ARTICLES

Quantum effect for an electric dipole

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The behavior of a particle possessing a permanent or induced electric dipole moment and interacting with external electromagnetic fields is described. For a special configuration of the fields, a quantum effect is obtained for a moving dipole. The experimental observation of this effect is within reach of atom or molecular interferometry. [S1050-2947(99)00704-0]

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I. INTRODUCTION

The electromagnetic (em) interaction is an important feature of several quantum effects such as the Aharonov-Bohm (AB) [1] effect, where charged particles moving with uniform velocity interact with the vector potential of a solenoid. In the Aharonov-Casher (AC) [2] effect, a magnetic dipole interacts with an electric field, while in the electrostatic effect of Matteucci and Pozzi [3] charged particles interact with an electric field. Another quantum effect is that considered by Colella, Overhauser, and Werner [4], where gravitational interaction is involved.

An approach to derive the quantum phase of an electric dipole has been proposed by Spavieri [5], who discusses the recent works of Wilkens [6] and of Wei *et al.* [7] on the same subject. As shown below, this approach may be elaborated to obtain a new quantum effect where moving particles possessing an electric dipole moment interact with em fields.

All these effects foresee an observable displacement of the interference pattern related to the phase shift of the wave function of the system,

$$\Delta\phi=\frac{1}{\hbar}\oint \mathbf{Q}\cdot d\mathbf{x}, \tag{1}$$

where the quantity $\mathbf{Q}$ is related to the canonical momentum of interaction. Most of them have either been already tested [8] or are within the possibility of experimental verification. Effects for quadrupoles or a higher order are unfeasible because they either require that the particle move in a medium or field strengths well beyond experimental reach [9].

In Sec. II we derive the phase of an electric dipole as an application of the AB effect to a system composed of two charges. In Sec. III we analyze the behavior of an electric dipole using a Lagrangian approach to obtain once more the same expression for phase and dedicate Sec. IV to some aspects of the observable phase shift. Finally, in Sec. V we devise a field configuration which leads to a quantum effect for an electric dipole and in Sec. VI we discuss its experimental verification.

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II. QUANTUM PHASE OF AN ELECTRIC DIPOLE

For the purpose of obtaining a quantum effect for an electric dipole, we first derive the phase directly from the AB phase and recall that in the AB effect the canonical momentum of interaction is given by $\mathbf{Q}=(q/c)\mathbf{A}$, where q is the electric charge and $\mathbf{A}$ is the vector potential of a solenoid. Thus, from expression (1),

$$\phi_{\mathrm{AB}}=\frac{q}{\hbar c}\int \mathbf{A}\cdot d\mathbf{x}. \tag{2}$$

An electric dipole of total mass $m=m_1+m_2$ moving with a nonrelativistic velocity $\mathbf{v}$ may be thought of as being composed of two charges $\pm q$ of mass m_1 and m_2 separated by the small distance $\mathbf{r}'=\mathbf{x}_1-\mathbf{x}_2$. Let the position of the center of mass be $\mathbf{x}=(m_1\mathbf{x}_1+m_2\mathbf{x}_2)/m$ and consider the expansion $\mathbf{A}(\mathbf{x}_i)\simeq\mathbf{A}(\mathbf{x})+(\mathbf{x}_i-\mathbf{x})\cdot\nabla\mathbf{A}$. A simple way to obtain the phase for an electric dipole consists in summing the AB phases of the two charges $\pm q$ in the dipole approximation

$$\begin{aligned}\phi&=\frac{q}{\hbar c}\int \mathbf{A}(\mathbf{x}_1)\cdot d\mathbf{x}_1-\frac{q}{\hbar c}\int \mathbf{A}(\mathbf{x}_2)\cdot d\mathbf{x}_2\\&=\frac{1}{\hbar c}\int (\mathbf{d}\cdot\nabla)A(\mathbf{x})\cdot d\mathbf{x},\end{aligned} \tag{3}$$

where $\mathbf{d}=q\mathbf{r}'$ is the electric dipole moment.

Result (3) represents the application of the AB phase (2) to an electric dipole and is derived below using a different approach.

