Quantum electrodynamics of strong-field Compton scattering and its ponderomotive and drift energies

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In earlier classical, semiclassical, and quantum electrodynamical treatments of the Compton scattering in strong fields the strong component of the radiation remained unchanged or forward scattered during interaction with the electron. In our analysis, along with the Compton scattering of a single photon the simultaneous nonforward elastic scattering of the other photons of the intense radiation is also taken into account, resulting in changed kinematical relations that enable the electron to absorb or emit a larger amount of energy up to the ponderomotive energy, in agreement with the experimental results. [S1050-2947(99)03506-4]

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I. INTRODUCTION

The interaction of strong radiation fields with truly free electrons has been treated in the past using classical [1–5], semiclassical [6–11], and quantum electrodynamical [12–14] methods, all under the assumption that the ''strong-field component'' remains unchanged or elastically forward scatters while one or several photons inelastically scatter out of the beam. The quantum electrodynamical calculations [12] predicted no changes in the kinematics of the Compton process while the semiclassical analyses asserted a minor frequency shift [8] that remained disputable until now. On the other hand, in all these considerations the quiver (oscillatory) energy of the electron in the radiation field, also called the ponderomotive energy [15], appeared without an explanation for now it could be converted into translational energy at four-momentum conservation. This question represents a timely subject of investigation [16]. The experiments, however, proved that the electrons, leaving the laser beam and becoming truly free, have a translational (drift) energy up to their quiver energy in the field [17,18].

In this paper we describe, in the framework of quantum electrodynamics, a multiphoton–free-electron interaction process in which, along with the Compton scattering of a single photon, the simultaneous nonforward elastic scattering (in the electron rest frame) of the other photons of the intense radiation is also taken into account. This process transfers only momentum but no energy between the field and the particle, enabling the electron to definitively absorb or emit a larger amount of energy up to its quiver energy at four-momentum conservation. Even if the calculations are elaborate, the results are worthwhile because the analyzed process explains the conversion of ponderomotive energy into translational energy. They also prove that quantum electrodynamics is able to produce classical results without using approximations in which the Planck constant is taken zero, $\hbar=0$, as well as to give correct results for high-order processes.

II. KINEMATICAL RELATIONS

In intense radiation various free-electron–multiphoton interaction processes appear with appropriate kinematics and probabilities; among them are, for example, the production of higher harmonics that has been treated in the framework of quantum electrodynamics in [19] and experimentally verified in [20], the absorption of an integral number of photons, analyzed in [21–25], the simultaneous elastic scattering of n photons [26], the strong-field Compton effect we aim to consider here, and many others.

In order to evaluate the transition probabilities per unit time and the differential cross sections of the strong-field Compton process described above, we set up the four-momentum conservation law that gives the necessary kinematical relations. Let us assume that a collimated single mode laser beam with linearly polarized photons of four-momenta $k_0=(\varepsilon_0/c,\vec{k}_0)$, $\varepsilon_0=\hbar\omega_0$, ω_0 being the angular frequency of the photons, interacts with a free electron initially at rest with four-momentum $p_0=(E_0/c,0)$, $E_0=m_0c^2$, where m_0 is the electron rest mass and c the speed of light. During interaction n photons elastically scatter into a final state with four-momentum $k=(\varepsilon_0/c,\vec{k})$, $|\vec{k}|=|\hat{k}_0|=\varepsilon_0/c$ under an angle of α relative to the propagation direction of the incident beam while simultaneously a single Compton photon with k_0 inelastically scatters into a final state with four-momentum $k_C=(\varepsilon_C/c,\vec{k}_C)$ under an angle of α_C, transferring an amount of energy $\Delta\varepsilon=\varepsilon_0-\varepsilon_C$ to the electron with final four-momentum $p=(E/c,\vec{p})$, as is shown in Fig. 1. We note that α and α_C do not necessarily lie in the same plane.

