Einstein-Podolsky-Rosen-correlated atomic ensembles

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(Received 22 May 1998)

A method for generating two entangled macroscopic ensembles of atoms via interaction with Einstein-Podolsky-Rosen-correlated light is proposed. The method is directly applicable for creating entanglement between, e.g., two distant atom traps. The proposed special type of partial entanglement is analyzed. [S1050-2947(99)09706-1]

PACS number(s): 03.65.Bz, 03.67.−a, 42.50.Ct, 42.50.Lc

The realization of quantum-correlated physical systems has become one of the most exciting playgrounds for the demonstration of the weirdness of physical reality. Entangled Einstein-Podolsky-Rosen (EPR) quantum states [1–4] quantum teleportation [6], and quantum logic gates [7] are prime examples of such systems. Most of the results to date involve demonstrations of such correlations either between the states of the electromagnetic (e.m.) field [1,2,6] or between quantum states within an atom or a molecule [7]. One exception where entangled single pairs of atoms are generated via interaction with a microwave cavity field is Ref. [3]. Another recent achievement is the demonstration of the entanglement between two trapped ions [4].

In the present paper we propose a method for the generation of the *distant EPR-correlated macroscopic* atomic ensembles. Our method uses free propagating EPR-correlated light to produce entangled atoms and can be viewed as a method of mapping the nonlocal correlations transmitted by the light onto atoms [5]. The proposed method does not require any action on the entire vacuum reservoir, a formidable experimental challenge, but merely utilizes Gaussian-shaped entangled beams.

We consider a subthreshold optical parametric oscillator (OPO) as a source of the EPR-correlated light [2,8]. EPR correlations in orthogonally polarized components of the OPO output generated via a type-II parametric process have been demonstrated experimentally [2]. Below we discuss a type-I process where correlations are generated between the different frequency components of the same polarization as in the original proposal [8].

Consider two output modes a_+ and a_- of the subthreshold OPO with central frequencies ω_+ and ω_-. Here $\omega_+ + \omega_- = \omega_p$ where ω_p is the pump frequency. According to [8], these modes possess EPR correlations between quadrature phase amplitudes $X_\pm(\varphi_\pm) = \frac{1}{2}[a_\pm e^{i\varphi_\pm} + a_\pm^\dagger e^{-i\varphi_\pm}]$. Namely, $[X_+(\varphi) - X_-(-\varphi)]^2 \to 0$ and $[X_+(\varphi) + X_-(\pi - \varphi)]^2 \to 0$ as the OPO approaches the threshold, meaning that the amplitudes and phases of the two field modes are quantum copies of each other.

The mapping of the entanglement of the field modes onto the two atomic ensembles proceeds as follows. The OPO output modes a_+, a_- and two classical fields $\alpha_+ e^{-i\varphi_+}, \alpha_- e^{-i\varphi_-}$, where the subscripts refer to the central frequencies ω_+ and ω_-, propagate through two separate clouds of V-type three level atoms. α_+, a_+ interact only with one cloud (ensemble +) and α_-, a_- interact only with the other (ensemble −) (Fig. 1). Such a selective interaction can be easily achieved for atoms by, e.g., choosing two appropriate hyperfine transitions. We also assume that the fields are completely absorbed by atoms; again a_+, α_+ are absorbed in the first cloud and a_-, α_- in the second. As demonstrated in [9], a quantum property of light such as squeezing can be efficiently transferred onto atomic degrees of freedom when the optical depth of the atomic ensemble for relevant frequency components of the input squeezed light is $\gg 1$.

