Generation of optical macroscopic quantum superposition states via state reduction with a Mach-Zehnder interferometer containing a Kerr medium

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A method for producing macroscopic quantum superposition states (generally known as Schrödinger cat) states for optical fields is presented. The proposed method involves two modes of the field interacting dispersively in a Kerr medium where one of the modes is an arm of a Mach-Zehnder interferometer and the other mode is external to it. If the external mode initially contains a macroscopic quantum state, such as a coherent state, and the vacuum and a single photon state are the inputs to the interferometer, the external field state becomes entangled with the number states associated with the two paths of the interferometer. Selective measurement at the output ports of the interferometer project the external mode into the desired cat states. It is pointed out that the method can also be used to generate cat states out of multimode states initially containing correlations. [S1050-2947(99)07705-7]

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Currently, there is much interest in generating quantum states consisting of superpositions of macroscopically distinguishable quantum states, colloquially known as Schrödinger cat states [1]. States of this sort, although perhaps best described as consisting of mesocopically, rather than macroscopically, distinct quantum states, have been generated in the context of cavity QED [2], in the quantized vibrational motion of a trapped ion [3], and in the radial excitation of an electron in a Rydberg state [4]. In the former two cases, the cat states consist of superpositions of phase-shifted coherent states of a one-dimensional harmonic oscillator of the bosonic (photon or phonon) degree of freedom. The states are generated by first creating an entangled state where the phase of the coherent state of the bosonic system has been correlated with the internal state of an atom or ion. Subsequent measurements of the atom or ion project the bosonic system onto the desired superpositions.

So far, the Schrödinger cat states have not been realized optically. A great variety of methods have been proposed for generating such states and many of the proposals have recently been reviewed by Buzek and Knight [5]. One of the earliest proposed methods was due to Yurke and Stoler [6] who showed that self-modulation of coherent light propagating in a Kerr medium could lead to superpositions of coherent states of the form $|\beta\rangle + i|-\beta\rangle$, where $|\beta\rangle$ is a coherent state. But in the cavity QED and ion trap experiments, in addition to the Yurke-Stoler states, superpositions of the form $|\beta\rangle \pm |-\beta\rangle$, the so-called even and odd coherent states, as well as states of the type $|\beta e^{i\phi}\rangle + e^{i\theta}|\beta e^{-i\phi}\rangle$, can be formed.

In this paper I present a method which, in principle, is capable of generating a great variety of optical cat states. As far as I am aware, the method has not previously been discussed. It has much in common with a procedure discussed in the context of cavity QED experiments that involve dispersive atom-field interactions followed by selective state reductive measurements on the atoms. In the present case, there is a Kerr-type dispersive interaction between two field modes, one mode being one arm of a Mach-Zehnder interferometer while the other mode, external to the interferometer, initially contains the field to be transformed into a cat state. State reduction occurs as a result of measurements at the output ports of the interferometer. It will be evident that this scheme is analogous to the cavity QED case but where interferometer now plays the role of an atom passing through the cavity. The scheme to be presented here is closely related to some proposals for optical quantum nondemolition (QND) measurements [7–9] but QND measurement is not used. Schemes involving QND measurements have been proposed for the generation of the Schrödinger cat states [10,11] but they are quite different than what I am proposing here. Also, Dakna, *et al.* [12] describe a method for the generation of Schrödinger catlike states via conditional photon number measurements in one of the outputs of a beam splitter where the input state is a squeezed vacuum. However, in this scheme it is necessary to have precise photon counting in order to distinguish between even and odd photon states. In contrast, the current proposal requires only the ability to detect a photon or not.

A schematic for the proposed method is given in Fig. 1. The procedure requires a standard Mach-Zehnder interferometer with a Kerr medium in one arm of the counterclockwise path and a phase shifter, essentially controlling the path length difference, in the clockwise path. The phase shift will be denoted θ. Another field mode, not part of the interferometer itself, passes through the Kerr medium. I let $\hat{a}$ be the annihilation operator of the field mode of the arm interferometer containing the Kerr medium and $\hat{b}$ that of the other mode passing through the medium. The dispersive Kerr interaction between these modes is given by the interaction Hamiltonian

$$\hat{H}_K = \hbar K \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b}, \tag{1}$$

where K is related to a third-order nonlinear susceptibility $\chi^{(3)}$. The self-modulation terms of each mode, containing the operators $(\hat{a}^\dagger \hat{a})^2$ and $(\hat{b}^\dagger \hat{b})^2$, have been ignored here.