A process in which n photons scatter into a single final state is chosen because its probability P is $n!$ times higher than the probability P_d of the n-photon scattering into different final states, $P=n!P_d$, as is shown, for example, in [27]. The four-momentum conservation law governing this interaction process is given by the formula

$$p_0+nk_0+k_0=p+nk+k_C. \tag{1}$$

Expressing p from Eq. (1) as a function of the other terms and squaring both sides, we readily arrive at an equation for n,

$$n^2k_0k-n(k_0-k_C)(k_0-k)-p_0(k_0-k_C)+k_0k_C=0,$$

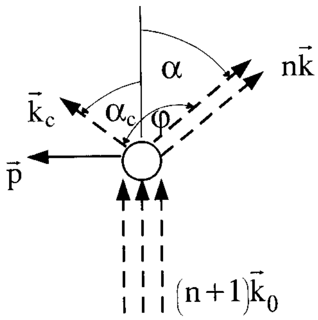

FIG. 1. Momentum vector conservation in the Compton scattering and simultaneous n-photon nonforward elastic scattering process. $\vec{k}_0$, $\vec{k}$, and $\vec{k}_C$ are the momentum vectors of the incident, scattered, and Compton photon while $\vec{p}$ is the final momentum vector of the electron. The angles α and α_C do not necessarily lie in the same plane.

where we took into account that $p^2=p_0^2$ and $k_0^2=k^2=k_C^2=0$ as well as that $p_0(k_0-k)=(E_0/c,0)(0,\vec{k}_0-\vec{k})=(0,0)$ in the electron rest frame.

The solution of this equation is

$$n=\frac{(k_0-k_C)(k_0-k)}{2k_0k}+\left\{\left[\frac{(k_0-k_C)(k_0-k)}{2k_0k}\right]^2+\frac{p_0(k_0-k_C)-k_0k_C}{k_0k}\right\}^{1/2}.$$

By inserting the components of the four-momenta k_0, k, k_C, and p_0 given above, we obtain

$$n=\frac{\varepsilon_C(\cos\alpha_C-\cos\varphi)}{2\varepsilon_0(1-\cos\alpha)}-\frac{1}{2}+\left\{\left[\frac{\varepsilon_C(\cos\alpha_C-\cos\varphi)}{2\varepsilon_0(1-\cos\alpha)}-\frac{1}{2}\right]^2+\frac{E_0\Delta\varepsilon-\varepsilon_0\varepsilon_C(1-\cos\alpha_C)}{\varepsilon_0^2(1-\cos\alpha)}\right\}^{1/2}, \tag{2}$$

φ being the angle between $\vec{k}$ and $\vec{k}_C$. At $\Delta\varepsilon_{Cm}\ll\Delta\varepsilon\lesssim\varepsilon_0$, where $\Delta\varepsilon_{Cm}\approx 2\varepsilon_0^2/E_0$ is the maximal energy transfer to the electron at ordinary weak-field Compton scattering, the final electron three-momentum, and thus its energy, is essentially determined by the n-photon scattering; in this case $\varepsilon_C/2\varepsilon_0\ll 1/2$ and

$$\frac{\varepsilon_C(\cos\alpha_C-\cos\varphi)}{2\varepsilon_0(1-\cos\alpha)}\ll\frac{1}{2}$$

even for small values of α since in this case $\varphi\to\alpha_C$ and hence

$$(\cos\alpha_C-\cos\varphi)\to 0,$$

as is seen from Fig. 1. We have also for the last term under the square root

$$\varepsilon_0\varepsilon_C(1-\cos\alpha_C)\ll E_0\Delta\varepsilon$$

at optical and near-optical frequencies since

$$E_0\Delta\varepsilon=m_0c^2\Delta\varepsilon\gg\varepsilon_0\varepsilon_C.$$

Thus deleting $\varepsilon_C(\cos\alpha_C-\cos\varphi)/2\varepsilon_0(1-\cos\alpha)$ relative to $\frac{1}{2}$ and $\varepsilon_0\varepsilon_C(1-\cos\alpha_C)$ relative to $E_0\Delta\varepsilon$, Eq. (2) is approximated to

$$n\approx\left[\frac{E_0\Delta\varepsilon}{\varepsilon_0^2(1-\cos\alpha)}+\frac{1}{4}\right]^{1/2}-\frac{1}{2}.$$