As shown in [9], a Fourier component of the collective atomic operator, $F_{1,-1}^\pm(t) = \Sigma |1\rangle_i^\pm \langle -1|_i^\pm$, of one of the excited atomic ensembles (summed over all atoms of the ensemble) obeys the following equation:

$$F_{1,-1}^\pm(\Delta) = \frac{\alpha_\pm e^{-i\varphi_\pm}}{\gamma - i\Delta} \{a_\pm(\Delta) + d_\pm(\Delta)\}. \tag{1}$$

Here the indices + and − refer to each of the atomic ensembles, respectively. $d(\Delta)$ and $a(\Delta)$ denote the Fourier components of the vacuum field operator and the input quantum field, while Δ is the detuning from the atomic resonance. The vacuum field in Eq. (1) describes the effect of spontaneous emission. Its presence limits the degree of quantum noise reduction as discussed below. The field $\alpha_\pm$ is in resonance with the atomic transition frequency of the ensemble ±. Also, γ is the spontaneous decay rate of the upper states ±. The spectral envelope of $a(\Delta)$ varies slowly across the band-

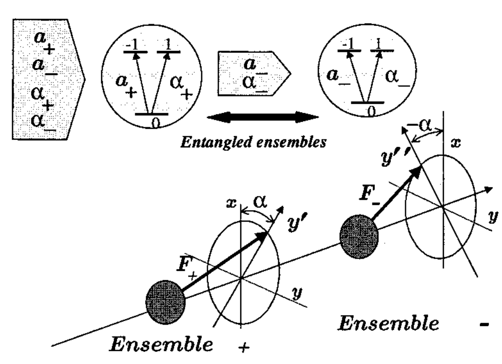

FIG. 1. Entangling two atomic clouds with EPR light. Level schemes of atoms in the two clouds and the four optical fields are shown (see comments in the text). Projections of the ensembles' collective spins F_+ and F_- on any pair of symmetric axes y' and y'' are entangled.

width γ, i.e., quantum correlations are broadband. Equation (1) holds under the assumption of the complete absorption of the field $a_\pm$ in the $\pm$ atomic ensemble. For an ensemble of atoms in states $|1\rangle^\pm$ and $|-1\rangle^\pm$, the (quasi)spin components can be defined as $F_x^\pm = \frac{1}{2}(F_{1,-1}^\pm + F_{-1,1}^\pm)$, $F_y^\pm = \frac{1}{2} i (F_{-1,1}^\pm - F_{1,-1}^\pm)$, $F_z^\pm = \frac{1}{2}(F_{11}^\pm - F_{-1,-1}^\pm)$. The fields are normalized so that $|\alpha|^2$ is the photon flux. Under the assumption of complete absorption the number of atoms in excited states $\pm$ will be given by $N_\pm = |\alpha_\pm|^2/\gamma$. The spin component (1) is therefore proportional to the square root of the number of atoms as expected, and the variances of the low-frequency ($\Delta \ll \gamma$) x components of the collective spin of each of the ensembles are

$$\begin{aligned}\delta\{F_x^\pm\}^2 &= \langle \tfrac{1}{4} F_z^\pm [\{a_\pm e^{-i\varphi_\pm} + a_\pm^\dagger e^{i\varphi_\pm}\}^2 + \{d + d^\dagger\}^2] \rangle \\ &= \langle F_z^\pm \rangle [\langle X_\pm^2(\varphi)\rangle + \tfrac{1}{4}], \\ \delta\{F_y^\pm\}^2 &= \langle \tfrac{1}{4} F_z^\pm [\{a_\pm e^{-i(\varphi_\pm + \pi/2)} + a_\pm^\dagger e^{i(\varphi_\pm + \pi/2)}\}^2 \\ &\quad + \{d + d^\dagger\}^2] \rangle \\ &= \langle F_z^\pm \rangle [\langle X_\pm^2(\varphi + \pi/2)\rangle + \tfrac{1}{4}]. \end{aligned} \tag{2}$$

In the last equations we used the fact that the vacuum noise d is phase insensitive, white and independent for both ensembles. Compared to the coherent spin state for which $\delta F_{x,y}^2 = \frac{1}{2}\langle F_z \rangle = \frac{1}{4} N$, the spin states (2) are more noisy because each of the fields $a_\pm$ taken separately is in a thermal state [10] and, therefore, $X_\pm^2(\varphi) \geqslant \frac{1}{4}$ for any φ. In fact, for an OPO pumped at the half-threshold power, $X_\pm^2(\varphi)$ is 30 times greater than for the coherent state. It is, therefore, evident that a probe interacting solely with either of the atomic ensembles "$+$" and "$-$" will only reveal the spin noise which greatly exceeds the quantum noise of the coherent spin state.

