of the Coulomb interaction, because the dimension of the muonic ion a_0/z

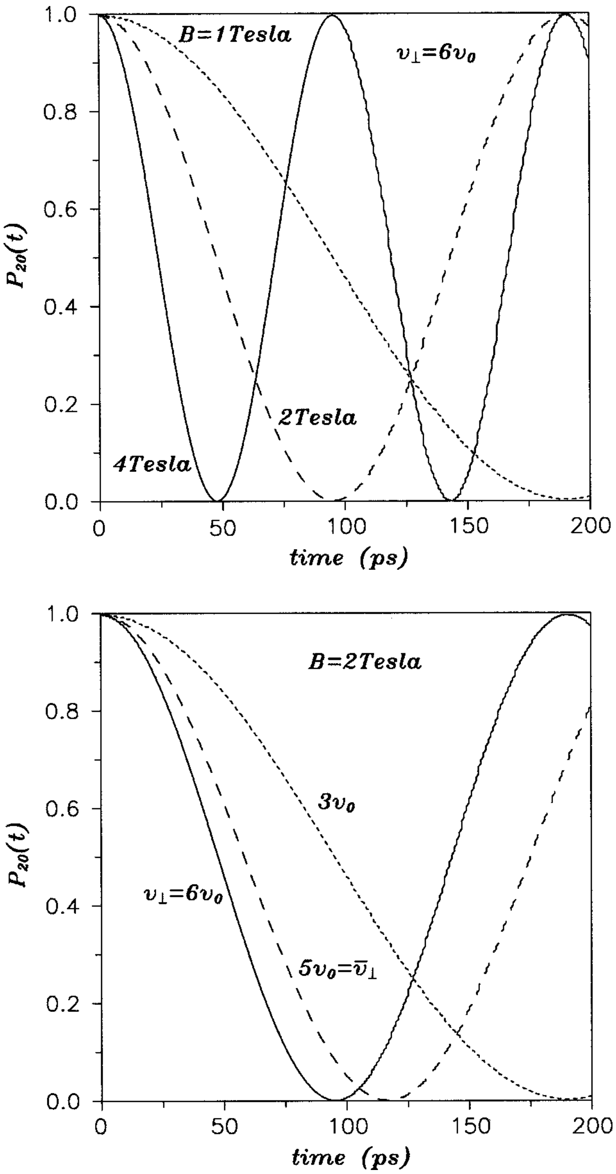

FIG. 2. Dependence of the time evolution of the population $P_{2s}(t)$ on the magnetic-field strength B (for fixed $v_\perp=6v_0$) and the $v_\perp$ component of the initial c.m. velocity v of the $\mu\mathrm{He}^+$ ion (for fixed $B=2$ T). The mean value $\overline{v_\perp}=\sqrt{2/3}\times 6v_0\simeq 5v_0$ corresponds to the initial kinetic energy 3.5 MeV of the $\mu\mathrm{He}^+$ ion emitting in dt fusion reactions.

$=\hbar^2/(m_\mu e^2 z)\simeq 2.6/z\times 10^{-11}$ cm is close to the Compton electron wavelength $\lambda_e=\hbar/m_e c\simeq 3.9\times 10^{-11}$ cm.

To evaluate the influence of the main relativistic effect on the magnetic field stimulated $2s$-$2p$ transitions we have integrated Eqs. (6)–(10) with the additional VP potential

$$\Delta U(r)=-\alpha\frac{2z}{3\pi r}\int_1^\infty\frac{\sqrt{x^2-1}}{x^2}\left(1+\frac{1}{2x^2}\right)e^{-2(r/\lambda_e)x}\,dx, \tag{15}$$

including in the Hamiltonian (7) an effective interaction between the muon and helium nucleus due to the virtual production of a single e^+e^- pair. The results presented in Fig. 3 demonstrate that the slowing down of the $2s$ state depopulation due to the relativistic $2s$-$2p$ splitting becomes considerable for times of the order $t\simeq 10^{-10}$ s for the chosen parameters of B and $v_\perp$.

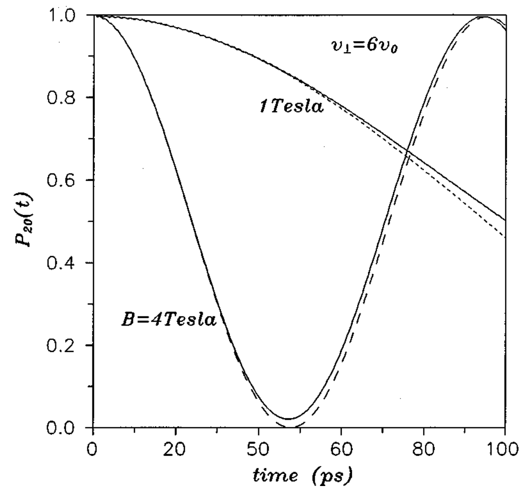

FIG. 3. Time evolution of the population $P_{2s}(t)$ with and without the relativistic splitting $\Delta E_{2S\text{-}2P}$. Broken curves have been calculated without the VP term (15), the solid lines correspond to calculations including the VP term in the effective Hamiltonian (7).