The minimal value of $E_0\Delta\varepsilon/\varepsilon_0^2(1-\cos\alpha)$ for optical and near-optical frequencies is about 10^4, giving $n\gtrsim 10^2$, which permits us to omit $\frac{1}{4}$ and $\frac{1}{2}$, resulting in, to a good approximation,

$$n\approx\left[\frac{E_0\Delta\varepsilon}{\varepsilon_0^2(1-\cos\alpha)}\right]^{1/2}. \tag{3}$$

From this formula the energy of the Compton photon is

$$\varepsilon_C=\varepsilon_0[1-n^2\varepsilon_0E_0^{-1}(1-\cos\alpha)], \tag{4}$$

which basically differs from the energy ε_{Cw} of the Compton scattered photon in weak fields,

$$\varepsilon_{Cw}=\varepsilon_0[1+\varepsilon_0E_0^{-1}(1-\cos\alpha_C)]^{-1} \tag{5}$$

as ε_C in Eq. (4) practically does not depend on α_C due to the decisively large amount of transferred three-momentum by the n elastically nonforward scattered photons. Since

$$\Delta\varepsilon/\varepsilon_0=n^2\varepsilon_0E_0^{-1}(1-\cos\alpha)<1,$$

formula (4) can also be written in an approximate form similar to Eq. (5),

$$\varepsilon_C\approx\varepsilon_0[1+n^2\varepsilon_0E_0^{-1}(1-\cos\alpha)]^{-1}, \tag{6}$$

noting, however, that α in Eq. (6) is the scattering angle of the n photons while α_C in Eq. (5) is the scattering angle of the Compton photon. When the electron has an appropriate initial momentum vector relative to the propagation direction of the incident beam, it can lose energy at simultaneous increase in the energy of the scattered photon, $\varepsilon_C>\varepsilon_0$. The kinematical analysis of this case can be carried out in the easiest way in a coordinate system in which the electron is at rest in its final state.

III. DIFFERENTIAL CROSS SECTIONS AND THRESHOLD BEAM INTENSITIES

Some important characteristics of the described electron-multiphoton process can be obtained starting with the Compton scattering and simultaneous two-photon nonforward elastic scattering, $n=2$, and generalizing the results for arbitrary number n. The appropriate Feynman diagrams are given in Fig. 2, where the solid lines represent the electron states while the dashed lines represent the incident and scattered photons. The shorter dashed line illustrates the scattered Compton photon; vertex y is a sum of two elements, 1 and 2, as is shown in Fig. 3. Figure 2 is divided into two groups, A and B. In A, a–d the number of absorbed and emitted quanta is equal in every even intermediate state, thus having a near-resonance character, in contrast to the diagrams in group B, where some or all the intermediate states lie farther from the initial state, contributing much less to the transition

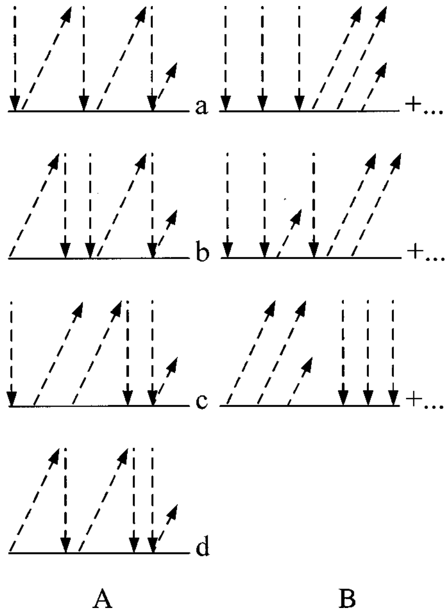

FIG. 2. Feynman diagrams for two-photon nonforward elastic scattering and simultaneous Compton scattering (dashed lines) from the electron (solid line).

