

Using parity kicks for decoherence control

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We show how it is possible to suppress decoherence using tailored external forcing acting as pulses. In the limit of infinitely frequent pulses decoherence and dissipation are completely frozen; however, a significant decoherence suppression is already obtained when the frequency of the pulses is of the order of the reservoir typical frequency scale. This method could be useful in particular to suppress the decoherence of the center-of-mass motion in ion traps. [S1050-2947(99)03606-9]

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I. INTRODUCTION

Decoherence is the process which limits our ability to maintain pure quantum states, or their linear superpositions. It is the phenomenon by which the classical world appears from the quantum one [1]. In more physical terms it is described as the rapid destruction of the phase relation between two, or more, quantum states of a system caused by the entanglement of these states with different states of the environment. The present widespread interest in decoherence is due to the fact that it is the main limiting factor for quantum information processing. We can store information, indeed, in two-level quantum systems, known as quantum bits (qubits), which can become entangled with each other, but decoherence can destroy any quantum superposition, reducing the system to a mixture of states, and the stored information is lost. For this reason decoherence control is now becoming a rapidly expanding field of investigation.

In a series of previous papers [2–5] we have faced the control of decoherence by actively modifying the system's dynamics through a feedback loop. This procedure turned out to be very effective, in principle [4], to slow down the decoherence of the only experiment [6], up to now, in which the decoherence of a mesoscopic superposition was detected. The main limiting aspect of this procedure is connected with the need for a measurement. In order to do the feedback in the appropriate way, one has first to perform a measurement and then the result of this measurement can be used to operate the feedback. However, any physical measurement is subject to the limitation associated with a nonunit detection efficiency. We have shown [5] that with detection efficiency approaching unity the quantum superposition of states stored in a cavity can be protected against decoherence for many decoherence times t_{dec} , where t_{dec} is defined as the cavity relaxation time divided by the average photon number [7].

We wish now to face the problem of eliminating the measurement in controlling the decoherence. We show, here, how it is possible to inhibit decoherence through the application of suitable open-loop control techniques to the system of interest, that is, by using appropriately shaped time-varying control fields. To be more specific, decoherence can be inhibited by subjecting a system to a sequence of very frequent *parity kicks*, i.e., pulses designed in such a way that

their effect is equivalent to the application of the parity operator on the system.

The paper is organized as follows. In Sec. II the parity kick method is presented in its generality and it is shown how decoherence and dissipation are completely frozen in the limit of infinitely frequent pulses. In Sec. III the method is applied to the case of a damped harmonic oscillator, such as, for example, a given normal mode of a system of trapped ions. In Sec. IV the numerical results corresponding to this case are presented, showing that a considerable decoherence suppression is obtained when the parity kick repetition rate becomes comparable to the typical time scale of the environment. In Sec. V the possibility of applying this scheme to harness the decoherence of the center-of-mass motion in ion traps is discussed.

II. THE GENERAL IDEA

Let us consider a generic open system, described by the Hamiltonian

$$H = H_A + H_B + H_{\text{int}}, \quad (1)$$

where H_A is the bare system Hamiltonian, H_B denotes the reservoir Hamiltonian, and H_{int} is the Hamiltonian describing the interaction between the system of interest and the reservoir, which is responsible for dissipation and decoherence. We shall now show that if the Hamiltonian (1) possesses appropriate symmetry properties with respect to parity, it is possible to actively control dissipation (and the ensuing decoherence) by adding suitably tailored time-dependent external forcing acting on system variables only. The new Hamiltonian becomes

$$H_{\text{tot}} = H + H_{\text{kick}}(t), \quad (2)$$

where

$$H_{\text{kick}}(t) = H_k \sum_{n=0}^{\infty} \theta(t - T - n(T + \tau_0)) \theta((n+1)(T + \tau_0) - t) \quad (3)$$

[$\theta(t)$ is the usual step function] is a time-dependent, periodic, *system operator* with period $T + \tau_0$ describing a train of

pulses of duration τ_0 separated by the time interval T . The stroboscopic time evolution is therefore given by the evolution operator

$$U(NT + N\tau_0) = [U_{\text{kick}}(\tau_0)U_0(T)]^N, \quad (4)$$

where $U_{\text{kick}}(\tau_0) = \exp\{-i\tau_0(H + H_k)/\hbar\}$ describes the evolution during the external pulse and $U_0(T) = \exp\{-iTH/\hbar\}$ gives the standard evolution between the pulses. We now assume that the external pulse is so strong that it is possible to neglect the standard evolution during the pulse, $H_k \gg H$, and then we assume that the pulse Hamiltonian H_k and the pulse width τ_0 can be chosen so as to satisfy the *parity kick condition*

$$U_{\text{kick}}(\tau_0) \simeq e^{-(i/\hbar)H_k\tau_0} = P, \quad (5)$$

where P is the system parity operator.

