Scheme to probe the decoherence of a macroscopic object

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We propose a quantum optical version of Schrödinger's famous gedanken experiment in which the state of a microscopic system (a cavity field) becomes entangled with and disentangled from the state of a massive object (a movable mirror). Despite the fact that a mixture of Schrödinger cat states is produced during the evolution (due to the fact that the macroscopic mirror starts off in a thermal state), this setup allows us to systematically probe the rules by which a superposition of spatially separated states of a macroscopic object decoheres. The parameter regime required to test environment-induced decoherence models is found to be close to those currently realizable, while that required to detect gravitationally induced collapse is well beyond current technology. [S1050-2947(99)01405-5]

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I. INTRODUCTION

Quantum mechanical superpositions of macroscopically distinguishable states of a macroscopic object decay rapidly due to the strong coupling of the object with its environment. This process is called environment-induced decoherence (EID) [1]. There are many assumptions involved in modeling the EID of a macroscopic object. For example, the nature of the coupling of a macroscopic object to its environment is generally taken to be a linear [2,3] or a nonlinear [4] function of the position operator of the object. Assumptions are also made about the environment. Based on these assumptions, various explicit formulas have been derived for the dependence of the decoherence time scale on various parameters of the system, its environment, and the spatial separation between the superposed components [1–4]. Obviously, the most appropriate model can be selected only through experimentation. Decoherence formulas relevant to the quantum optical domain [5] are now beginning to be tested experimentally [6]. As far as quantum objects bearing mass are concerned, decoherence has been investigated for the motional states of ions in a trap [7]. There have also been other interesting suggestions for testing decoherence experimentally [8–10]. However, as yet, no one has managed to test the the rules of decoherence of a superposition of spatially separated states of a macroscopic object. This is, presumably, because of the implicit assumption that one actually needs to prepare a superposition of distinct states of a macroscopic object in order to probe the rules of its decoherence. Such a superposition is extremely difficult to prepare because of the difficulty of obtaining a macroscopic system in a pure quantum state. In this paper, we propose a scheme that will allow us to probe the rules of decoherence of a superposition of states of a macroscopic object without actually creating such a superposition. We also show that it requires experimental parameters that are close to the potentially realizable domain.

Besides EID, there also exist a set of collapse models [11,12] developed to resolve the measurement problem of quantum mechanics. Whether such a mechanism is really necessary or whether some reformulation of quantum mechanics such as the histories approach [13] suffices, is an open question [14]. Some experiments to detect such mechanisms have been suggested [9,15], and some pre-existing experimental data have been analyzed [16,17]. In particular, atomic interferometry experiments provide a great potential to put bounds on such models [16]. However, there is no direct evidence for their existence. We calculate the parameter regime required if our experiment is to probe such models, and show that this is a much more difficult task than probing EID.

Our experiment is based on applying the ideas used by Schrödinger in his famous gedanken experiment involving a cat [18] to a certain quantum optical system. Obvious differences arise as our setup is meant to be realistic. We begin by recapitulating Schrödinger's technique and describing qualitatively what happens when such a technique is applied to our setup.

II. SCHRÖDINGER'S METHOD FOR CREATING MACROSCOPIC SUPERPOSITIONS

The basic idea used by Schrödinger to create macroscopic superpositions was to entangle the states of a microscopic and a macroscopic system [18]. It is easy to put the state of a microscopic system (which follows quantum mechanics beyond any controversy) into a superposition of distinct states. Subsequently, this system is allowed to interact with a macroscopic system to propel it to macroscopically distinct states corresponding to the different superposed states of the microscopic system. In Schrödinger's case, the microscopic system was a radioactive atom, while the macroscopic system was a cat. In this paper we propose to apply exactly the Schrödinger technique to a cavity field (a microscopic system) coupled to a movable mirror (a macroscopic system). However, there are differences of such a realistic version of Schrödinger's thought experiment from his original version. We will enumerate these problems below and outline why our experiment can still achieve its target (testing the decoherence of superpositions of states of a macroscopic object) despite being quite different from Schrödinger's original experiment.