probability than those in group A. Hence, when determining the threshold incident laser beam intensity above which Compton scattering with energy change $\Delta\varepsilon$ becomes probable, group B may be temporarily neglected. All the diagrams in group A can be constructed using the elements x and $\overline{x}$ and Compton scattering element y given in Fig. 3. The diagram summation can be carried out first pairing a, b, c, and d in such a way that they differ only in their first element, $(a,b)=(x_1x_2y,\overline{x_1}x_2y)$ and $(c,d)=(x_1\overline{x_2}y,\overline{x_1}\overline{x_2}y)$ and summing over them, which gives $(x_1+\overline{x_1})x_2y$ for $(a+b)$ and $(x_1+\overline{x_1})\overline{x_2}y$ for $(c+d)$, and, finally, $(x_1+\overline{x_1})(x_2+\overline{x_2})y$ for $(a+b+c+d)$. Generalizing this procedure for a number n one obtains

$$(a+b+c+d+\cdots)$$
$$=(x_1+\overline{x_1})(x_2+\overline{x_2})(x_3+\overline{x_3})\cdots(x_n+\overline{x_n})y. \quad (7)$$

One has to note here that these elastic absorption-emission processes result in ''dressing'' the originally free electron when interacting with the strong field, attaining in this way a quiver (ponderomotive) energy as we shall see below.

The transition probability per unit time P' for arbitrary n is given by the well-known formula [28]

$$P'=(2\pi/\hbar)|M_{0f}|^2\rho_f, \quad (8)$$

where the matrix element is

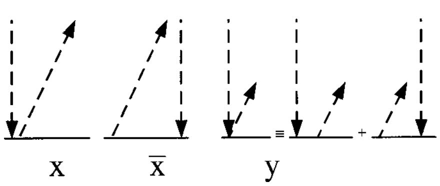

FIG. 3. Elements from which diagrams in group A in Fig. 2 are constructed.

$$M_{0f}=\sum_{g=1}^{2^{n+1}}\sum \frac{\Pi_{r=0}^{2n+1}H_{rg-(r+1)g}}{\Pi_{r=1}^{2n+1}(\mathcal{E}_0-\mathcal{E}_{rg})} \quad (2n+2\equiv f), \quad (9)$$

in which $\mathcal{E}_0$ is the energy of the initial state of the interacting system and $\mathcal{E}_r$ the energy of its rth intermediate state. Summation Σ_g is over all different near-resonance diagrams and Σ over all states within one r intermediate state. Later we shall also sum over the initial and final spin states of the electron, S_0 and S. The partial matrix elements H in Eq. (9) are, for the absorption of the n_rth photon,

$$H_{r-(r+1)}=-e[\mu_0c^4\hbar^2(n_i-n_r+1)/2\varepsilon_0V]^{1/2}(U^*_{r+1}\alpha_iU_r), \quad (10)$$

and for the emission of the n_rth quantum,

$$H_{r-(r+1)}=-e[\mu_0c^4\hbar^2n_r/2\varepsilon_0V]^{1/2}(U^*_{r+1}\alpha_fU_r), \quad (11)$$

while for the emission of the Compton photon

$$H_{r-(r+1)}=-e[\mu_0c^4\hbar^2/2\varepsilon_cV]^{1/2}(U^*_{r+1}\alpha_cU_r), \quad (12)$$