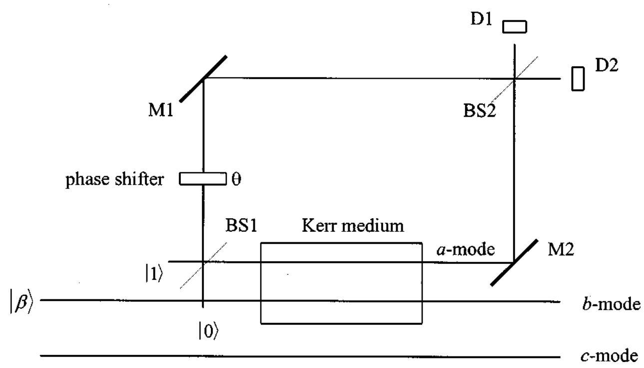

FIG. 1. Schematic for the proposed method of generating Schrödinger cat states with a Mach-Zehnder interferometer with a Kerr medium in one arm. BS1 and BS2 are beam splitters. The clockwise path of the interferometer contains a delay given rise to a phase shift denoted θ. Mode a is the arm of the interferometer containing the Kerr medium while mode b, also passing through the Kerr medium, initially contains a coherent state. Mode c is included for cases where the state to be transformed into a cat state is a two-mode correlated state, there being correlations between the photon states of modes b and c. $D1$ and $D2$ are the photon detectors at the output ports of the interferometer.

This can be justified by choosing resonances of the Kerr medium in an appropriate way as has been shown by Imoto *et al.* [7].

To begin describing the process of forming cat states, I assume that single photon and vacuum states enter the input ports of the first splitter, as indicated in Fig. 1, and that mode b contains a coherent state $|\beta\rangle$. Adopting the Schrödinger picture throughout, it can be seen that just after the first beam splitter (BS1), the state of the system is

$$\frac{1}{\sqrt{2}}(|1,0\rangle + i|0,1\rangle)|\beta\rangle, \tag{2}$$

where the interferometer number states are labeled according to the scheme where the number of photons in the clockwise path is followed by the number in the counterclockwise path. The interferometer states are entangled, consisting of a superposition of the states for a photon propagating along two different paths. In this arm of the interferometer, the counterclockwise states are of mode a, the mode passing through the Kerr medium. Just before the mirrors, as a result of the Kerr interaction and the phase shifter, the state of the system is

$$\frac{1}{\sqrt{2}}(e^{i\theta}|1,0\rangle|\beta\rangle + i|0,1\rangle|\beta e^{-i\phi}\rangle), \tag{3}$$

where $\phi = Kl/v$ and where l is the length of the Kerr medium and v is the velocity of light in the medium. Note that this is a three mode entangled state now involving the mode initially containing the coherent state and the interferometer states. The unitary operator that created this additional entanglement out of the state given in Eq. (2) is, of course,

$$\hat{U}_K = \exp(-i\phi\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}). \tag{4}$$

This operator, acting on the state of Eq. (2), produces, in the language of quantum information theory [13], a kind of controlled-NOT with respect to the phase shifting of the coherent state. That is, a vacuum state in mode a is correlated to the non-phase-shifted coherent state in mode b whereas a one photon state in the former is correlated with a phase-shifted coherent state in the latter. Now, at the mirrors, $M1$ and $M2$, both beams suffer a $\pi/2$ phase shift amounting to an overall irrelevant phase factor. Finally, the second beam splitter, BS2, causes the transformations,

$$|1,0\rangle \Rightarrow \frac{1}{\sqrt{2}}(|1,0\rangle + i|0,1\rangle),$$
$$|0,1\rangle \Rightarrow \frac{1}{\sqrt{2}}(|0,1\rangle + i|1,0\rangle), \tag{5}$$

where the number states are now labeled such that where the first is of the beam emerging horizontally from the beam splitter while the second is of the vertical beam. Thus just after the second beam splitter, the state of the system is

$$\tfrac{1}{2}[|1,0\rangle(e^{i\theta}|\beta\rangle - |\beta e^{-i\phi}\rangle) + i|0,1\rangle(e^{i\theta}|\beta\rangle + |\beta e^{-i\phi}\rangle)]. \tag{6}$$