Firstly, as yet, no technique exists to put a macroscopic oscillator in a pure coherent state (though some progress has been made in cooling of such objects [19]). Thus, unlike the

cat in Schrödinger's original experiment, the macroscopic mirror cannot start off in a pure state, and in general, will start off in a thermal state. So only a mixture of Schrödinger's cat states can ever be created through unitary evolutions. So the primary aim of Schrödinger's experiment (creating macroscopic superpositions) cannot be achieved this way. However, our aim is to test the decoherence of states of a macroscopic object and not to create a pure Schrödinger's cat state. An important point to realize is that the former *can* be done *without* necessarily doing the latter. We shall show that the state of the cavity field (the system on which we actually perform our measurements) at the end of our experiment is same *irrespective* of whether the macroscopic object (the movable mirror) coupled to it starts off in a thermal state or a coherent state. This is due to the specific nature of the coupling between the field and the mirror. Thus the mixture of Schrödinger's cat states produced has the same observational consequences as a pure Schrödinger's cat as far as our scheme is concerned.

Secondly, it appears that the decoherence of a superposition of states of a macroscopic mirror (which can never be made as isolated as the cat in Schrödinger's thought experiment) will be too fast to detect. To circumvent this, one must note that the decoherence rate of a certain superposition of states of an object increases with the spatial separation between these states [1–3]. So the amount by which the decoherence rate increases due to the macroscopic nature (large mass) of the object can always be offset by reducing the spatial separation between the superposed states.

Thirdly, in Schrödinger's case, the coupling between radioactivity and the cat involved highly nonlinear biological processes. So even a small trigger of radioactive decay was sufficient (via the breaking of the poison vial) to produce as much of a change in a cat as killing it. Can we get such a nonlinear coupling to produce a drastic change in the state of the macroscopic mirror from small changes in the state of the cavity field? The answer to this is that it is really not necessary to have a drastic change in the state of the macroscopic mirror. Even a superposition of macroscopically nondiscernible states is sufficient to produce a detectable rate of decoherence if the the mirror is sufficiently macroscopic.

III. THE SYSTEM UNDER CONSIDERATION

We consider a microcavity with one fixed and one movable mirror. The cavity contains a single mode of an electromagnetic field (frequency ω_0 and annihilation operator denoted by a) that couples to the movable mirror (which is treated as a quantum harmonic oscillator of frequency ω_m and annihilation operator denoted by b). This system has already been studied quite extensively [20–22] and the relevant Hamiltonian [20] is

$$H=\hbar\omega_0 a^\dagger a+\hbar\omega_m b^\dagger b-\hbar g a^\dagger a(b+b^\dagger), \tag{1}$$

where

$$g=\frac{\omega_0}{L}\sqrt{\frac{\hbar}{2m\omega_m}}, \tag{2}$$

and L and m are the length of the cavity and mass of the movable mirror, respectively. For the moment we consider the system to be totally isolated. If the field inside the cavity was initially in a number state $|n\rangle_c$ and the mirror was initially in a coherent state $|\beta\rangle_m$, then at any later time t the mirror will be in the coherent state [21]

$$|\phi_n(t)\rangle_m=|\beta e^{-i\omega_m t}+\kappa n(1-e^{-i\omega_m t})\rangle_m, \tag{3}$$

where $\kappa=g/\omega_m$. Thus, in effect the mirror undergoes an oscillation with a frequency ω_m and an amplitude dependent on the Fock state inside the cavity. This feature of the mirror dynamics gives us the basic idea of the paper. A superposition of two different Fock states is created inside the cavity so that the mirror is driven to an oscillation of different amplitude corresponding to each of these Fock states. As the mirror is a macroscopic object, this situation can be regarded as a version of Schrödinger's cat experiment. Of course, in practice, only a mixture of several Schrödinger's cats is created because the mirror starts off in a thermal state instead of starting in a single coherent state $|\beta\rangle_m$.