III. LAGRANGIAN FORMULATION FOR THE PHASE

Let us consider a dipole that moves with velocity $\mathbf{v}$ in the presence of a time-independent scalar potential $\Phi(\mathbf{x})$ and a vector potential $\mathbf{A}(\mathbf{x})$. In our simple nonrelativistic model, the two charges of the dipole are held together by internal forces and the corresponding self-interaction potential $U(r')$ may depend on the relative coordinate r'.

A. The Lagrangian

For a charge q the standard gauge-independent interaction Lagrangian is $-q\Phi(\mathbf{x})+c^{-1}\mathbf{v}\cdot q\mathbf{A}(\mathbf{x})$. Its application to the two charges of the dipole leads to the Lagrangian

$$L=\tfrac{1}{2}m_1\dot{\mathbf{x}}_1^2+\tfrac{1}{2}m_2\dot{\mathbf{x}}_2^2-q[\Phi(\mathbf{x}_1)-\Phi(\mathbf{x}_2)]+(q/c)[\dot{\mathbf{x}}_1\cdot\mathbf{A}(\mathbf{x}_1)-\dot{\mathbf{x}}_2\cdot\mathbf{A}(\mathbf{x}_2)]-U(r'),$$

which may be written as a function of the position of the center of mass $\mathbf{x}$ and relative coordinates $\mathbf{r}'$. In the dipole approximation, with $\mathbf{v}=\frac{1}{2}(m_1\dot{\mathbf{x}}_1+m_2\dot{\mathbf{x}}_2)/m$, our Lagrangian becomes

$$L=\frac{1}{2}m\mathbf{v}^2+\frac{1}{2}m_r(\dot{\mathbf{r}}')^2+\frac{1}{c}\mathbf{v}\cdot[(q\mathbf{r}'\cdot\boldsymbol{\nabla})\mathbf{A}]+\frac{1}{c}q\dot{\mathbf{r}}'\cdot\mathbf{A}-(q\mathbf{r}'\cdot\boldsymbol{\nabla})\Phi-U(r'),\tag{4}$$

where $m_r=m_1m_2/(m_1+m_2)$ is the reduced mass and Φ and $\mathbf{A}$ are evaluated at $\mathbf{x}$.

The canonical momentum for the center of mass reads

$$\mathbf{P}=\frac{\partial L}{\partial\mathbf{v}}=m\mathbf{v}+\frac{1}{c}(q\mathbf{r}'\cdot\boldsymbol{\nabla})\mathbf{A}=m\mathbf{v}+\mathbf{Q}.\tag{5}$$

Expression (4) leads to the equation of motion

$$\frac{d}{dt}(m\mathbf{v})=\boldsymbol{\nabla}\left\{-(\mathbf{d}\cdot\boldsymbol{\nabla})\Phi+\frac{1}{c}\dot{\mathbf{d}}\cdot\mathbf{A}+\mathbf{v}\cdot\mathbf{Q}\right\}-\frac{d}{dt}\mathbf{Q}.\tag{6}$$

With $d/dt=\partial_t+\mathbf{v}\cdot\boldsymbol{\nabla}$, the identities $\boldsymbol{\nabla}(\mathbf{v}\cdot\mathbf{Q})=(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{Q}+\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{Q})$, and $\boldsymbol{\nabla}\times\boldsymbol{\nabla}\Phi=0$, the right-hand side (RHS) of Eq. (6) assumes the form $(\mathbf{d}\cdot\boldsymbol{\nabla})(-\boldsymbol{\nabla}\Phi)+c^{-1}\boldsymbol{\nabla}(\dot{\mathbf{d}}\cdot\mathbf{A})+\mathbf{v}\times[\boldsymbol{\nabla}\times\mathbf{Q}]-\partial_t\mathbf{Q}$. By making use of Eq. (5), the RHS becomes $(\mathbf{d}\cdot\boldsymbol{\nabla})(-\boldsymbol{\nabla}\Phi-c^{-1}\partial_t\mathbf{A})+c^{-1}\{\mathbf{v}\times[\boldsymbol{\nabla}\times(\mathbf{B}\times\mathbf{d})]-\mathbf{B}\times\dot{\mathbf{d}}\}$, and in terms of fields, Eq. (6) reads

$$\frac{d}{dt}(m\mathbf{v})=(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{E}+\frac{1}{c}\mathbf{v}\times[\boldsymbol{\nabla}\times(\mathbf{B}\times\mathbf{d})]-\frac{1}{c}\mathbf{B}\times\dot{\mathbf{d}}.\tag{7}$$

The same result can be obtained starting from the expression of the Lorentz force applied to the two charges and making use of the dipole approximation.

