Parametric down-conversion for light beams possessing orbital angular momentum

J. Arlt, K. Dholakia, L. Allen, and M. J. Padgett
School of Physics & Astronomy, University of St. Andrews, North Haugh, St. Andrews, Fife KY16 9SS, Scotland
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We investigate the spontaneous parametric down-conversion for light beams possessing orbital angular momentum. The experimental results indicate that the orbital angular momentum is not conserved within the classical light fields. This is in contrast to second harmonic generation where the conservation of orbital angular momentum leads to a well-defined mode transformation. We attribute this difference in behavior to the loss of spatial coherence within each of the down-converted fields. [S1050-2947(99)01205-6]

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A circularly polarized light beam possesses spin angular momentum attributable to the spin of the individual photons. Light beams can also carry orbital angular momentum associated with their azimuthal phase structure. It was shown theoretically in 1992 that light beams with an azimuthal phase term $e^{il\phi}$, of which Laguerre-Gaussian (LG) modes are an example, have a well-defined orbital angular momentum of $l\hbar$ per photon [1]. Experimentally the orbital angular momentum of light beams has been used to rotate microscopic particles trapped in optical tweezers [2,3]. To confirm the quantitative orbital angular momentum content of beams with an azimuthal phase dependence of $e^{il\phi}$, such rotation has been compared to that induced by the spin angular momentum of a circularly polarized light beam [4].

Light beams carrying orbital angular momentum are conveniently described in terms of Laguerre-Gaussian modes. As they form a complete orthonormal basis set, any paraxial beam can be described as a superposition of such modes. Laguerre-Gaussian modes are characterized by two mode indices l and p, where l is the number of 2π cycles in phase around the circumference and $p+1$ is the number of radial nodes. Their amplitude u_p^l is given by

$$u_p^l \propto e^{-ikr^2/2R} e^{-r^2/w^2} e^{-i(2p+l+1)\Psi} \times e^{-il\phi}(-1)^p (r\sqrt{2}/w)^l L_p^l(2r^2/w^2), \qquad (1)$$

where R is the wave-front radius of curvature, w is the radius for which the Gaussian term falls to $1/e$ of its on-axis value, Ψ is the Gouy phase, and $L_p^l(x)$ is a generalized Laguerre polynomial.

Although frequency doubling of beams with phase singularities was first reported in 1993 [5], it was only subsequently that the doubling of LG modes of various order was investigated in the context of orbital angular momentum of light [6,7]. It was found that a mode with azimuthal index l generates a second harmonic field with an $e^{i2l\phi}$ phase term. Thus a mode with $l\hbar$ orbital angular momentum per photon produces a frequency doubled field with $2l\hbar$ orbital angular momentum per photon. As the number of photons in the second harmonic field is only half that of the depleted photons of the fundamental beam, the orbital angular momentum within the light fields is conserved. Berzanskis *et al.* obtained similar results by looking at sum-frequency generation of beams of the same frequency but differing azimuthal mode structure [8]. They also investigated the parametric amplification of beams with optical vortices and found that topological charge is conserved in the process. A vortex with charge l in their experiment is associated with an azimuthal phase $e^{il\phi}$ and this result can, therefore, also be interpreted as the conservation of orbital angular momentum for parametric amplification.

In this paper we discuss parametric down-conversion of light beams with orbital angular momentum. Parametric down-conversion is a nonlinear process in which an incoming pump field generates two new fields, named signal and idler, which must fulfill both the conservation of energy and phase-matching conditions within the nonlinear crystal [9]. In the degenerate case, when the signal and idler fields have the same frequency ($\omega_{\text{Signal}} = \omega_{\text{Idler}} = \omega_{\text{Pump}}/2$), parametric down-conversion appears to be exactly the inverse process of second harmonic generation. If the orbital angular momentum within the light fields is conserved, it should be divided equally between signal and idler. For pump modes with an even mode index $2l$, one therefore might expect that the fluorescence would form a mode with $l\hbar$ of orbital angular momentum per photon. For pump modes with an odd mode index l, or for fluorescence away from degeneracy, down-conversion would result in beams with noninteger multiples of $\hbar$ orbital angular momentum per photon. Such beams are perfectly feasible, but are not circularly symmetric nor do they propagate in a structurally stable fashion [10].