where $n_r=1,2,3$, etc. for $r=0,1,2$, etc. U are the spinors from Dirac's wave equation with Dirac's matrix $(\beta,\vec{\alpha})$. In the above formulas e is the electric charge, μ_0 is the magnetic permeability of the vacuum, $\alpha_i=\vec{\alpha}\cdot\vec{e}_i$, $\alpha_f=\vec{\alpha}\cdot\vec{e}_f$, with $\vec{e}_i$ and $\vec{e}_f$ being the polarization vectors of the incident and scattered photons, respectively. For the scattered Compton photon $\alpha_C=\vec{\alpha}\cdot\vec{e}_C$. In our calculations $I_i=n_i\varepsilon_0c/V$ is the incident and $I_n=n\varepsilon_0c/V$ the scattered light intensity, n_i being the number of incident photons in volume V. M_{0f} in Eq. (9) can also be written in the form

$$M_{0f}=M_nM_c,$$

with

$$M_n=\sum_{g=1}^{2^n}\sum \frac{\Pi_{r=0}^{2n-1}H_{rg-(r+1)g}}{\Pi_{r=1}^{2n}(\mathcal{E}_0-\mathcal{E}_{rg})} \quad (13)$$

and

$$M_c=\sum_{g=1}^{2}\sum \frac{\Pi_{r=2n}^{2n+1}H_{rg-(r+1)g}}{(\mathcal{E}_0-\mathcal{E}_{(2n+1)g})}. \quad (14)$$

Following closely the method of calculation in [28], a factor Q_g appears in the numerator of M_n for every diagram as a generalization of a simple absorption-emission process,

$$Q_g=K_{2n}\alpha_{i,f}\cdots K_3\alpha_{i,f}K_2\alpha_{i,f}K_1\alpha_{i,f}, \quad (15)$$

where $\alpha_{i,f}\equiv\alpha_i\equiv\vec{\alpha}\cdot\vec{e}_i$ for the absorption of an incident photon with polarization vector $\vec{e}_i$, or $\alpha_{i,f}\equiv\alpha_f\equiv\vec{\alpha}\cdot\vec{e}_f$ for the emission of a scattered quantum. For r an odd number,

$$K_r=E_0(1+\beta)\pm1\varepsilon_0+r\vec{\alpha}(\vec{k}_0-\vec{k})c\pm\vec{\alpha}\left\{\begin{matrix}\vec{k}\\ \vec{k}_0\end{matrix}\right\}c, \quad (16a)$$

with $+$ and $\vec{k}$ for the x_r element in the diagram and $-$ and $\vec{k}_0$ for $\overline{x_r}$. K_r for r an even number is

$$K_r=E_0(1+\beta)+r\vec{\alpha}(\vec{k}_0-\vec{k})c. \quad (16b)$$

In the denominator of M_n we have, without the signs of the quantities $(2E_0\varepsilon_0)$—which will be incorporated in the numerator—a factor

$$\approx(2E_0\varepsilon_0)^n\prod_{r=1}^{n}[-r^2c^2(\vec{k}_0-\vec{k})^2]. \tag{17}$$

The summation of the factors Q_g in Eq. (15) is carried out by the summation of diagrams (7). So $(x_1+\overline{x_1})$ in Eq. (7) gives in $Q_n=\Sigma_g Q_g$ a factor

$$K_2\alpha_i(-K_1)\alpha_f+K_2\alpha_f K_1\alpha_i$$

in which the $-$ is taken as the sign of the appropriate $(2E_0\varepsilon_0)$ in Eq. (17), with the result

$$\approx 2E_0\varepsilon_0(\vec{e}_i\cdot\vec{e}_f)(1+\beta)$$

and similarly for $(x_2+\overline{x_2})$ and so on. In M_c in Eq. (14) a factor Q_C is present,

$$Q_C=\varepsilon_C^{-1}\alpha_C K_1\alpha_i(1+\beta)-\varepsilon_0^{-1}\alpha_i K_2\alpha_C(1+\beta), \tag{18}$$