It is now possible to see that such a time-dependent modification of the system dynamics is able to perfectly protect the system dynamics and completely inhibit decoherence whenever the following general conditions are satisfied:

$$PH_A P = H_A, \quad (6)$$

$$PH_{\text{int}} P = -H_{\text{int}}, \quad (7)$$

that is, the system Hamiltonian is parity invariant and the interaction with the external environment anticommutes with the system parity operator. To be more specific, one has that in the ideal limit of continuous parity kicks, that is,

$$\begin{aligned} T + \tau_0 &\rightarrow 0, \\ N &\rightarrow \infty, \end{aligned} \quad (8)$$

$$t = N(T + \tau_0) = \text{const.},$$

the pulsed perturbation is able to eliminate *completely* the interaction with the environment and therefore all the physical phenomena associated with it, i.e., energy dissipation, diffusion, and decoherence. To see this it is sufficient to consider the evolution operator during two successive parity kicks, which, using the parity kick condition of Eq. (5), can be written as

$$U(2T + 2\tau_0) = P e^{-(i/\hbar)HT} P e^{-(i/\hbar)HT}. \quad (9)$$

Since

$$P e^{-(i/\hbar)HT} P = e^{-(i/\hbar)PHPT} = e^{-(i/\hbar)(H_A + H_B - H_{\text{int}})T}, \quad (10)$$

one has that the time evolution after two successive kicks is driven by the unitary operator

$$U(2T + 2\tau_0) = e^{-(i/\hbar)(H_A + H_B - H_{\text{int}})T} e^{-(i/\hbar)(H_A + H_B + H_{\text{int}})T}. \quad (11)$$

This expression clearly shows how the pulsed perturbation is able to ‘‘freeze’’ the dissipative interaction with the environment: the application of two successive parity kicks alternatively changes the sign of the interaction Hamiltonian between system and reservoir. Therefore one expects that, in

the limit of continuous kicks, this sign inversion becomes infinitely fast and the interaction with the environment averages exactly to zero.

This fact can be easily shown just using the definition of the continuous kick limit. In fact, in this limit

$$\begin{aligned} U(t) &= \lim_{T + \tau_0 \rightarrow 0} [e^{-(i/\hbar)(H_A + H_B - H_{\text{int}})T} \\ &\quad \times e^{-(i/\hbar)(H_A + H_B + H_{\text{int}})T}]^{t/(2T + 2\tau_0)}, \end{aligned} \quad (12)$$

which, using just the definition of the exponential operator, yields

$$U(t) = e^{-(i/\hbar)(H_A + H_B)t}. \quad (13)$$

This means that in the ideal limit of continuous parity kicks, the interaction with the environment is completely eliminated and only the free uncoupled evolution is left.

Let us briefly discuss the physical interpretation of this result. The continuous kick limit (8) is formally analogous to the continuous measurement limit usually considered in the quantum Zeno effect (see, for example, [8,9]) in which a stimulated two-level transition is inhibited by a sufficiently frequent sequence of laser pulses. However, this is only a mathematical analogy because at the physical level one has two opposite situations. In fact, during the pulses, in the quantum Zeno effect the interaction with the environment (i.e., the measurement apparatus) prevails over the internal dynamics, while in the present situation the interaction with the reservoir is practically turned off by the externally controlled internal dynamics [see Eq. (5)].

This parity kick idea has instead a strong relationship with spin echo phenomena [10] in which the application of appropriate rf pulses is able to eliminate much of the dephasing in nuclear magnetic resonance spectroscopy experiments. These rf pulses realize a sort of time reversal and a similar situation takes place here in the case of parity kicks. In fact the relevant evolution operator is given by the unitary operator governing the time evolution after two pulses $U(2T + 2\tau_0)$ of Eq. (11), in which the standard evolution during a time interval T is followed by an evolution in which the interaction with the environment is time reversed for the time T . If the time interval T is comparable to the time scale at which dissipative phenomena take place, this time reversal is too late to yield appreciable effects, while if T is sufficiently small the very frequent reversal of the interaction with the reservoir may give an effective freezing of any dissipative phenomena.

Two very recent papers [11] have considered a generalization of this parity kick method and have shown that a complete decoupling between system and environment can be obtained for a generic system if one considers an appropriate sequence of infinitely frequent kicks, realizing a symmetrization of the evolution with respect to a given group. The present parity kick method is, in fact, equivalent to symmetrizing the time evolution of the open system with respect to the group \mathcal{Z}_2 , composed by the identity and the system parity operator P . Here we focus only on this case because the experimental realization of the parity kicks of Eq. (5) is quite easy in many cases and moreover the applicability con-

ditions of the present method, Eqs. (6) and (7), are satisfied by many interesting physical systems.

The continuous kick limit is only a mathematical idealization of no practical interest. However, it shows that there is in principle no limitation in the decoherence suppression one can achieve using parity kicks. Moreover, it clearly shows that one has to use the most possible frequent pulses in order to achieve a significative inhibition of decoherence. Therefore the relevant question from a physical point of view is to determine at which values of the period $T + \tau_0$ one begins to have a significant suppression of dissipation and decoherence. We shall answer this question by considering in the next section a specific example of experimental interest, that is, a damped harmonic oscillator, representing, for instance, a normal mode of a system of trapped ions, or an electromagnetic mode in a cavity.