Consider now the probe light which propagates through both ensembles and interacts with the total spin $F = F^+ + F^-$ (Fig. 1). The probe is resonant with transitions from the ± 1 states to some upper state. As demonstrated in [11], the probe will measure the quantities $\delta F_{x,y}^2$, which can be written using Eqs. (2) as

$$\begin{aligned}\delta F_x^2 = \delta\{F_x^+ + F_x^-\}^2 &= \langle \tfrac{1}{8} F_z [\{a_+ e^{-i\varphi_+} + a_+^\dagger e^{i\varphi_+} + a_- e^{-i\varphi_-} \\ &\quad + a_-^\dagger e^{i\varphi_-}\}^2 + 2\{d + d^\dagger\}^2] \rangle \\ &= \tfrac{1}{2}\langle F_z\rangle [\langle \{X_+(\varphi_+) + X_-(\varphi_-)\}^2\rangle + \tfrac{1}{2}], \end{aligned} \tag{3}$$

$$\begin{aligned}\delta F_y^2 = \delta\{F_y^+ + F_y^-\}^2 &= \langle \tfrac{1}{8} F_z [\{a_+ e^{-i(\varphi_+ + \pi/2)} + a_+^\dagger e^{i(\varphi_+ + \pi/2)} \\ &\quad + a_- e^{-i(\varphi_- + \pi/2)} + a_-^\dagger e^{i(\varphi_- + \pi/2)}\}^2 \\ &\quad + 2\{d + d^\dagger\}^2] \rangle \\ &= \tfrac{1}{2}\langle F_z\rangle [\langle \{X_+(\varphi_+ + \pi/2) + X_-(\varphi_- + \pi/2)\}^2\rangle + \tfrac{1}{2}]. \end{aligned} \tag{4}$$

It is assumed in Eqs. (3) and (4) that the two ensembles have equal mean z spin components. Hence, $\langle F_z \rangle = 2\langle F_z^\pm \rangle$ is the total mean spin of the ensembles. The conditions under which the probe is sensitive to either F_x or to F_y are specified in [11]. For the vacuum input modes $a_\pm$, the equality $\langle \{X_+ + X_-\}^2 \rangle = \frac{1}{2}$ holds true, independent of the choice of the phases. Therefore, $\delta F_x^2 = \delta F_y^2 = \frac{1}{2}\langle F_z \rangle$ which indicates that the total spin of the two atomic ensembles is in a coherent state due to the statistical independence of the individual atomic spins of the two ensembles, as one might expect.

If, however, the fields $a_\pm$ are the EPR-correlated output fields of the parametric amplifier, and if the phases of the coherent fields are chosen properly, we obtain, from Eqs. (3) and (4),

$$\delta F_x^2 = \delta\{F_x^+ + F_x^-\}^2 = \tfrac{1}{4}\langle F_z \rangle \tag{5}$$

for $\varphi_+ + \varphi_- = \pi$ and

$$\delta F_y^2 = \delta\{F_y^+ + F_y^-\}^2 = \tfrac{1}{4}\langle F_z \rangle \tag{6}$$

for $\varphi_+ + \varphi_- = 0$. The above equations prove that the *macroscopic ensembles* $+$ and $-$ *are entangled*. The entanglement is evidenced by the fact that the variance of the sum of the quantities belonging to the $+$ and $-$ ensembles is less (in this case a factor of 2 less) than the quantum limit corresponding to the uncorrelated ensembles. Imperfect entanglement, i.e., the nonzero variances, is due to spontaneous decay of atomic states.