with

$$K_1=E_0(1+\beta)+1\varepsilon_0+\vec{\alpha}(\Delta\vec{p}_n+\vec{k}_0)c$$

and

$$K_2=E_0(1+\beta)-1\varepsilon_C+\vec{\alpha}(\Delta\vec{p}_n-\vec{k}_C)c,$$

where $\Delta\vec{p}_n=n(\vec{k}_0-\vec{k})$ is the momentum of the n nonforward elastically scattered photons transferred to the electron. Finally we have

$$M_{0f}\sim(U_f^*Q_CQ_nU_0)\approx(4E_0\varepsilon_0)^n(\vec{e}_i\cdot\vec{e}_f)^n(U_f^*Q_CU_0). \tag{19}$$

One can now sum over the initial and final spin states of the initially unpolarized electron [28], to obtain

$$\begin{aligned}|M_{0f}|^2&\sim\left(\frac{1}{2}\right)S_0S|U_f^*Q_CQ_nU_0|^2\\&=[4E_0\varepsilon_0(\vec{e}_i\cdot\vec{e}_f)]^{2n}\,\mathrm{Tr}[Q_c^+(H_e+E)Q_C(1+\beta)]\\&\quad\times(8E)^{-1},\end{aligned} \tag{20}$$

where Tr denotes the trace of the matrix and H_e the Hamiltonian of the electron in its final state. From Eq. (19) follows that the transition probability per unit time P' in Eq. (8) appears in the form of a product

$$P'=P_nP_c', \tag{21}$$

where P_n is the probability of the nonforward elastic scattering of the n photons and P_C' is the Compton scattering probability per unit time (with altered kinematics relative to the weak-field case). Taking the factors that stand before $(U_{r+1}^*\alpha_{i,f}U_r)$ from Eqs. (10) and (11), as well as the value in Eq. (17), Q_n from Eq. (19), and, with full ‘‘dressing’’ of the electron, $n+1=n_i$, we arrive at

$$P_n\approx\left[\frac{r_0^2 16\pi^2c^2\hbar^4(\vec{e}_i\cdot\vec{e}_f)^2I_i^2}{\varepsilon_0^4(\Delta\varepsilon)^2}\right]^n\equiv K^n, \tag{22}$$

where we used Stirling's formula for $n!$, r_0 being the classical electron radius. Obviously, P_n has a maximal value for $(\vec{e}_i\cdot\vec{e}_f)^2=1$. At optical radiation the minimal value of n, from Eq. (3), is $n_{\min}>10^2$ and thus P_n practically differs from zero only if $K\to1$ in Eq. (22). This condition gives from Eq. (22) the threshold intensity I_C above which the process has a measurable value; at $(\vec{e}_i\cdot\vec{e}_f)^2\approx1$,

$$I_i=I_C=\varepsilon_0^2\Delta\varepsilon(r_04\pi c\hbar^2)^{-1}. \tag{23}$$

Formula (23) gives the energy $\Delta E=\Delta\varepsilon$, definitively absorbed by the electron in this process, as a linear function of the incident beam intensity I_i,

$$\Delta E=\Delta\varepsilon=4\pi I_ic\hbar^2r_0/\varepsilon_0^2=4\pi I_ir_0c/\omega_0^2, \tag{24}$$

representing the translational energy of the Compton scattered electron with $\Delta\varepsilon<\varepsilon_0$. On the other hand, Eq. (24) is the well-known formula for the ponderomotive energy of the electron oscillating in the electromagnetic wave. It is important to point out that Eq. (24) is a classical quantity obtained in the framework of quantum electrodynamics without the approximation $\hbar=0$. We emphasize that the above results refer to a single scattering event only, after which the electron is detected. At certain laser pulse shapes and intensities the electron can suffer a number of scatterings before leaving the field; in such adiabatic cases the final electron energy will also depend on the intensity distribution in space-time. Then the process of the conversion of the ponderomotive energy into translational energy requires a further, detailed analysis that we plan to present in future, including the comparison of different differential cross sections for different scattering processes at different intensities.