III. PARITY KICKS FOR A DAMPED HARMONIC OSCILLATOR

Let us consider a harmonic oscillator with bare Hamiltonian

$$H_A = \hbar \omega_0 a^\dagger a, \quad (14)$$

describing, for example, a given normal mode of a system of ions in a Paul trap or an electromagnetic mode in a cavity. In these two cases the relevant environmental degrees of freedom can be described in terms of a collection of independent bosonic modes [12]

$$H_B = \sum_k \hbar \omega_k b_k^\dagger b_k, \quad (15)$$

representing the elementary excitations of the environment (in the cavity mode case they are simply the vacuum electromagnetic modes). Moreover, the interaction with the environment is usually well described by the following bilinear term in which the ‘‘counter-rotating’’ terms are dropped [13,14]:

$$H_{\text{int}} = \sum_k \hbar \gamma_k (a b_k^\dagger + a^\dagger b_k). \quad (16)$$

In these cases time evolution is usually described in the frame rotating at the bare oscillation frequency ω_0 in which the effective total Hamiltonian of Eq. (1) becomes

$$H = H'_B + H_{\text{int}}, \quad (17)$$

where

$$H'_B = \sum_k \hbar (\omega_k - \omega_0) b_k^\dagger b_k. \quad (18)$$

It is immediately seen that in this example the conditions of Eqs. (6) and (7) for the application of the parity kick method are satisfied. More generally, conditions (6) and (7) are satisfied whenever H_A is an even function of a and a^\dagger and H_{int} is an odd function of a and a^\dagger . In the harmonic oscillator

case the pulsed perturbation realizing the sequence of parity kicks can be simply obtained as a train of π -phase shifts, that is,

$$H_{\text{kick}}(t) = E(t) a^\dagger a, \quad (19)$$

where $E(t) = E_0 \sum_{n=0}^{\infty} \theta(t - T - n(T + \tau_0)) \theta((n+1)(T + \tau_0) - t)$ and the pulse height E_0 and the pulse width τ_0 satisfy the condition

$$E_0 \tau_0 = (2n+1) \pi \hbar, \quad n \text{ integer}. \quad (20)$$

To see how large the pulse repetition rate has to be in order to achieve a relevant decoherence suppression, first of all one has to compare the pulsing period $T + \tau_0$ with the relevant time scales of the problem. The harmonic oscillator dynamics is characterized by the free oscillation frequency ω_0 and by its energy decay rate γ (which is a function of the couplings γ_k); the bath is instead characterized by its ultraviolet frequency cutoff ω_c which essentially fixes the response time of the reservoir and generally depends on the system and bath considered. A reservoir is usually much faster than the system of interest and this means that one usually has $\omega_c \gg \gamma$. Typically the reduced dynamics of the system of interest is described in terms of effective master equations which are derived using the Markovian approximation (see, for example, [13]) which means assuming the limit $\omega_c \rightarrow \infty$; however, the existence of a finite cutoff ω_c is always demanded on physical grounds [15] and this parameter corresponds, for example, to the Debye frequency in the case of a phonon bath.

From the preceding section, it is clear that the pulsing period $T + \tau_0$ has to be much smaller than $1/\gamma$, otherwise the change of sign of the interaction Hamiltonian is realized when a significant transfer of energy and quantum coherence from the system into the environment has already taken place. At the same time it is easy to understand that the condition $\omega_c (T + \tau_0) \ll 1$ is a sufficient condition for a significant suppression of dissipation, since in this case the sign of the interaction H_{int} changes with a rate much faster than every frequency of the bath oscillators; each bath degree of freedom becomes essentially decoupled from the system oscillator and there is no significant energy exchange, i.e., no dissipation. Therefore the relevant question is, for which value of the pulsing period $T + \tau_0$ within the range $[1/\gamma, 1/\omega_c]$ does one begin to have a relevant inhibition of dissipation?

We shall answer this question by studying in particular the time evolution of a Schrödinger cat state, that is, a linear superposition of two coherent states of the oscillator of interest

$$|\psi_\varphi\rangle = N_\varphi (|\alpha_0\rangle + e^{i\varphi} |-\alpha_0\rangle), \quad (21)$$

where

$$N_\varphi = \frac{1}{\sqrt{2 + 2e^{-2|\alpha_0|^2} \cos \varphi}}.$$

We consider this particular example because these states are the paradigmatic quantum states in which the progressive

effects of decoherence and dissipation caused by the environment are well distinct and clearly visible [6]. From the general proof of the preceding section it is evident that whenever one finds a significant suppression of decoherence in the Schrödinger cat case, this implies that the system-reservoir interaction is essentially averaged to zero and that one gets a significant system-environment decoupling in general.

Describing the evolution of such a superposition state in the presence of the dissipative interaction with a reservoir of oscillators which is initially at thermal equilibrium at $T=0$ is quite simple. In fact, it is possible to use the fact that a tensor product of coherent states retains its form at all times when the evolution is generated by the Hamiltonian (17), that is,

$$|\alpha_0\rangle \otimes \prod_k |\beta_k(0)\rangle \rightarrow |\alpha(t)\rangle \otimes \prod_k |\beta_k(t)\rangle, \quad (22)$$

where the time-dependent coherent amplitudes satisfy the following set of linear differential equations:

$$\begin{aligned} \dot{\alpha}(t) &= -i \sum_k \gamma_k \beta_k(t), \\ \dot{\beta}_k(t) &= -i(\omega_k - \omega_0)\beta_k(t) - i\gamma_k \alpha(t). \end{aligned} \quad (23)$$