As shown in [11], the quantum spin noise of a collection of cold atoms in a magneto-optical trap (MOT) can be readily observed in the probe noise spectrum. To demonstrate the entanglement described in the present paper two atom clouds precooled in separate MOT's can be excited with the EPR beams, as in Fig. 1 (the trapping fields may have to be turned off). The probe resonant with a transition with the state F as the lower one can be used to measure the suitable projections of the collective spin of the two clouds. While each of the clouds has a rather uncertain projection of its collective spin on any axis perpendicular to the direction of the excitation, the projections of the two spins on x and y axes are entangled. This entanglement will manifest itself in the reduction of the quantum spin noise below the quantum limit reached in [11].

An additional insight into the proposed multiatom entanglement can be gained by considering a Stern-Gerlach type of spin measurement performed on atoms of the two ensembles. By neglecting spontaneous emission in this state analysis we assume perfect correlations, i.e., δF_y^2 (or δF_x^2) $= 0$.

We will now trace the connection between the individual atomic spin states of the $+$ and $-$ atoms, on the one hand, and the collective spin state on the other. Suppose the spins are projected along the axis corresponding to the maximal entanglement (zero collective spin variance), e.g., x, with the choice of phases $\varphi_+ + \varphi_- = \pi$. For N uncorrelated atoms of which $\frac{1}{2}N$ belongs to each of the $+$ and $-$ ensembles, the state can be written as $|\Psi\rangle_{\mathrm{coh}} = (|\frac{1}{2}\rangle + |-\frac{1}{2}\rangle)^N$. Here we consider, for simplicity, spin-$\frac{1}{2}$ atoms. The resulting collective spin is characterized by a binomial distribution with zero mean value and a variance $\delta F_x^2 = \frac{1}{4}N = \frac{1}{2}\langle F_z \rangle$. A value of the variance less than that suggests correlations between the individual spins of the two ensembles. In particular, $\delta F_y^2 = 0$ implies that the number of atoms in state $|\frac{1}{2}\rangle$ in both ensembles is equal to the number of atoms in state $|-\frac{1}{2}\rangle$. Such a state of N atoms can be written as

$$|\Psi\rangle_{\mathrm{EPR}}=a_0|\tfrac{1}{2},\tfrac{1}{2},\ldots,\tfrac{1}{2}\rangle^{+}*|-\tfrac{1}{2},-\tfrac{1}{2},\ldots,-\tfrac{1}{2}\rangle^{-}$$
$$+a_1|-\tfrac{1}{2},\tfrac{1}{2},\ldots,\tfrac{1}{2}\rangle^{+}*|\tfrac{1}{2},-\tfrac{1}{2},\ldots,-\tfrac{1}{2}\rangle^{-}+a_2|$$
$$-\tfrac{1}{2},-\tfrac{1}{2},\ldots,\tfrac{1}{2}\rangle^{+}*|\tfrac{1}{2},\tfrac{1}{2},\ldots,-\tfrac{1}{2}\rangle^{-}+\cdots+a_{N/2}|$$
$$-\tfrac{1}{2},-\tfrac{1}{2},\ldots,-\tfrac{1}{2}\rangle^{+}*|\tfrac{1}{2},\tfrac{1}{2},\ldots,\tfrac{1}{2}\rangle^{-}$$

plus all possible permutations within each bracket. Each bracket contains $\frac{1}{2}N$ terms; the first bracket of each term describes the $+$ ensemble while the second bracket describes the $-$ ensemble. The number of $|\frac{1}{2}\rangle$ and $|-\frac{1}{2}\rangle$ atoms in each term is the same. The state has to be averaged over the statistical distribution $p(N)$. Note, however, that fluctuations of N of the order of $\sqrt{N}$ affect the entanglement only to the order of $1/\sqrt{N}$. As the entangled ensembles are prepared by addressing atoms collectively, there is no way to identify which particular pairs of atoms in the two ensembles are entangled. Therefore, the spin measurement of a randomly picked pair of atoms from the two ensembles will produce a small degree of entanglement, of the order of N^{-1}. However, if we measure the *collective spins* of the ensembles (count all the atoms), we obtain, using $|\Psi\rangle_{\mathrm{EPR}}$, the equality $F_x^+=-F_x^-$, as a result of the individual spin entanglement.