IV. THE PROPOSED SCHEME

We propose to start with the cavity field prepared in the initial superposition of Fock states

$$|\psi(0)\rangle_c=\frac{1}{\sqrt{2}}(|0\rangle_c-|n\rangle_c). \tag{4}$$

Methods of preparing the cavity field in such states has been described in Refs. [23–25]. When discussing experimental parameters we will choose $n=1$, which is the simplest to prepare. The initial state of the movable mirror will be taken to be a thermal state at some temperature T, and is given by the density operator

$$\rho_m=\frac{1}{\pi\bar{n}}\int(|\beta\rangle\langle\beta|)_m\exp(-|\beta|^2/\bar{n})d^2\beta, \tag{5}$$

where

$$\bar{n}=\frac{1}{e^{\hbar\omega_m/k_B T}-1}, \tag{6}$$

and $|\beta\rangle_m$ represents a coherent state of the mirror corresponding to amplitude β and k_B is the Boltzmann constant. Equation (3) and the initial states given by Eqs. (4) and (5) imply that at any time t, in the absence of any external environment, the joint density operator describing the cavity mode and the mirror is given by

$$\rho(t)_{c+m}=\frac{1}{2\pi\bar{n}}\int(\rho(t)_{00}-\rho(t)_{0n}-\rho(t)_{n0}+\rho(t)_{nn})_{c+m}\times\exp(-|\beta|^2/\bar{n})d^2\beta, \tag{7}$$

where

$$\rho(t)_{00}=(|0\rangle\langle 0|)_c\otimes(|\phi_0(t)\rangle\langle\phi_0(t)|)_m, \tag{8a}$$

$$\rho(t)_{0n}=(|0\rangle\langle n|)_c\otimes(|\phi_0(t)\rangle\langle\phi_n(t)|)_m e^{-i\kappa^2 n^2(\omega_m t-\sin\omega_m t)}, \tag{8b}$$

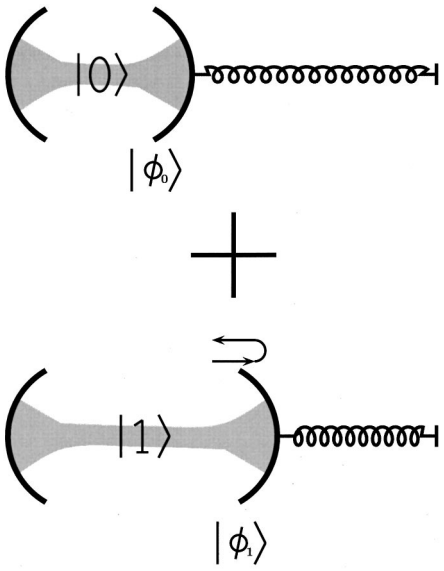

FIG. 1. The proposed optomechanical version of Schrödinger's thought experiment: the quantized single mode field is modified by the harmonic motion of one of the mirrors. If the mirror started in a single coherent state, then the result is an entangled state of the mirror motion and the cavity field, here labeled by $|0\rangle|\phi_0\rangle$ and $|1\rangle|\phi_1\rangle$. Given that the initial state of the mirror is a thermal state, only a mixture of several such optomechanical cats with different mean positions is produced.

$$\rho(t)_{n0}=(|n\rangle\langle 0|)_c\otimes(|\phi_n(t)\rangle\langle\phi_0(t)|)_m e^{i\kappa^2 n^2(\omega_m t-\sin\omega_m t)}, \tag{8c}$$

$$\rho(t)_{nn}=(|n\rangle\langle n|)_c\otimes(|\phi_n(t)\rangle\langle\phi_n(t)|)_m. \tag{8d}$$

The phase factors $e^{\pm i\kappa^2 n^2(\omega_m t-\sin\omega_m t)}$ in Eqs. (8b) and (8c) derive from a Kerr-like term in the time evolution operator corresponding to the Hamiltonian H, which has been evaluated in Refs. [20,21]. Note that there are absolutely no assumptions involved while writing Eq. (7). However, the coherent state basis expansion of the initial thermal state of the mirror [Eq. (5)] has been used for a specific purpose. The terms $\rho(t)_{00}$, $\rho(t)_{0n}$, $\rho(t)_{n0}$, and $\rho(t)_{nn}$ appear in Eq. (7) only if such an expansion is made. The effect of decoherence on such terms is already well studied [2,3] and the specific form of Eq. (5) allows us to simply utilize these known results.