Therefore, using Eq. (22), one has the following time evolution for the Schrödinger cat state (21):

$$\begin{aligned} |\psi_\varphi\rangle \otimes \prod_k |0_k\rangle &\rightarrow N_\varphi \left(|\alpha(t)\rangle \otimes \prod_k |\beta_k(t)\rangle \right. \\ &\left. + e^{i\varphi} |-\alpha(t)\rangle \otimes \prod_k |-\beta_k(t)\rangle \right). \end{aligned} \quad (24)$$

$$U(2NT + 2N\tau_0) = [e^{-(i/\hbar)H'_B\tau_0} e^{-(i/\hbar)(H'_B - H_{RWA})T} e^{-(i/\hbar)H'_B\tau_0} e^{-(i/\hbar)(H'_B + H_{RWA})T}]^N \quad (27)$$

[we have neglected only the system-reservoir interaction during the parity kick and therefore we have added two terms $e^{-(i/\hbar)H'_B\tau_0}$ in Eq. (27) with respect to Eq. (11)].

Due to the general result (22), it is convenient to express the state of the whole system in terms of a vector $(\alpha(t), \dots, \beta_k(t), \dots)$ whose zeroth component is given by the amplitude $\alpha(t)$ and whose k th component is given by the amplitude $\beta_k(t)$, $k=1, 2, \dots$. In this way the formal solution of the set of linear equations (23) can be expressed as

$$\begin{pmatrix} \alpha(t) \\ \vdots \\ \beta_k(t) \\ \vdots \end{pmatrix} = A(\{\gamma_k\}, t) \begin{pmatrix} \alpha_0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}, \quad (28)$$

where $A(\{\gamma_k\}, t)$ is a matrix whose first matrix element $A(\{\gamma_k\}, t)_{00}$ is given by the inverse Laplace transform

The corresponding reduced density matrix of the oscillator of interest at time t , $\rho(t)$, is given by

$$\begin{aligned} \rho(t) &= N_\varphi^2 \{ |\alpha(t)\rangle \langle \alpha(t)| + |-\alpha(t)\rangle \langle -\alpha(t)| + D(t) \\ &\times [e^{-i\varphi} |\alpha(t)\rangle \langle -\alpha(t)| + e^{i\varphi} |-\alpha(t)\rangle \langle \alpha(t)|] \}, \end{aligned} \quad (25)$$

i.e., it is completely determined by the coherent amplitude $\alpha(t)$ which is a decaying function describing dissipation of the oscillator's energy and by the function $D(t)$ describing the suppression of quantum interference terms, whose explicit expression is

$$D(t) = \prod_k \langle \beta_k(t) | -\beta_k(t) \rangle = \exp \left\{ -2 \sum_k |\beta_k(t)|^2 \right\}. \quad (26)$$

These results for the time evolution of the Schrödinger cat are valid both in the presence and in the absence of the external pulsed driving. In fact, the only difference lies in the fact that in the absence of kicks one has the standard evolution driven by $\exp\{-iHt/\hbar\}$, where H is given by Eq. (17), while in the presence of parity kicks one has a stroboscopic-like evolution driven by a unitary operator analogous to that of Eq. (11)

$$A(\{\gamma_k\}, t)_{00} = \mathcal{L}^{-1} \left[\frac{1}{z + K(z)} \right], \quad (29)$$

where

$$K(z) = \sum_k \frac{\gamma_k^2}{z + i(\omega_k - \omega_0)}. \quad (30)$$

All the other matrix elements can be expressed in terms of this matrix element $A(\{\gamma_k\}, t)_{00}$ in the following way:

$$\begin{aligned} A(\{\gamma_k\}, t)_{0k} &= A(\{\gamma_k\}, t)_{k0} \\ &= -i\gamma_k \int_0^t ds e^{-i(\omega_k - \omega_0)s} A(\{\gamma_k\}, t-s)_{00}, \end{aligned} \quad (31)$$

$$A(\{\gamma_k\}, t)_{kk'} = \delta_{kk'} e^{-i(\omega_k - \omega_0)t} - \gamma_k \gamma_k' \int_0^t ds e^{-i(\omega_k - \omega_0)(t-s)} \\ \times \int_0^s ds' e^{-i(\omega_k' - \omega_0)(s-s')} A(\{\gamma_k\}, s')_{00}. \quad (32)$$

In the presence of kicks, the stroboscopic evolution is determined by the consecutive application of the evolution operator for the elementary cycle lasting the time interval $2T + 2\tau_0$ [see Eq. (27)], during which one has a standard evolution for a time T , followed by an uncoupled evolution for a time τ_0 , then a ‘reversed’ evolution for a time T in which the interaction with the environment has the opposite sign and another uncoupled evolution for the time τ_0 at the end. It is easy to understand that in terms of the vector of coherent amplitudes, the evolution in the presence of parity kicks can be described as

$$\begin{pmatrix} \alpha(2NT + 2N\tau_0) \\ \vdots \\ \beta_k(2NT + 2N\tau_0) \\ \vdots \end{pmatrix} = [F(\tau_0)A(\{-\gamma_k\}, T)F(\tau_0)A(\{\gamma_k\}, T)]^N \begin{pmatrix} \alpha_0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}, \quad (33)$$