If detector $D1$ ($D2$) and not $D2$ ($D1$) fires, then the state $|1,0\rangle$ ($|0,1\rangle$) is detected thus projecting the b mode into the (unnormalized) state,

$$|\beta\rangle \mp e^{-i\theta}|\beta e^{-i\phi}\rangle. \tag{7}$$

These, of course, are the Schrödinger cat states. If the Kerr medium can be sufficiently long or if K is sufficiently large such that $\phi = \pi$ and if $\theta = 0$, then the states of Eq. (7) become the odd and even coherent states $|\beta\rangle \mp |-\beta\rangle$, respectively, and for $\theta = \pi/2$ they are Yurke-Stoler states mentioned above. The probabilities of obtaining the states of Eq. (7), as functions of θ and ϕ are given by

$$P_{\mp}(\theta,\phi) = \tfrac{1}{2}\{1 \mp \exp[-|\beta|^2(1-\cos\phi)] \times \cos(\theta + |\beta|^2 \sin\phi)\}. \tag{8}$$

Of course, these are also the probabilities of detecting the interferometer output states $|1,0\rangle$ and $|0,1\rangle$, respectively.

Once the cat states are produced, they could be detected via homodyne techniques [14]. The signature of the cat states will be interference fringes in the statistics of the photocurrent of the detector. This should be contrasted with cases where no measurement is made at the output of the interferometer and mode b is then just a statistical mixture of coherent states given by the density operator

$$\rho_b = \tfrac{1}{2}(|\beta\rangle\langle\beta| + |\beta e^{-i\phi}\rangle\langle\beta e^{-i\phi}|), \tag{9}$$

obtained by tracing over the interferometer states in Eq. (6). No interference fringes should appear in this case. Unfortunately, the detection of the expected fringes for the cat states has been hampered by the low efficiencies of the detectors. The issue of detecting optical cat states with currently available equipment, in spite of low detector efficiencies, has recently been discussed at length by Montina and Arrechi [15].

The procedure outlined here for generating cat states is not limited to producing superpositions of states for a single mode field. It can be used to generate cat states for multimode fields that initially contain correlations between the modes. Examples of such correlated states are the two-mode SU(1,1) coherent states generated from the vacuum by a nondegenerate parametric amplifier [16], the SU(2) coherent states generated from a vacuum and number state in a frequency converter [17], and the pair coherent states generated from a competitive two-photon process [18]. These correlated two-mode states possess many nonclassical properties. Schrödinger cat states constructed out of these states have been discussed elsewhere [19,20]. Suffice it to say that the states often exhibit nonclassical properties enhanced over those of their constituent states. Here I will only consider the case of the cat state constructed from the pair coherent state—the pair cat state [19].

The pair coherent state with equal average photon numbers in each mode is given by [18]

$$|\xi\rangle=\frac{1}{\sqrt{I_0(2\xi)}}\sum_{n=0}^{\infty}\frac{\xi^n}{n!}|n\rangle_b|n\rangle_c, \tag{10}$$

where mode c has been introduced. This mode c is not directed though the Kerr medium but mode b is as before. I_0 is a modified Bessel function. $|\xi\rangle$ is an eigenstate of the two-mode annihilation operator $\hat{a}\hat{b}$ and of the number difference operator $\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b}$ with eigenvalues ξ and 0, respectively. Following the above procedure with $|\xi\rangle$ the initial state in modes b and c, the (unnormalized) pair cat states,

$$|\xi\rangle\mp e^{-i\theta}|\xi e^{-i\phi}\rangle, \tag{11}$$

may be formed. Properties of the specific cases with $\theta=0$, $\pi/2$, and $\phi=\pi$ have been discussed in Ref. [19]. (The pair and pair cat states have also been discussed in the context of the quantized vibrational motion of a trapped ion by Gou, Steinbach, and Knight [21].) Obviously, this scheme can be used for the generation of superposition states out of other initially correlated multimode field states. I should point out that in Ref. [19], a proposal for generating the pair cat states was put forward in which a *both* modes of a pair coherent state undergo a Kerr interaction of the form given in Eq. (1). No state reduction was involved so only the two-mode analog of the Yurke-Stoler state [6] could be formed. In contrast, the present scheme allows for a great variety of pair cat states including even and odd pair cat states.