The quantum character of the above described entanglement as opposed to the ''Bertlmann socks''–type classical correlations [12] is supported by the following argument. Let us choose the phases of the atomic excitation in Eqs. (3) and (4) to be $\varphi_+=\varphi_-=0$. Then, according to Eq. (6), the y spin projections of the $+$ and $-$ ensembles are entangled. However, we can also devise an experiment with a *random choice of the projection axes*. Suppose one measures the spin projection of the $+$ ensemble on the axis y' tilted at some angle α_+ with regard to the y axis (Fig. 1). If the spin of the $-$ ensemble is projected on the axis y'' tilted at an angle α_- with regard to y, the result of the measurement $F_{y'}^{+}+F_{y''}^{-}$ is still described by Eq. (4) with the substitution $\varphi_\pm\rightarrow\alpha_\pm$. We conclude from Eq. (6) that with an arbitrary choice of y', the entanglement between the spins of the two ensembles is present provided that y'' is chosen according to $\alpha_++\alpha_-=\pi$, i.e., provided that y' and y'' are symmetric around x (Fig. 1).

If the entanglement is not perfect [due, for example, to the spontaneous emission as in Eqs. (5) and (6)], an admixture of uncorrelated states should be added to $|\Psi\rangle_{\mathrm{EPR}}$. Imperfect absorption of the entangled light (optical depth OD$\gtrsim 1$) will also lead to degradation of the atomic entanglement proportionally to $1-\exp(-\mathrm{OD})$, in a way similar to the spin squeezing degradation described in [9]. However, with the current trapping technology this factor can be very close to unity.

Entanglement of the kind described above is robust against decoherence. As evidenced by Eq. (1), the spectral width of the spin noise is of the order of γ. Consequently, the decoherence time of the entangled state is of the order of the lifetime of the atomic state γ^{-1}. This is because it is essentially a pairwise entanglement, although it involves many particles. Therefore, it decays much slower than a maximal multiparticle entanglement of the kind $|00\ldots00\rangle+|11\ldots11\rangle$, and can serve as an example of a partial entanglement which is optimal for particular applications. Note, for example, that the state $|\Psi\rangle_{\mathrm{EPR}}$ bears some similarity to the ''optimal spectroscopic partially entangled state'' discussed in [13].

In summary, we propose a way to entangle two atomic ensembles. Three features of the proposal should be emphasized: the atomic entanglement is nonlocal, it is produced via mapping of the propagating light onto atoms, and finally the entangled ensembles are macroscopic. Our mapping strategy uses weak coupling of a free propagating field with an optically thick multiatom ensemble as opposed to the strong coupling between a cavity field and a single atom used before [3,14]. Recently, multiatom nonclassical states [9,15] have attracted considerable attention. The present proposal considers the generation of yet another type of such states. The mapping of the entanglement carried by light onto atoms is relevant for various quantum information protocols, for overcoming quantum limits in atomic time standards, and for the development of quantum memory. Such mapping is a natural development of quantum cryptography with EPR photons [16] towards the realization of the storage of quantum information. Observation of this entanglement for short-lived atomic states looks experimentally feasible in view of the recent spectroscopic detection of multiatom spin noise at the quantum level [11]. Although in this paper we consider a specific type of entanglement, in principle, our method can be generalized to map other types of entanglement of light onto atoms.

I am grateful to H. J. Kimble and K. Mølmer for stimulating discussions. This research has been supported by the Danish Natural Sciences Research Council.

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