where $F(\tau_0)$ is the diagonal matrix associated to the free evolution driven by H'_B and is given by $F(\tau_0)_{i,j} = \delta_{i,j} \exp\{-i(\omega_i - \omega_0)\tau_0\}$. We have seen that the quantities of interest are [see Eq. (25)] $\alpha(t)$ and $D(t)$; for the amplitude of the two coherent states one has

$$\alpha(t) = A(\{\gamma_k\}, t)_{00} \alpha_0 \quad (34)$$

in the absence of kicks and

$$\alpha(t) = \{[F(\tau_0)A(\{-\gamma_k\}, T)F(\tau_0)A(\{\gamma_k\}, T)]^N\}_{00} \alpha_0 \quad (35)$$

in the presence of kicks ($t = 2NT + 2N\tau_0$). As for the function $D(t)$, it is clear that each amplitude $\beta_k(t)$ in Eq. (26) is proportional to α_0 and therefore one can write

$$D(t) = \exp\{-2|\alpha_0|^2 \eta(t)\}, \quad (36)$$

where

$$\eta(t) = \sum_k |A(\{\gamma_k\}, t)_{k0}|^2 \quad (37)$$

in the absence of kicks and

$$\eta(t) = \sum_k \{|[F(\tau_0)A(\{-\gamma_k\}, T)F(\tau_0)A(\{\gamma_k\}, T)]^N\}_{k0}|^2 \quad (38)$$

in the presence of kicks ($t = 2NT + 2N\tau_0$).

IV. NUMERICAL RESULTS

In the description of dissipative phenomena one always considers a continuum distribution of oscillator frequencies in order to obtain an irreversible transfer of energy from the system of interest into the reservoir. Moreover, most often, also the Markovian assumption is made which means assuming an infinitely fast bath with an infinite frequency cutoff ω_c . This case of a standard vacuum bath in the Markovian limit is characterized by an infinite, continuous, and flat distribution of couplings [13]

$$\gamma(\omega)^2 = \frac{\gamma}{2\pi} \quad \forall \omega, \quad (39)$$

which implies

$$K(z) = \frac{\gamma}{2}. \quad (40)$$

Using Eqs. (34), (31), and (37), we get

$$\alpha_{\text{Mark}}(t) = \alpha_0 e^{-\gamma t/2}, \quad (41)$$

$\eta_{\text{Mark}}(t)$

$$= \sum_k \frac{\gamma_k^2 (1 + e^{-\gamma t} - e^{-i(\omega_k - \omega_0)t - \gamma t/2} - e^{i(\omega_k - \omega_0)t - \gamma t/2})}{\gamma^2/4 + (\omega_k - \omega_0)^2}. \quad (42)$$

The last sum in the expression for $\eta_{\text{Mark}}(t)$ has to be evaluated in the continuum limit, that is, replacing the sum with an integral over the whole real ω axis and using $\gamma_k^2 \rightarrow \gamma/2\pi$. The result is the standard vacuum bath expression for the decoherence function $\eta(t)$ [2],

$$\eta_{\text{Mark}}(t) = 1 - e^{-\gamma t}. \quad (43)$$

From the general expressions of the above section it is clear that it is not possible to solve the dynamics in the presence of parity kicks in simple analytical form. We are therefore forced to solve numerically the problem, by simulating the continuous distribution of bath oscillators with a large but finite number of oscillators with closely spaced frequencies. To be more specific, we have considered a bath of 201 oscillators, with equally spaced frequencies, symmetrically distributed around the resonance frequency ω_0 , i.e.,

$$\omega_k = \omega_0 + k\Delta, \quad (44)$$

$$\Delta = \frac{\omega_0}{100} \quad (45)$$

$$k_{\text{max}} = \frac{\omega_0}{\Delta} = 100 \Rightarrow \omega_k^{\text{max}} = 2\omega_0, \quad (46)$$

$$k_{\text{min}} = -k_{\text{max}} = -100 \Rightarrow \omega_k^{\text{min}} = 0, \quad (47)$$

and we have considered a constant distribution of couplings similar to that associated with the Markovian limit

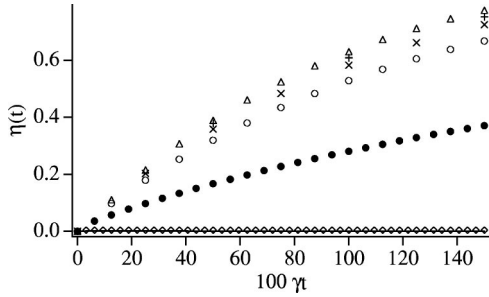


FIG. 1. Time evolution of the decoherence function $\eta(t)$ [see Eqs. (37) and (38)] for different values of the time interval between two kicks T : \triangle , no kicks; $+$, $\omega_c T=50$; \times , $\omega_c T=25$; \circ , $\omega_c T=12.5$; \bullet , $\omega_c T=6.25$; \diamond , $\omega_c T=3.125$; full line, $\omega_c T=1.5625$.