As a last example of the kinds of states that can be formed with this type of interaction, dispensing with the interferometer, suppose that both inputs to the Kerr medium are coherent states: $|\alpha\rangle_a|\beta\rangle_b$. If the medium length is such that $\phi=\pi$, it can be shown that the coherent states evolve into (apart from normalization)

$$(|\alpha\rangle+|-\alpha\rangle)|\beta\rangle+(|\alpha\rangle-|-\alpha\rangle)|-\beta\rangle, \tag{12}$$

where the ordering of the states for modes a and b is obvious. Equivalently, this state can be written as

$$|\alpha\rangle(|\beta\rangle+|-\beta\rangle)+|-\alpha\rangle(|\beta\rangle-|-\beta\rangle). \tag{13}$$

Projection onto a coherent state in one of the modes produces an even or odd cat state in the other mode. The projections can be done via a homodyne technique as recently discussed by Yurke and Stoler [22]. This technique is insensitive to the photon number but *is* sensitive to the sign of the coherent state. Only the even and odd cat states can be produced in this case.

It is now time to address possible difficulties with the proposed scheme. The main difficulty is associated with obtaining large phase space separations of the coherent states in the superpositions. Maximal separation occurs for the case with $\phi=\pi$. This requires either a large Kerr nonlinearity or a long glass fiber. Currently available Kerr nonlinearities are small. Using the estimates of Sanders and Milburn [23], it can be shown that to produce a phase shift on the order of π for an optical frequency of about $\omega\approx5\times10^{14}$ rad/sec would require a of length $\approx$3000 km. But in a fiber of that length, dissipative effects will be of great importance and will cause a decoherence of the desired superposition states. But if materials with a high $\chi^{(3)}$ become available, the fiber could be shorter and thus the effects of dissipation could be minimized. Agarwal [24] has suggested that the Kerr nonlinearity could be enhanced by enclosing the medium in a cavity with a low transmission rate. Also, Schmidt and Imamolgu [25] have discussed the scheme to obtain giant Kerr nonlinearities from electromagnetically induced transparency and Franson [26] has discussed the enhancement of optical quantum gates by cooperative effects involving pairs of atoms. It is important to note that many of the proposed QND schemes [7–9] also require Kerr media with large nonlinearities. Even if the required nonlinearities for obtaining the maximal separation cannot be achieved, it would still be of interest to generate states with nonmaximal separation. Suppose, for example, that the states $|\beta\rangle-|\beta e^{-i\phi}\rangle$ are formed in the proposed experiment and that ϕ is small. In terms of the Fock states, this state is given as

$$\exp(-|\beta|^2/2)\sum\frac{\beta^n}{\sqrt{n!}}[1-\exp(-in\phi)]|n\rangle. \tag{14}$$

Even though ϕ is small, there will be values of n where $n\phi$ is close to, for example, 2π (or any even multiple of π) in which case the photon number probability distribution will go to near zero. For example, if $\overline{n}=|\beta|^2\approx2\pi/\phi$, then at $n\approx\overline{n}$ there will be a near vanishing of the probability of obtaining the n photon state and the original Poisson distribution of the coherent state will be bifurcated near its center. There could be many such oscillations, or ''holes,'' in the photon number distribution depending on the values of β and ϕ. Oscillations of this sort can be found in, for example, the displaced number states [27]. States possessing such oscillations are known to be highly nonclassical. The oscillations can be interpreted as arising from interference in phase space [28]. Haroche and collaborators have discussed the possibility of generating the maximally separated states in cavity QED [2(b)] but, so far, only the nonmaximally separated states have been generated in the experiments [2(a)]. Properties of a class of nonmaximally separated states have been studied by Schleich, Pernigo, and Le Kien [29].

Lastly, it is necessary to deal with the issue of the generation of the single photon state that is required as the input to the interferometer. Generating this state may be the most challenging aspect of this proposal. But some time ago, Hong and Mandel [30] generated a single photon state from spontaneous nondegenerate parametric down conversion. In this process, photons are produced in correlated pairs. Photodetection of the photon in one of the beams state reduces the other beam to a single photon state. Single photons produced in this way have been used in a number of experiments [31].

In summary, I have presented a scheme for the generation of Schrödinger cat states for optical fields. The method uses a Kerr medium in one arm of a Mach-Zehnder interferometer and state reduction at the output ports. Cat states for single mode fields as well as correlated multimode fields can be formed with the proposed procedure.

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