We note that at $\Delta\varepsilon=\varepsilon_0$ the scattering process transforms into photon absorption by the free electron that has been analyzed in detail in [21–23] with possible applications in [24,25].

At near-threshold intensities $I_i\approx I_C$ the electron is accelerated with $\Delta\varepsilon$ very nearly normal to the propagation direction of the incident photons. For higher intensities $I_i\gg I_C$ the electron can also absorb the same amount of energy $\Delta\varepsilon$, being accelerated under a smaller angle relative to the propagation direction of the incident photons (while the n photons scatter under larger angle α); in this case formula (22) changes into

$$P_n\approx\left[\frac{r_0^2 32\pi^2c^2\hbar^4(\vec{e}_i\cdot\vec{e}_f)^2I_iI_n}{\varepsilon_0^4(\Delta\varepsilon)^2}\right]^n\equiv K^n, \tag{22$'$}$$

where I_n is the intensity of the scattered photons, $I_n=n\varepsilon_0c/V$, $n\ll n_i$. At $K\to1$ one obtains now from Eq. (22$'$) the threshold intensity; for $(\vec{e}_i\cdot\vec{e}_f)^2\approx1$,

$$I_i=I_C\approx\pi(\Delta\varepsilon)^2/4r_0^2\hbar n. \tag{23$'$}$$

At increasing incident light intensities above I_C the probability P_n cannot rise above the value of 1 because at above-threshold intensities forward scattering [12–14] and multiple reabsorption [29] will damp the near-resonance processes given in Fig. 2, group A, $a-d$ and the interaction will proceed through processes given in Fig. 2, group B with probability $P_n=1$ in a wide range of above-threshold light intensities. This means, in fact, that momentum transfer between the radiation field and the electron occurs with certainty, as is the case with electrons in electrostatic fields. The second factor in $P'=P_nP'_C$ in Eq. (21) is

$$P'_C=(2\pi/\hbar)|M_C|^2\rho_f, \tag{25}$$

where

$$\rho_f=\left|\frac{\partial\mathcal{E}_f}{\partial E}\right|^{-1}\rho_e=\left(\frac{2\Delta\varepsilon}{n\varepsilon_0}\right)\frac{E(2E_0\Delta\varepsilon)^{1/2}V\,d\Omega_e}{(2\pi\hbar c)^3} \tag{26}$$

is the density of final energy states of the interacting system, ρ_e being the density of final electron energy states and $d\Omega_e$ the solid angle into which the electron scatters. With the use of Eqs. (12), (18), (20), (25), and (26) we get

$$\begin{aligned}P'_C\approx{}& r_0^2I_i\varepsilon_0^{-1}[2\Delta\varepsilon(2E_0\Delta\varepsilon)^{1/2}/n\varepsilon_0^2\varepsilon_C]\\ &\times[4E_0(\vec{e}_i\cdot\vec{e}_C)^2+2\varepsilon_C^{-1}(\varepsilon_0\varepsilon_C^{-1}-1)(\vec{e}_C\Delta\vec{p}_nc)^2\\ &+2\varepsilon_0^{-1}(1-\varepsilon_C\varepsilon_0^{-1})(\vec{e}_i\Delta\vec{p}_nc)^2]d\Omega_e,\end{aligned} \tag{27}$$

where $(\Delta\vec{p}_nc)^2=n^2(\vec{k}_0-\vec{k})^2c^2=2E_0\Delta\varepsilon$.