The situation described by Eq. (7) between times $t=0$ and $t=2\pi/\omega_m$ is a mixture of several Schrödinger's cat states of the type depicted in Fig. 1 (where the value of n has been taken to be equal to 1). Equation (3) implies that at $t=2\pi/\omega_m$, all states $|\phi_n(t)\rangle_m$ will evolve back to $|\beta\rangle_m$ irrespective of n. Thus the mirror will return to its original thermal state [given by Eq. (5)] and the state of the cavity field will be disentangled from the mirror. In the absence of any EID, this state will be

$$|\psi(2\pi/\omega_m)\rangle_c=\frac{1}{\sqrt{2}}(|0\rangle_c-e^{i\kappa^2 n^2 2\pi}|n\rangle_c). \tag{9}$$

In reality, two sources of decoherence will be present. The first one is the decoherence due to photons leaking from the cavity and the second one is EID of the motional state of the mirror. The aim of this paper is to show how the rate of the second type of decoherence can be determined. To simplify our analysis, we shall take $n=1$ (i.e, the initial state inside the cavity is $(1/\sqrt{2})(|0\rangle_c-|1\rangle_c)$). First consider the case when no photon happens to leak out of the cavity up to a time t. If the damping constant of the cavity mirror is γ_a, then the probability for this to happen is $\frac{1}{2}(1+e^{-\gamma_a t})$ [26]. In this case, the amplitude of the state $|1\rangle_c|\phi_1(t)\rangle_m$ is suppressed with respect to the state $|0\rangle_c|\phi_0(t)\rangle_m$ by a factor $e^{-\gamma_a t/2}$. In addition to this form of decoherence, there is EID of the mirror's motional state. This has already been studied quite extensively, the basic result being a rapid decay of those terms in the density matrix that are off diagonal in the basis of Gaussian coherent states [2,3]. However, the diagonal terms in the coherent state basis are hardly affected on the same time scale (in fact, it has been shown that in the case of a harmonic oscillator, coherent states emerge as the most stable states under decoherence [27]). Quite independent of any specific model of decoherence, the satisfactory emergence of classicality would require the off diagonal terms in a coherent state basis to die much faster than the diagonal terms as coherent states are the best candidates for classical points in phase space. Thus, EID of the mirror's motional state decreases the coherence between the states $|0\rangle_c|\phi_0(t)\rangle_m$ and $|n\rangle_c|\phi_n(t)\rangle_m$ because $|\phi_0(t)\rangle_m$ and $|\phi_n(t)\rangle_m$ are spatially separated coherent states of the mirror's motion. Let the average rate of this decoherence be Γ_m. Then the state of the system at time t is given by

$$\rho(t)_{c+m}=\left(\frac{1}{1+e^{-\gamma_a t}}\right)\frac{1}{2\pi\bar{n}}\int[\rho(t)_{00}-e^{-(\gamma_a/2+\Gamma_m)t}\rho(t)_{01}-e^{-(\gamma_a/2+\Gamma_m)t}\rho(t)_{10}+e^{-\gamma_a t}\rho(t)_{11}]_{c+m}\times\exp(-|\beta|^2/\bar{n})d^2\beta, \tag{10}$$

where the symbols $\rho(t)_{ij}$ denote states as given by Eqs. (8a)–(8d). Equation (10) shows that at time $t=2\pi/\omega_m$ the states of the cavity field and the mirror become dynamically disentangled. Now consider the complementary case (i.e., when a photon actually leaks out of the cavity between times 0 and t). The total probability for this to happen is $\frac{1}{2}(1-e^{-\gamma_a t})$. As soon as the photon leaks out, the state of the cavity field goes to $(|0\rangle\langle 0|)_c$ and its state becomes completely disentangled from the state of the mirror. Moreover, the mirror does not interact with the cavity field any more as the interaction is proportional to the number operator of the cavity field. Thus its state remains disentangled from the state of the cavity field at all times after the photon leakage. Adding both the cases (photon loss and no photon loss) with respective probabilities, one gets the state of the cavity field at time $t=2\pi/\omega_m$ to be

$$\rho(2\pi/\omega_m)_c=\frac{(2-e^{-2\gamma_a\pi/\omega_m})}{2}|0\rangle\langle 0|_c-\frac{e^{-\gamma_a\pi/\omega_m}}{2}e^{-\Gamma_m 2\pi/\omega_m}(e^{i\kappa^2 2\pi}|1\rangle\langle 0|_c+e^{-i\kappa^2 2\pi}|0\rangle\langle 1|_c)+\frac{e^{-2\gamma_a\pi/\omega_m}}{2}|1\rangle\langle 1|_c. \tag{11}$$

Thus by probing the state of the cavity field at time $t=2\pi/\omega_m$, it should be possible to determine the value of Γ_m and this gives the rate of decoherence of spatially separated states of the macroscopic mirror.