Figure 1 shows the experimental setup. The pump beam was produced by a commercial frequency-doubled Nd:YVO laser which gives 100 mW of cw light at a wavelength of 532 nm. Computer-generated holograms were used to produce beams with an azimuthal phase term $e^{il\phi}$, carrying $l\hbar$ orbital angular momentum per photon. The blazed transmission holograms were manufactured on holographic film and

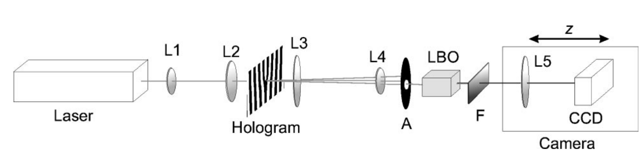

FIG. 1. Experimental setup: Lenses L1 and L2 expand the laser beam to avoid damage to the hologram; L3 and L4 collimate the beam generated by the hologram; the aperture *A* removes the unwanted diffraction orders; *F* is a filter blocking the pump beam; and the lens L5 images the beams onto a CCD array.

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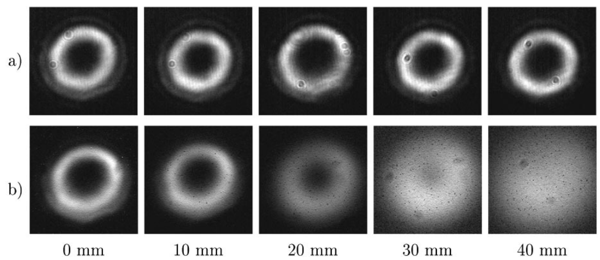

FIG. 2. (a) The profile of the green pump beam for $l=2$ and (b) the profiles of the down-converted infrared beam for a range of positions behind the backface of the nonlinear crystal. Note that the grayscale for the last two infrared profiles has only half the range of the others.

had an efficiency of about 25% at 532 nm [11]. The generated beam had an annular intensity profile similar to a Laguerre-Gaussian mode with radial index $p=0$. Decomposition of the beam in terms of Laguerre-Gaussian modes showed that about 80% of its energy was carried in the mode with $p=0$; the remainder was distributed among modes with higher radial index p but the same azimuthal index l [12]. The beam was then collimated with a spot size of about $w_0=360$ μm and passed through the down-conversion crystal that was a 20-mm long lithium triborate (LBO) crystal cut for type I noncritical phase matching. The temperature was set to 148 °C to achieve degenerate output at 1064 nm. The generated signal and idler fields are indistinguishable as they have similar wavelength and the same polarization. The intensity profile of the down-converted light was investigated using a cooled charge-coupled-device (CCD) array (Meade Pictor 216XT, 336×242 pixels). The camera unit, which is an imaging lens and the CCD array, was moved along the propagation direction to image the beam profiles in different planes behind the crystal. The quantum efficiency of the CCD array at our signal wavelength ($\lambda=1064$ nm) was only about 0.1%. As the signal was very weak, about 10 pW, fairly long integration times of 1 to 3 min were required.

Figure 2 shows the recorded intensity profiles for both pump and down-converted beams at various positions after the nonlinear crystal for a pump beam with an azimuthal index $l=2$. One might expect such a pump beam to give rise to down-converted beams with azimuthal index $l=1$ if parametric down-conversion were the inverse of frequency doubling. Although the collimated pump beam propagates in a structurally stable fashion, the down-converted light changes its form as it propagates. In the vicinity of the nonlinear crystal the intensity profile of the down-converted light resembled that of the pump beam with a clearly distinguishable intensity null on the beam axis. But further away from the crystal the ring became ill-defined and at about 40 mm behind the crystal, the on-axis zero could no longer be distinguished.

Figure 3 shows the parametric fluorescence generated by a pump beam with azimuthal index $l=1$. The intensity profiles display the same qualitative behavior as for $l=2$. The wide tuning range of the LBO crystal also allowed the investigation of phase matching away from degeneracy, where it is possible to distinguish signal and idler field by their difference in frequency. We looked at down-conversion at a

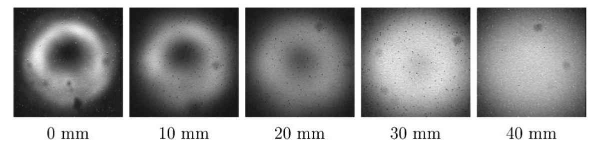

FIG. 3. Profiles of the down-converted beams generated by a pump beam with $l=1$ for a range of positions behind the backface of the nonlinear crystal.

frequency ratio of 2:1 for signal and idler, that is, $\lambda_{\text{Signal}}=798$ nm and $\lambda_{\text{Idler}}=1596$ nm. As the CCD camera was not sensitive at the idler wavelength we could look at the signal field on its own, but this showed the same qualitative behavior as in the degenerate case.