In order to assess the maximal value of the differential cross section for the electron scattering nearly in the direction of the polarization vector of the incident photons, $d\sigma_C/d\Omega_e$, we shall assume that the incident light intensity I_i reaches the threshold value of I_C for which $\Delta\varepsilon\approx\varepsilon_0$, $\varepsilon_C\ll\varepsilon_0$. At threshold intensities $P'=P_nP'_C=P'_C$ as $P_n=1$. Besides this, P'_C is maximal when the direction of $\vec{e}_C$ is parallel with $\Delta\vec{p}_n$. Under these conditions

$$\frac{d\sigma_C}{d\Omega_e}\approx\left(\frac{r_0^2}{137}\right)\left(\frac{E_0}{2E_k}\right)^{1/2}\left(\frac{\varepsilon_0}{\varepsilon_C}\right)^3, \tag{28}$$

where $E_k=\Delta E=\Delta\varepsilon$ is the kinetic energy of the electron in its final state. Formula (28) is similar to the differential cross section for inverse bremsstrahlung $d\sigma_B/d\Omega_e$ when the free electron receives momentum—but no energy—from a proton and simultaneously absorbs a single photon from the incident laser beam, becoming accelerated in the direction of the polarization vector of the incident photons [28],

$$\frac{d\sigma_B}{d\Omega_e}=\left(\frac{r_0^2}{137}\right)\left(\frac{E_0}{2E_k}\right)^{1/2}. \tag{29}$$

The difference is in the factor $(\varepsilon_0/\varepsilon_C)^3$ in Eq. (28). This similarity is reasonable since in our case only the interchange of momentum between the proton and the electron at inverse bremsstrahlung is substituted by the interchange of momentum between the electron and the radiation field itself. Equation (28) has, nevertheless, a much higher value than Eq. (29) because $(\varepsilon_0/\varepsilon_C)^3\gg1$. As Eq. (28) diverges for $\varepsilon_C\to0$, its validity extends to the value when Eq. (28) reaches the value of the differential cross section for the absorption of a photon by the free electron, $d\sigma_1/d\Omega_e$, given in [22],

$$\frac{d\sigma_1}{d\Omega_e}=r_0^2\left(\frac{E_0}{2E_k}\right)^{1/2}\left(\frac{16E_0}{\varepsilon_0}\right)\quad(E_k=\varepsilon_0) \tag{30}$$

because for sufficiently small ε_C the Compton photon can be treated as a part of the background noise. Comparing Eq. (28) to Eq. (30), one obtains the validity condition for Eq. (28),

$$\varepsilon_C\geqslant\varepsilon_0\left(\frac{\varepsilon_0}{137E_0 16}\right)^{1/3}.$$

For photon energies of $\varepsilon_0=1$ eV,

$$\varepsilon_C\geqslant\varepsilon_0\times10^{-3},$$

which means that the scattered photon may be sufficiently soft that

$$\frac{d\sigma_C}{d\Omega_e}\approx\frac{d\sigma_1}{d\Omega_e}\approx10^9\frac{d\sigma_B}{d\Omega_e}.$$

We emphasize that Eq. (27) may be made finite for $\varepsilon_C\to0$ by renormalization, as is given in detail for double Compton scattering in [30] and thus we shall not consider it here.

IV. CONCLUSIONS

It is shown, in the framework of nonlinear quantum electrodynamics, that a multiphoton–free-electron interaction process, in which the Compton scattering occurs simultaneously with the elastic nonforward scattering of the other photons of the intense radiation, explains the conversion of the ponderomotive energy of the electron—oscillating in the radiation field—into translational energy at four-momentum conservation. The energy absorbed by the free electron in this process is much higher than that in weak fields and extends up to the energy of one photon when instead of the Compton effect photon absorption occurs.

The obtained energy absorbed by the electron as a linear function of the incident radiation intensity is a classical quantity, showing that nonlinear quantum electrodynamics is able to produce classical results, in which the Planck constant is absent. The derived formulas may be used, for example, in the analysis of the above-threshold ionization of atoms in strong fields, in plasma physics, astrophysics, and elsewhere.

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