We should pause here to note that the state of the cavity field at $t=2\pi/\omega_m$ is completely *independent* of the initial thermal state [given by Eq. (5)], in which the mirror starts off. This feature is very important for our proposal. It implies that the effects on the cavity field will be the same *irrespective* of whether the mirror started off in a mixture of coherent states (like a thermal state) or in a single coherent state. This makes the imprint of the demise of a single Schrödinger's cat state on the cavity field identical to the imprint made by the demise of a mixture of several such states.

The simplest method to determine the value of Γ_m is to pass a single two level atom (which interacts resonantly with the cavity field) in its ground state $|g\rangle$ through the cavity at time $t=2\pi/\omega_m$, such that its flight time through the cavity is half a Rabi oscillation period. The state of the cavity will get mapped onto the atom with $|e\rangle$ replacing $|1\rangle_c$ and $|g\rangle$ replacing $|0\rangle_c$ in Eq. (11). Then the probability of the atom to be in the state $|+\rangle=1/\sqrt{2}(|g\rangle+|e\rangle)$ after it exits the cavity is

$$P(|+\rangle\langle+|)=\tfrac{1}{2}[1-e^{-(\pi/\omega_m)(\gamma_a+2\Gamma_m)}\cos\kappa^2 2\pi]. \quad (12)$$

From the above equation it is clear that determining the probability of the exiting atom to be in the state $|+\rangle$ will help us to determine the decoherence rate Γ_m if the order of magnitude of Γ_m can be made greater than or about the same as that of γ_a. Another requirement is that Γ_m must be of the same order as ω_m or even lower. Otherwise changes in $P(|+\rangle\langle+|)$ due to changes in Γ_m would be too small to observe. Of course, if one initially started with a general state $1/\sqrt{2}(|0\rangle_c-|n\rangle_c)$ of the cavity field, then more general tomography schemes [28] will have to be used.

V. A HEURISTIC FORMULA FOR THE AVERAGE DECOHERENCE RATE

We now proceed to estimate Γ_m in terms of the physical parameters of our system to illustrate the importance of this experiment from the point of view of testing the decoherence of a macroscopic object. According to the models of references [1] and [2], a superposition of coherent states spatially separated by a distance Δx decoheres (when Δx is greater than the thermal de Broglie wavelength $\lambda_{th}=\hbar/\sqrt{2mk_B\theta}$) on a time scale,

$$t_D=\frac{\hbar^2}{2m\gamma_m k_B\theta(\Delta x)^2}, \quad (13)$$

where m and γ_m stand for the mass and damping constant of the object under consideration and θ is the temperature of the enclosure where the object is placed. In our case, the spatial separation between the coherent states $|\phi_0(t)\rangle_m$ and $|\phi_n(t)\rangle_m$ varies with time as

$$\Delta x(t)=\sqrt{\frac{\hbar}{2m\omega_m}}2\kappa n(1-\cos\omega_m t). \quad (14)$$

Assuming, for the time being, that the decoherence process was entirely environment-induced, one can use Eqs. (13) and (14) to calculate the average value of the decoherence rate Γ_m to be

$$\Gamma_m=\frac{1}{(2\pi/\omega_m)}\frac{4\hbar\kappa^2n^2}{2m\omega_m}\frac{2m\gamma_m k_B\theta}{\hbar^2}\int_0^{2\pi/\omega_m}(1-\cos\omega_m t)^2dt$$

$$=\frac{3n^2\omega_0^2}{L^2\omega_m^4 m}k_B\theta\gamma_m, \quad (15)$$

where the value of κ has been substituted.