The calculated rates of the $2s$-$2p$ and $3s$-$3p$-$3d$ mixing exceed, at least by two orders of magnitude, the rates of the resonance muonic molecule formation ($\sim 10^8$–10^9 s^{-1}) [1–4] and are comparable with other transition rates of the $\mu\mathrm{He}^+$ in the process of the deceleration in a dense deuterium-tritium mixture (transitions due to inelastic collisions, Auger transitions, Stark mixing [18,19,25,26]). Transitions between different n, stimulated by the driven field of the order of a few tesla, are much slower then sublevel transitions with the same n. Thus, for low density with suppression of the collisional transitions such that only the radiative transitions $nl\rightarrow n'l'$ between some known states remained essential [18,19], one can drive the sublevels population P_{nl} by varying the strength of the applied magnetic field. Different modeling of the $\mu\mathrm{He}^+$ ion time evolution in the dt mixture shows the strong dependence of the muonic ions x-ray yield from the $2s$-$2p$ population (especially for $K\alpha$ radiation). Some estimations show also an influence of these populations on the reactivation rate R [18,19,27]. Both parameters, reactivation rate R and $K\alpha$ lines intensity, are under extensive experimental investigation so far [3,20,21,28]. Thus, the experimental achievement in μCF makes it possible to analyze directly the influence of the driving magnetic field to the $K\alpha$ x-ray production and the reactivation rates R by measuring these parameters at low densities in the presence of external magnetic fields. The possibility to create a well-defined mixing of the l sublevels in such kind of experiments looks especially valuable, because different hypotheses about the $2s$-$2p$ mixing have been used so far in modeling the $\mu\mathrm{He}^+$ time evolution [18,19] due to a considerable variation for the estimates of the $2s$-$2p$ Stark mixing by different authors [19,25–27].

IV. SELF-IONIZATION OF HIGHLY EXCITED FAST MUONIC HELIUM IONS IN STRONG MAGNETIC FIELDS

In Refs. [5,6] the self-ionization process for ions in the presence of an external magnetic field has been predicted and

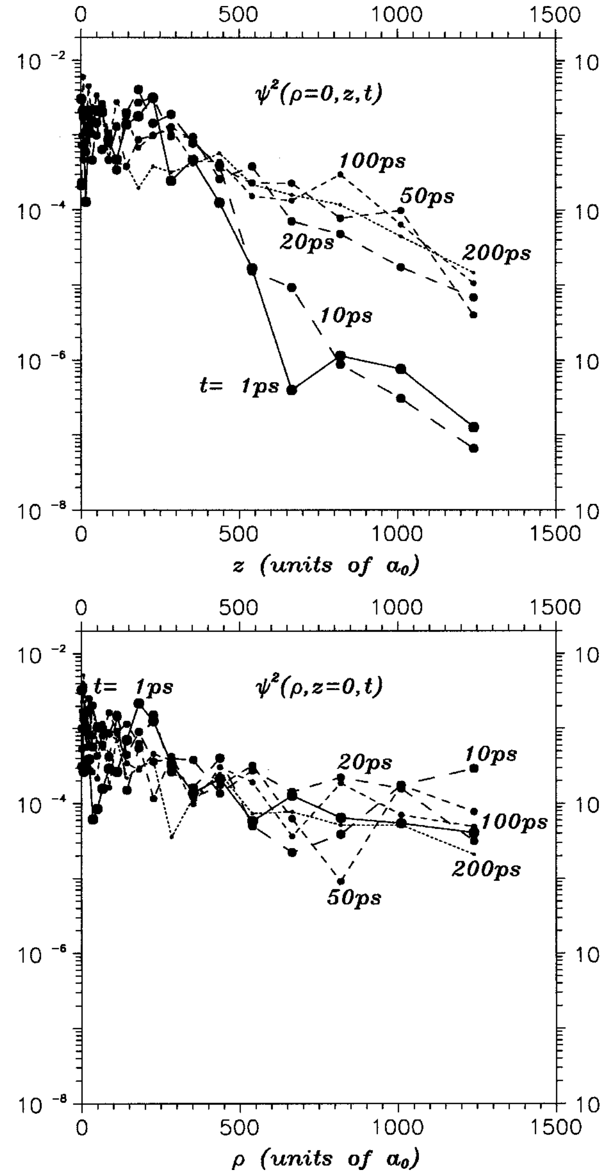

FIG. 4. Time evolution of the muon densities $[\psi(\rho=0,z,t)]^2$ and $[\psi(\rho,z=0,t)]^2$ starting with the initially populated state $nlm=15,0,0$ for $B=4\times10^3$ T. The distances are given in muonic atomic units $a_0=2.56\times10^{-11}$ cm.