$$\gamma_k^2 = \frac{\gamma \Delta}{2\pi} \quad \forall k. \quad (48)$$

Considering a finite number of bath oscillators has two main effects with respect to the standard case of a continuous Markovian bath. First of all, the adoption of a discrete frequency distribution with a fixed spacing Δ implies that all the dynamical quantities are periodic with period $T_{\text{rev}} = 2\pi/\Delta$ [16] which can therefore be considered a sort of “revival time.” It is clear that our numerical solution will correctly describe dissipative phenomena provided that we focus only on not too large times (say $t \leq \pi/\Delta$). Moreover, the introduction of a finite cutoff ($\omega_c = 2\omega_0$ in our case) implies a modification of the coupling spectrum $\gamma(\omega)$ at very high frequency with respect to the infinitely flat distribution of the Markovian treatment [see Eq. (39)]. This fact will manifest itself in a slight modification of the exponential behavior shown by Eqs. (41) and (43) at very short times ($t \approx \omega_c^{-1}$) [16]. We have verified both facts in our simulations. However, to facilitate the comparison between the dynamics in the presence of parity kicks with the standard case of a Markovian bath, we have chosen the parameters of our simulation so that, within the time interval of interest, $0.1/\gamma < t < 3/\gamma$ say, we found no appreciable difference between the standard Markovian bath expressions (41) and (43) and the corresponding general expression for a discrete distribution of oscillators (34) and (37).

Let us now see what is the effect of parity kicks on decoherence, by studying first of all the behavior of the decoherence function $\eta(t)$ for different values of the pulsing period $T + \tau_0$. As discussed above, we expect an increasing decoherence suppression for decreasing values of $T + \tau_0$ and this is actually confirmed by Fig. 1, showing $\eta(t)$ for different values of T (we have chosen $\tau_0 = 0$ for simplicity in the numerical simulation). In fact, the increase of $\eta(t)$ is monotonically slowed down as the time interval between two successive kicks T becomes smaller and smaller. Moreover, Fig. 1 seems to suggest that decoherence inhibition becomes very efficient [$\eta(t) \approx 0$] when the kick frequency $1/T$ becomes comparable to the cutoff frequency ω_c . To better clarify this important point we have plotted in Fig. 2 the value of the decoherence function at a fixed time as a function of the time between two successive kicks T . In Fig. 2(a) the decoherence function at half relaxation time $\eta(t=0.5/\gamma)$ is plotted as a function of $\omega_c T/2\pi$, while in Fig. 2(b) the decoherence func-

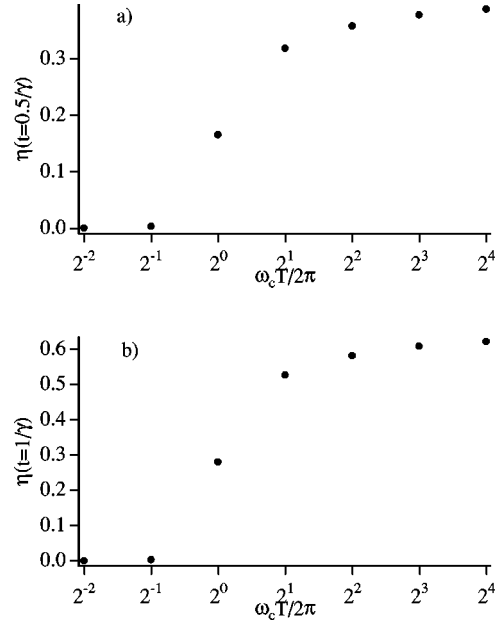


FIG. 2. Decoherence function $\eta(t)$ at a fixed time t versus $\omega_c T/2\pi$: (a) refers to $t=0.5/\gamma$ and (b) refers to $t=1/\gamma$.

tion at one relaxation time $\eta(t=1/\gamma)$ is plotted again as a function of $\omega_c T/2\pi$. In both cases one sees a quite sharp transition at $\omega_c T/2\pi = 1$: *decoherence is almost completely inhibited as soon as the kick frequency $1/T$ becomes larger than the cutoff frequency $\omega_c/2\pi$* . Therefore we can conclude that by forcing the dynamics of a harmonic oscillator with appropriate parity kicks one is able to inhibit decoherence almost completely. This decoherence suppression becomes significant when the kick frequency $1/T$ becomes of the order of the typical reservoir time scale, i.e., the cutoff frequency $\omega_c/2\pi$.

A similar conclusion has been reached by Viola and Lloyd [17], who have considered a model for decoherence suppression in a spin-boson model very similar in spirit to that presented here. In their paper they have considered a single 1/2-spin system coupled to an environment and they have shown how decoherence can be suppressed by a sequence of appropriately shaped pulses. They have shown that the 1/2-spin decoherence can be almost completely suppressed provided that the pulse repetition rate is at least comparable with the environment frequency cutoff. Similar results have been obtained in a more recent paper [18] in which a generalization to more qubits and more general sequences of pulses is considered. However, despite the similarity of our approach to decoherence control to that of these papers, there are important differences between the present paper and Refs. [17,18]. First of all, we have considered a harmonic oscillator instead of a 1/2-spin system, but above all, we have considered a *dissipative* bath, that is, a zero temperature reservoir inducing not only decoherence (i.e., quantum information decay), but also dissipation (i.e., energy decay). In Ref. [17] on the contrary, there is only decoherence and the 1/2-spin system energy is conserved. The more general nature of the present model helps in clarifying a main point of this impulsive method to combat decoherence: parity kicks do not simply suppress decoherence but tend to completely inhibit *any interaction between system*