VI. CONSTRAINTS ON THE PARAMETERS

For our scheme to be successful in testing the decoherence of a macroscopic object, and our method of analysis to be valid, certain parameter constraints have to be satisfied. The first constraint comes from Eq. (12). The decoherence rate to be measured, Γ_m, has to be made greater than or about the same order as that of the other decoherence rate γ_a. An associated requirement is that Γ_m must be of the same order as ω_m or even lower. This is because, in order to be able to measure a finite decoherence rate, we have to have only partial decoherence. If Γ_m is much greater than ω_m, then the decoherence will be too fast and essentially complete before even one oscillation period of the mirror and thereby not measurable. Thus we have,

Constraint 1

$$\omega_m\sim\Gamma_m\geqslant\gamma_a. \quad (16)$$

The next constraint is required for our heuristic treatment of the decoherence of the mirror to be valid. The use of Eq. (13) is valid only when Δx is greater than the thermal de Broglie wavelength $\lambda_{th}=\hbar/\sqrt{2mk_B\theta}$. Using the expression for λ_{th} in Eq. (14), we get

Constraint 2

$$\frac{\omega_0^2}{L^2\omega_m^4 m}k_B\theta\gg 1. \quad (17)$$

A final constraint comes from the fact that the spatial separation Δx between the superposed peaks must be greater than or at least of the same order as the width of a single peak. This is a requirement for two reasons: firstly, for the validity of our heuristic treatment of decoherence and secondly, for the components of the Schrödinger's cat to be sufficiently separated in space (i.e., at least as much separated than the spatial width of each component of the Schrödinger's cat). As the width of each of the components of the cat is simply equal to the width of a coherent state, using Eq. (14) and the fact that $n=1$, we get

Constraint 3

$$\kappa\geqslant 1. \quad (18)$$

We should stress that while Constraint 1 will be a *necessary* constraint in any analysis of our system, Constraints 2 and 3 really arise due to our method of analysis. If we could calculate the decoherence rate when the superposed wave

packets almost overlap each other, then neither Constraints 2 or 3 would be needed. But in that case, the decrease in the decoherence rate may be so much that Constraint 1 becomes difficult to achieve. We leave the analysis of this domain for the future. We now proceed to propose a set of parameters which satisfy the above constraints and which are fairly close to those currently realizable.

VII. EXPERIMENTAL PARAMETER REGIMES SATISFYING THE CONSTRAINTS

At first, let us briefly state the available ranges of the various parameters involved in our experiment as far as the technology stands today. The frequency of mechanical oscillators ($\omega_m/2\pi$ in our case) is normally in the KHz domain, but can be made to rise up to a GHz [29]. However, in the case of such high frequencies, the mass of the oscillators is very small (about 10^{-17} kg [29]). The mass m of the movable mirror has no upper restriction, but is bounded on the lower side by the requirement of having to support the beam waist of an electromagnetic field mode. This means that the masses of mirrors for microwave cavities should be no smaller than about 0.1 g while those for optical cavities can go as low as 10^{-15} kg. The length L of the cavity can be no lower than 1 cm in the microwave domain but can be as low as 1 μm in the optical domain. In fact, optical cavities with a length of the order of 10 μm already exist [30]. While there is essentially no limit to how high the mechanical damping rate γ_m of the moving mirror can be made, there is a lower limit (not necessarily a fundamental limit, but the best achievable in current experiments). Oscillating cavity mirrors with $\omega_m/2\pi \sim 10$ kHz and Q factor $\sim 10^6$ have been fabricated [31]. We will take the corresponding $\gamma_m \sim 10^{-2}$ s^{-1} to be a lower limit on the value of the mechanical damping constant. The lowest temperature θ to which a macroscopic mirror has been cooled as yet, is about 0.5 K [32]. As far as the damping constant γ_a due to leakage of photons from the cavity is concerned, the lowest values are 10^7 s^{-1} for optical cavities of $L \sim 10$ μm (with stationary mirrors) [30], 10^6 s^{-1} for optical cavities of $L \sim 1$ cm (with a moving mirror) [31], and 10 s^{-1} for microwave cavities of $L \sim 1$ cm (with a stationary mirror) [32].