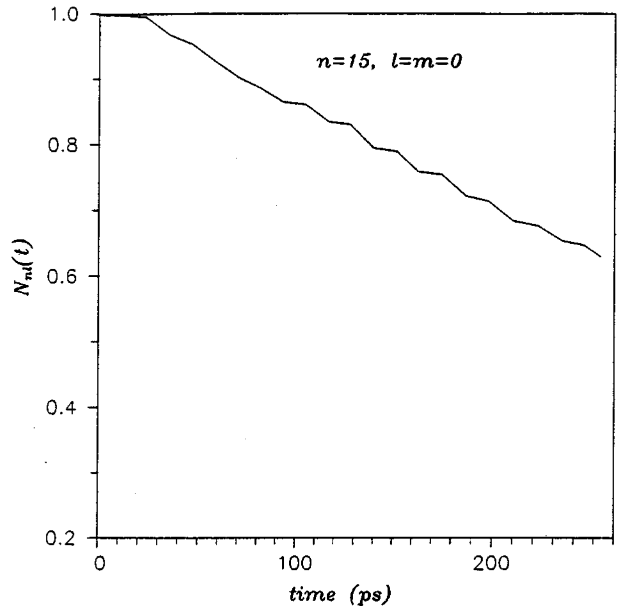

FIG. 5. Norm $N_{nlm}(t)$ decay of the initially populated state $nlm=15,0,0$ for $B=4\times10^3$ T.

demonstrated by solving the classical equations of motion for Rydberg states of the one-electron He^+ ion. Here we discuss briefly the possibility to ionize the fast highly excited μHe^+ ion by a strong magnetic field. Actually, the muon kinetic energy in the direction of the magnetic field $\boldsymbol{B}$ can become large enough in order to ionize the system through the energy transfer from the c.m. to the internal motion according to Eq. (14). Perpendicular to the magnetic field the muonic motion is finite because of the confining property of the magnetic field. This is a principal difference of the self-ionization in the presence of a magnetic field compared to the classical ionization by an external electric field.

In the present calculations (the initial population is chosen to be $P_{15s}(t=0)=1$, i.e., the $n=15$, $l=m=0$ state) we used a more detailed grid over $r\in[0,r_m=1500a_0]$ (400 grid points) as compared to the calculations for the low-lying states $n=2,3$. The boundary r_m was chosen approximately 10 times exceeding the initial value of the mean radius $\langle r\rangle$ of the μHe^+ ion in the $n=15$, $l=m=0$ state. Following Refs. [15,29] we have used an absorbing boundary condition at the point $r=r_m$. It permits to prevent the artificial reflection of the muon flux from the grid boundary as well as allows an estimation of the ionization rate by analyzing the decay of the muon norm $N_{15s}(t)=\int|\psi(\boldsymbol{r},t)|^2\,d\boldsymbol{r}$ with time [29].

The calculated time evolution of the muon densities in the z direction of $\boldsymbol{B}$ and the perpendicular direction are presented in Figs. 4 for the strong field $B=4\times10^3$ T. They demonstrate considerable spreading of the muon density in the z direction for $t\geqslant2\times10^{-11}$ s. The evaluated norm $N_{15s}(t)$ (Fig. 5) gives the following estimate $t_I\simeq10^{-10}$ s for the order of the self-ionization time. This shows that the self-ionization is at least one order of magnitude faster than the process of the muonic molecule resonant formation ($\sim10^{-8}$–10^{-9} s [1–4]) and potentially may be useful for increasing the muon stripping from the highly excited states ($n\sim15$) of the $(\mu He^+)_n$ ions. However, it is at the time not clear how to make this process efficient with respect to the increase of the reactivation rate R. Actually, the $(\mu He^+)_n$ ions are formed in reaction (11) mainly in low-lying excited states (only a few percents of the muonic ions are in $n>5$ [1–4]) for which the self-ionization is much slower. Our estimates, made for the same field strength $B=4\times10^3$ T, show a fast increase of the self-ionization time with decreasing n. Particularly, for $n=5$ the self-ionization time already exceeds the critical value $\sim10^{-8}$ s defined by the resonant muonic molecule formation. In principle it may be compensated by increasing the strength of the applied field to $B>4\times10^3$ T, which is, however, orders of magnitude beyond the currently available fields at high magnetic-field facilities [30]. The so far known mechanisms [27] also do not permit an efficient excitation of the fast muonic ions.

V. CONCLUSIONS

In this work we analyzed the transitions stimulated by external magnetic fields from excited states of the fast μHe^+

ions. The results have been obtained within an approach developed for treating nonadiabatic two-body charged system in external magnetic fields on a mixed quantum-semiclassical level.

It was shown that the present-day laboratory fields of the order of a few tesla may stimulate strong $2s$-$2p$ mixing as well as l mixing of the sublevels for $n>2$ in the μHe^+ ions formed in μCF. The effect of the magnetic field stimulating the $2s$-$2p$ transitions can be analyzed experimentally by measuring the dependence of the intensities of the $K\alpha$ lines from the μHe^+ on the field strength. It is also interesting to analyze experimentally the influence of this effect on another important μCF parameter, the reactivation rate R, due to the possible dependence of the value R on the $2s$-$2p$ mixing in μHe^+ [25,26]. The possibility of the self-ionization process for fast μHe^+ ions in highly excited states has been demonstrated for strong magnetic fields.

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