Clearly, the absence of an on-axis intensity minimum is inconsistent with the down-converted light having an $e^{il\phi}$ phase structure. More detailed examination of the phase structure of the down-converted beam was achieved by interfering it with a sheared image of itself. Within our experimental setup this was accomplished by introducing a blazed phase grating into the system after the crystal. About 60% of the transmitted light was diffracted into the first order and most of the rest was in the undiffracted beam. The camera lens recombined the first order and the undiffracted beam on the CCD array. If the grating was positioned in the object plane of the camera, the beams overlapped completely. When the grating was moved away from the object plane, the lateral shear between the beams was increased. For complete overlap, the degree of spatial coherence of the beams was irrelevant and straight line interference fringes were formed with a spacing determined by the intersection angle of the beams. However, if the two beams were sheared, no interference fringes were observed (Fig. 4). This shows that each of the down-converted beams is spatially incoherent.

Although optical vortices have been associated with apparently random laser speckle patterns [13], the low spatial coherence of the down-converted beams shows that they do not have a well-defined orbital angular momentum. Consequently, we conclude that within the process of spontaneous parametric down-conversion, orbital angular momentum is not conserved as an observable property within the classical light beams. This is in direct contrast to the process of second harmonic generation [6] or sum frequency mixing [8] where the conservation of orbital angular momentum gives rise to a mode transformation. It is well known that spin angular mo-

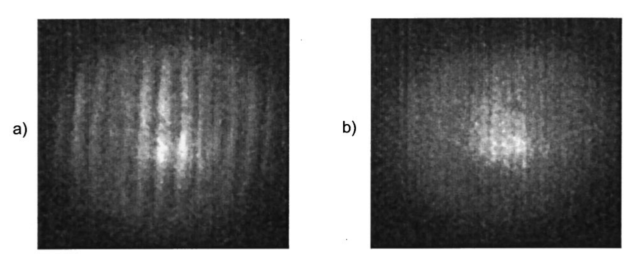

FIG. 4. Profile of the interference between a down-converted beam and a sheared image of itself. (a) The beams overlapped completely and produced interference fringes. (b) When the beams were laterally sheared by approximately 60 μm, which corresponds to roughly a third of the fringe period in Fig. 4(a), the interference fringes disappeared.

mentum is not conserved within the light during nonlinear frequency conversion. For example, type II phase matched second harmonic generation with circularly polarized light yields a linearly polarized frequency doubled beam. We assume that, as for circularly polarized light, the angular momentum in our experiment might be transferred to the nonlinear crystal.

Our observation that spontaneous parametric down-conversion yields signal and idler beams, which individually have low spatial coherence, is consistent with results of Ribeiro *et al.* [14] and Ghosh *et al.* [15]. They demonstrate that for a fundamental Gaussian mode (0,0) pump beam the down-converted light has low spatial coherence. The subsequent spread of the down-converted beam is a function of the phase-matching acceptance cone. Ghosh showed, too, that a higher-order correlation relating to the behavior of nonclassical light also occurs. However, the implications of the low spatial coherence with respect to the conservation of the orbital angular momentum has not previously been considered.

The spatial incoherence of the signal and idler fields stems from the nature of the three wave interaction. In three wave interactions in which one or more fields build up from noise, the phases of the fields are constrained by the equation

$$\phi_3 - \phi_1 - \phi_2 = \pm \frac{\pi}{2}, \tag{2}$$

where the sign depends on the direction of the energy transfer between the three fields [16]. In sum frequency mixing, ϕ_1 and ϕ_2 are defined by the two input beams. It follows that the phase of the generated field ϕ_3 is delineated and the spatial coherence of the pump beams is transferred to the generated wave. However, in parametric down-conversion, only one of the fields, that of the pump beam, has an externally defined phase ϕ_3. Consequently, although at any point there is a well-defined phase relationship between the signal and idler fields, neighboring positions in either of the beams have no fixed phase relationship. Hence the spatial coherence of the pump beam is not transferred to the down-converted beams.

Although the orbital angular momentum for spontaneous parametric down-conversion is not conserved within the classical light beams, other parametric processes may be expected to behave differently. For example, it seems likely that if optical feedback were to be introduced to one or both of the down-converted fields, the mode selectivity associated with the cavity may well favor the oscillation of self-reproducing signal and idler modes. Spatial overlap with the annular Laguerre-Gaussian pump mode would presumably favor annular signal and idler modes. Thus it is possible that these generated modes could themselves be Laguerre-Gaussian possessing the corresponding orbital angular momentum.

Our experiment indicates that, unlike second harmonic generation, orbital angular momentum is not conserved in the spontaneous parametric down-conversion of a classical light beam.

Note added in proof. Our experiment examines the orbital angular momentum of down-converted classical beams. However, a recent experiment by Mair and Zeilinger [17] shows that at the single-photon level the orbital angular momentum is conserved for an $l=0$ pump beam.

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