Now let us examine a parameter regime in which all our constraints are satisfied. We first look at optical cavities ($\omega_0/2\pi \sim 10^{15}$ Hz). For optical cavities we can choose $L \sim 10$ μm [30]. We choose $m = 1$ mg, $\gamma_m = 10^{-2}$ s^{-1}, $\omega_m/2\pi = 10$ kHz, and $\theta = 0.1$ K. With this choice of parameters,

$$\frac{\omega_0^2}{L^2\omega_m^4 m} k_B\theta \sim 10^6 \tag{19}$$

and

$$\kappa \sim 1. \tag{20}$$

So both Constraints 2 and 3 are satisfied. Also, we have

$$\Gamma_m \sim \omega_m \sim 10^4 \text{ s}^{-1}. \tag{21}$$

Thus, in order to satisfy Constraint 3, we require $\gamma_a \leqslant 10^4$ s^{-1}. While this value of γ_a is only three orders of magnitude removed from the best reflectivity for stationary mirrors and five orders of magnitude removed from the best reflectivity for moving mirrors in the optical domain at present, all the other parameters assumed here are well within the experimentally accessible domain. We don't see any *fundamental* reason why the reflectivity of the moving mirror cannot be increased by a few orders of magnitude, as the mirror is quite macroscopic (of milligram mass). Note that in the above case the position separation Δx between the superposed components is *really* tiny (of the order of 10^{-16} m!), yet even this is sufficient to cause an observable rate of decoherence. This is because the macroscopic nature of the moving mirror implies that even this minute separation is much larger than the thermal de Broglie wavelength. We note that one is allowed to increase the mass of the moving mirror to about 100 mg, if the length of the cavity is decreased to 1 μm. Our constraints will still have exactly the same values as above when this change is made. However, it seems that 100 mg is probably the largest mass the mirror in our experiment can possibly have. Note that mirror masses of 1 mg–100 mg are well within experimentally accessible domains as mirror masses of the order of 10 mg have already been used in optical bistability experiments [33].

The above choice of parameters was entirely motivated by an attempt to keep the parameters as close as possible to an existing optical cavity with a moving mirror experiment [31]. Our constraints require the mirror reflectivity of this experiment to be improved in order for our proposal to be a success. However, another alternative is to keep the values of mirror reflectivity the same as in existing experiments but move on to a mirror oscillating at a much higher frequency. Let us choose $\omega_m \sim \gamma_a \sim 10^7$ s^{-1} (though this value of γ_a is for the best existing static mirror). To make $\Gamma_m \sim 10^7$ and satisfy constraint 1, we require one to choose low $L \sim 10$ μm, low $m \sim 10^{-15}$ kg, temperature $\theta \sim 10$ K and high $\gamma_m \sim 100$ s^{-1}. The frequency of the cavity mode is kept the same ($\omega_0/2\pi \sim 10^{15}$ Hz). This choice also satisfies Constraints 2 and 3 as

$$\frac{\omega_0^2}{L^2\omega_m^4 m} k_B\theta \sim 10^5 \tag{22}$$

and

$$\kappa \sim 1. \tag{23}$$

Among the basic changes made here from existing experiments, the temperature and higher γ_m will only be too easy to achieve. However, a cavity mirror with a very tiny mass of 10^{-15} kg should be difficult to fabricate. But mechanical resonators of much lower mass have already been fabricated [29]. Moreover, there is nothing of *principle* that excludes such a mirror for an optical cavity because it can still support an optical beam waist. Besides, small masses are also required for mechanical resonators of very high frequencies [29] as in this case. One might also think that the very small time period of mirror oscillation (10^{-7} s) may be a barrier to the tomography of the cavity field using atoms. But cesium atoms with lifetime ~ 10 ns should be useful for this purpose.

We should now proceed to examine the prospects of implementing our experiment successfully in the microwave domain. In this domain the lowest possible values of L and m are already fixed to be 1 cm and 0.1 g. Thus Constraint 3 implies that the maximum value $\omega_m/2\pi$ can take is 10^{-2} Hz. But this clearly makes Constraints 1 and 2 impossible to satisfy unless

$$\theta\gamma_m < 10^{-14}\ \mathrm{K\,s^{-1}}. \tag{24}$$

This value of the $\theta\gamma_m$ product lies well outside potentially realizable domains. Thus strictly speaking, an experiment of the kind proposed here would not achieve success in the microwave domain. The only way it could would be to use a much smaller mass for the oscillating mirror. As such a mirror will not be able to support a microwave cavity mode, we would have to introduce it as a small mechanical resonator inside a cavity with fixed mirrors. This should be quite an interesting but different problem to study, because the cavity field–mechanical resonator interaction Hamiltonian in this case may be different.