and environment. This means that energy dissipation is also suppressed; in more intuitive terms, the external pulsed driving is perfectly “phase matched” to the system dynamics, so that any transfer of energy and quantum coherence between system and bath is inhibited. The fact that decoherence suppression in our model is always associated with the suppression of dissipation can be easily shown using the fact that the evolution of the whole system is unitary. In particular, this implies that the matrix $A(\{\gamma_k\}, t)$, driving the evolution in the absence of kicks [see Eq. (28)], and the matrix $[F(\tau_0)A(\{-\gamma_k\}, T)F(\tau_0)A(\{\gamma_k\}, T)]^N$, driving the evolution in the presence of kicks [see Eq. (33)], are unitary matrices and therefore subject to the condition

$$|L_{00}|^2 + \sum_k |L_{k0}|^2 = 1, \quad (49)$$

where L_{ij} denotes any of the two above mentioned matrices. Now, using the reduced density matrix of Eq. (25), it is easy to derive the mean oscillator energy

$$\langle H_A(t) \rangle = \hbar \omega_0 |\alpha(t)|^2 \frac{1 - \cos \varphi e^{-2|\alpha_0|^2}}{1 + \cos \varphi e^{-2|\alpha_0|^2}}. \quad (50)$$

From Eqs. (34) and (35) one therefore derives that the normalized oscillator mean energy is given by

$$\frac{\langle H_A(t) \rangle}{\langle H_A(0) \rangle} = |A(\{\gamma_k\}, t)_{00}|^2 \quad (51)$$

in the absence of kicks and

$$\frac{\langle H_A(t) \rangle}{\langle H_A(0) \rangle} = |\{[F(\tau_0)A(\{-\gamma_k\}, T)F(\tau_0)A(\{\gamma_k\}, T)]^N\}_{00}|^2 \quad (52)$$

in the presence of kicks ($t = 2NT + 2N\tau_0$). Now, by considering Eqs. (37) and (38) and the unitarity condition (49), it is immediate to get the following simple relation between decoherence and dissipation:

$$\frac{\langle H_A(t) \rangle}{\langle H_A(0) \rangle} = 1 - \eta(t), \quad (53)$$

which is valid both in the presence and in the absence of kicks. This equation simply shows that when decoherence is suppressed [$\eta(t) \sim 0$] the oscillator energy is conserved.

A qualitative demonstration of the ability of the kick method to suppress dissipation and decoherence is provided by Fig. 3. (a) shows the Wigner function of an initial odd cat state with $\alpha_0 = \sqrt{5}$, $\varphi = \pi$; (b) shows the Wigner function of the same cat state evolved for a time $t = 1/\gamma$ in the presence of parity kicks with $\omega_c T = 3.125$ and (c) the Wigner function of the same state again after a time $t = 1/\gamma$, but evolved in the absence of kicks. This elapsed time is ten times the decoherence time of the Schrödinger cat state, $t_{\text{dec}} = (2\gamma|\alpha_0|^2)^{-1}$ [7], i.e., the lifetime of the interference terms in the cat state density matrix in the presence of the usual vacuum damping. As is shown by (c), the cat state has begun to lose its energy and has completely lost the oscillating part of the Wigner function associated to quantum interference. This is no

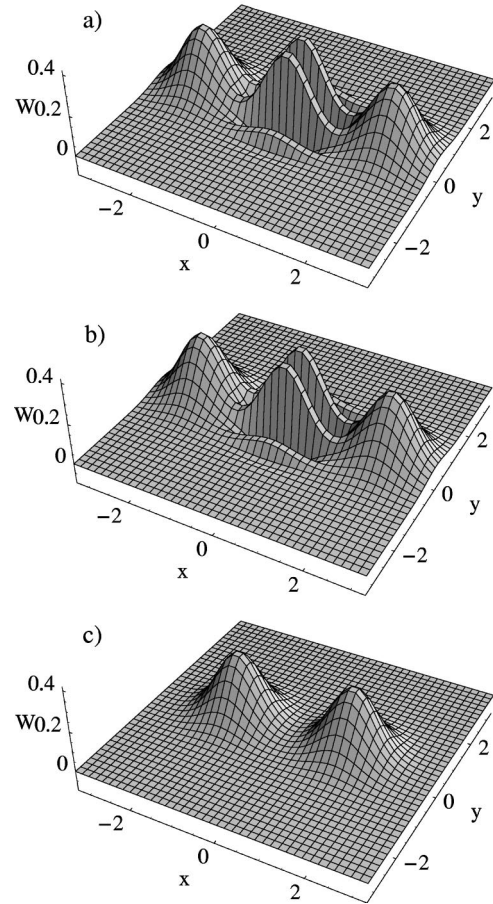


FIG. 3. (a) Wigner function of the initial odd cat state, $|\psi\rangle = N_{-}(|\alpha_0\rangle - |-\alpha_0\rangle)$, $\alpha_0 = \sqrt{5}$; (b) Wigner function of the same cat state evolved for a time $t = 1/\gamma$ ($t = 10t_{\text{dec}}$), in the presence of parity kicks ($\omega_c T = 3.125$); (c) Wigner function of the same state after a time $t = 1/\gamma$, but evolved in the absence of kicks.

longer true in the presence of parity kicks: (b) shows that, after $t = 10t_{\text{dec}}$, the state is almost indistinguishable from the initial one and that the quantum wiggles of the Wigner function are still well visible.

V. CONCLUDING REMARKS ON THE POSSIBLE EXPERIMENTAL APPLICATIONS

The numerical results presented above provides a clear indication that perturbing the dynamics of an oscillator with an appropriate periodic pulsing can be a highly efficient method for controlling decoherence. However, we have seen that a significant decoherence suppression is obtained only when the pulsing frequency $1/(T + \tau_0)$ becomes comparable to the cutoff frequency of the reservoir. This fact poses some limitations on the experimental applicability of the proposed method. For example, the parity kick method is certainly unfeasible, at least with the present technologies, in the case of optical modes in cavities. In this case, realizing a parity kick is not difficult in principle since impulsive phase shifts can be obtained using electro-optical modulators or a dispersive interaction between the optical mode and a fast crossing atom [19]. The problem here is that it is practically impossible to make these kicks sufficiently frequent. In fact, we can assume, optimistically, that the frequency cutoff ω_c ,

even though larger, is of the order of the cavity mode frequency ω_0 (this is reasonable, since the relevant bath modes are those nearly resonant with the frequency ω_0). This means $\omega_c/2\pi \approx 10^{14} - 10^{15}$ Hz and it is evident such high values for $1/(T + \tau_0)$ are unrealistic both for electro-optical modulators and for dispersive atom-cavity mode interactions. A similar situation holds for an electromagnetic mode in a microwave cavity like that studied in the Schrödinger cat experiment of Brune *et al.* [6]. In this experiment the dispersive interaction between a Rydberg atom and the microwave mode is used for the generation and the detection of the cat state and therefore the realization of π -phase shifts is easy. The problem again is to have sufficiently fast and frequent π -phase shifts: in this case the mode frequency is $\omega_0 = 51$ GHz and therefore one would have a good decoherence suppression for $T + \tau_0 \approx 10^{-10}$ sec. This is practically impossible because it implies quasi-relativistic velocities for the crossing Rydberg atom and an unrealistic very high dispersive interaction in order to have a π -phase shift.

The situation is instead different in the case of ions trapped in harmonic traps. In this case, in fact, the free oscillation frequency ω_0 , and therefore the cutoff frequency ω_c too, are usually much smaller and it becomes feasible to realize fast and frequent π -phase shifts. Let us consider, for example, the case in which the oscillator mode of the preceding sections is the center-of-mass motion of a collection of trapped ions in a Paul trap, like that considered in the experiments at NIST in Boulder [14]. In this case the free oscillation frequency is of the order of $\omega_0/2\pi \approx 10$ Mhz and a frequent sequence of parity kicks could be obtained in principle by appropriately pulsing at about 10 Mhz the static potential applied to the end segments of the rods confining the ions along the z axis [14]. The duration τ_0 and the intensity E_0 of the pulses have to be tailored so as to have the parity kick condition $E_0\tau_0 = \pi\hbar$ [see Eq. (20)] and this implies having a very good control of the pulse area.

Controlling the decoherence of the center-of-mass mode is crucial for the realization of quantum information processing with trapped ions. In fact, even though only the higher frequency vibrational modes will be used for quantum gate transitions involving the ion motion (as has already been done in [20] where the deterministic entanglement of the internal states of two ions has been achieved by manipulating the stretching mode), suppressing decoherence of the center-

of-mass motion is still important because the heating of the center-of-mass motion partially couples also with the other vibrational stretching modes [14].

The parity kick method could be used even in the case in which the center-of-mass mode itself is used as a quantum bus for the realization of quantum gates. However, in this case, the use of parity kicks cannot be applied to protect against decoherence of all kinds of quantum gates. In fact, the parity kicks tend to average to zero any term in the Hamiltonian which is an odd function of the bosonic operators a and a^\dagger , and this means that the parity kicks could average to zero just the system dynamics we want to protect from decoherence, such as, for example, some gate operations in an ion-trap quantum computer. For example, it is easy to see that all the gate operations involving *first* red or blue sideband transitions, which implies having *linear* terms in a and a^\dagger in the system Hamiltonian, such as, for example, the Cirac-Zoller controlled-NOT (C-NOT) gate [21] and the C-NOT gate experimentally realized at NIST by Monroe *et al.* [22], tend to be averaged to zero by frequent parity kicks. To overcome this problem, it is, however, sufficient to restrict to quantum gates based on carrier transitions, which involve functions of $a^\dagger a$ only, and are not therefore affected by parity kicks, such as, for example, the one-pulse gate proposed by Monroe *et al.* [23].

Therefore, the parity kick method presented here could be very useful to achieve a significant decoherence control in ion traps designed for quantum information processing. However, we have to notice that our considerations are based on the model of Sec. III which is the one commonly used to describe dissipation, even though the actual sources of decoherence in ion traps have not been completely identified yet [14]. For example, Refs. [24,25] have considered the effect of fluctuations in the various experimental parameters, yielding no appreciable energy dissipation in the center-of-mass motion but only off-diagonal dephasing.

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