VIII. PARAMETER REGIME REQUIRED TO TEST GRAVITATIONALLY INDUCED COLLAPSE THEORIES

We may describe the parameters used in the estimations so far as being potentially realizable. Let us now identify the range of parameters that would be required if one intends to extend the scope of our experiment to test the gravitationally induced objective reduction (OR) models of the type proposed by Penrose and Diosi [12]. According to this model the decoherence rate will be

$$\gamma_{OR} \sim \frac{E}{\hbar}, \tag{25}$$

where E is the mean field gravitational interaction energy. We will examine only the case in which $\Delta x < R$, where R is the dimension of the object, as this is the easiest to achieve experimentally. In the case of a spherical geometry of the mirror (we use such an assumption just for an estimate), $E \sim Gm^2(\Delta x)^2/R^3$. Using the expression for Δx from Eq. (14) and substituting R^3 by m/D, where D is the density of the object, one gets

$$\gamma_{OR} \sim \frac{n^2\omega_0^2}{L^2\omega_m^4 m} G\hbar D. \tag{26}$$

Comparison of Eqs. (15) and (26) shows that decoherence rates according to EID and OR have exactly the same dependence on parameters L, m, ω_m, ω_0, and n and, therefore, one cannot distinguish between these models by varying any of these parameters (of course, this statement is true only for a spherical geometry of the mirror). In order to reduce the effect of EID to such an extent that effects of OR become prominent, one needs

$$G\hbar D > k_B\theta\gamma_m. \tag{27}$$

Taking the density D of a typical solid to be of the order of 10^3 kg m^{-3}, one gets

$$\theta\gamma_m < 10^{-19}\ \mathrm{K\ s^{-1}}. \tag{28}$$

Currently, temperature of a macroscopic object can be brought down to at most 0.1 K and a fairly optimistic estimate of γ_m is 10^{-2} s^{-1} (a mechanical oscillator that dissipates its energy in about 100 s). Thus an improvement of the product $\theta\gamma_m$ by sixteen orders of magnitude would be necessary to test OR using our scheme.

IX. CONCLUSIONS

In conclusion, we note that the experiment we have proposed just applies Schrödinger's method to a realistic system of a cavity field and a macroscopic moving mirror. Of course, to achieve Schrödinger's original aim (creating a macroscopic superposition), our scheme will have to be combined with a scheme that prepares the mirror in a pure coherent state. However, for testing the rules of decoherence of a macroscopic object, our scheme is sufficient. A special feature of our scheme is that the microscopic system (i.e., the cavity field) that creates the mixture of Schrödinger's cats is itself being used later as a kind of meter to read the decoherence that the mirror undergoes while the two systems are entangled. We believe that this is a canonical system for systematic probing of decoherence and offers an extensive scope of further work from both theoretical and experimental points of view. Modeling the EID of our system starting from the very first principles (assuming different types of coupling and environment) is necessary to check the accuracy of formula (15). A variant of our setup in which a small mechanical resonator is introduced inside a cavity should be an interesting problem to study. There can be an entire range of masses for such a mechanical oscillator introduced inside a cavity: starting from trapped ions [7] to trapped molecules and nanoparticles [34], to the smallest mechanical resonators that can be fabricated [29]. There can also be other variants of our proposal such as extending schemes in which an atom trapped in a cavity produces Fock states [25] to include the effects of a moving mirror.

The experimental challenge is in either of the two directions: to improve the reflectivity of existing macroscopic mirrors or to decrease the mass of the mirrors without decreasing the existing reflectivity. There is nothing of *principle*, which prohibits increasing the reflectivity of a macroscopic mirror, nor any *fundamental* connection between mirror reflectivity and mass (as long as the mirror can support the beam waist). So we do not see any real obstacle in progress directed at the possible realization of our proposal.

We would like to end with a note clarifying the exact relevance of our experiment. It is much more than detecting the presence of a thermal environment around the system. We are really interested in detecting *how* this environment causes the demise of the coherence between superposed spatially separated states of a macroscopic object. This is interesting, because *irrespective* of any role it plays in the foundations of quantum mechanics, thermal environment-induced decoherence is a *real* phenomenon yet to be systematically probed in the macroscopic domain. As far as the relevance of mentioning OR in this paper is concerned, it is mainly to emphasize the degree of technological improvement necessary in order to bring such effects into the observable do-

main. This technological feat should be taken up as a challenge unless shown to be impossible by some fundamental principle of physics.

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