The canonical momentum for relative coordinates reads

$$\mathbf{P}'=\frac{\partial L}{\partial\dot{\mathbf{r}}'}=m_r\dot{\mathbf{r}}'+q\mathbf{A}.\tag{8}$$

Proceeding in the same manner as for the derivation of Eq. (7), one obtains the equation of motion

$$\frac{d}{dt}(m_r\dot{\mathbf{r}}')=q\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)-\boldsymbol{\nabla}'U(r')\tag{9}$$

for the momentum and

$$\frac{d}{dt}\mathbf{L}=\mathbf{d}\times\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right),\tag{10}$$

for the angular momentum $\mathbf{L}=\mathbf{r}'\times(m_r\dot{\mathbf{r}}')$. The term $\mathbf{E}'\simeq\mathbf{E}+c^{-1}\mathbf{v}\times\mathbf{B}$ in Eqs. (9) and (10) represents the electric field experienced by the dipole in its rest frame.

In the two equations of motion (7) and (9) the variables $\mathbf{r}'$ and $\mathbf{x}$ are coupled and it is cumbersome to find a simple solution for $q\mathbf{r}'=\mathbf{d}$ except for special cases. For example, if the dipole moves in a region of space where the fields are uniform, $(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{E}+c^{-1}\mathbf{v}\times[\boldsymbol{\nabla}\times(\mathbf{B}\times\mathbf{d})]=0$ in Eq. (7) and this reads $(d/dt)(m\mathbf{v}+c^{-1}\mathbf{B}\times\mathbf{d})=0$. In this case, the quantity $\mathbf{v}=-(1/mc)\mathbf{B}\times\mathbf{d}+\text{const}$ may be substituted in Eq. (9), which becomes an equation in the variable $\mathbf{r}'$ only. In most cases the solution of this equation represents a bound oscillatory motion and will contain terms of the type $\sin(\omega t+\vartheta)$ or $\cos(\omega t+\vartheta)$, where ω is the frequency and ϑ is a constant phase. The average of a dynamical variable is obtained by performing the average over the phase constants and one may expect that the variable $\mathbf{d}=q\mathbf{r}'$ oscillated about the constant average equilibrium position

$$\langle\mathbf{d}\rangle_\vartheta\simeq\mathbf{d}_0\simeq\alpha(\mathbf{E}+c^{-1}\mathbf{v}\times\mathbf{B})+q\overline{\mathbf{x}},\tag{11}$$

where α is the polarizability and $\overline{\mathbf{x}}$ is the equilibrium position in the absence of external fields. For the purpose of this paper, in a nonrelativistic approximation, one may take $\mathbf{d}_0\simeq\alpha\mathbf{E}$ for induced dipoles or (with $\mathbf{E}=0$) $\mathbf{d}_0=\overline{\mathbf{x}}$ for permanent dipoles.

A drastic simplification for the Lagrangian (4) is obtained by neglecting the oscillations or by using for $\mathbf{d}$ its average $\mathbf{d}_0$ over the phase constant ϑ, where $\mathbf{d}_0$ is parallel to the field $\mathbf{E}+c^{-1}\mathbf{v}\times\mathbf{B}$ so that

$$\mathbf{d}_0\times\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)=0.\tag{12}$$

Because of Eq. (11), $\mathbf{d}_0$ is constant and independent of $\mathbf{x}$ only when the fields are uniform. In this case $\mathbf{d}$ is no longer a dynamical variable and, with $\mathbf{E}_0=-\boldsymbol{\nabla}\Phi$, the Lagrangian assumes the simple form

$$L_0=\frac{1}{2}m\mathbf{v}^2+\frac{1}{c}\mathbf{v}\cdot[(\mathbf{d}_0\cdot\boldsymbol{\nabla})\mathbf{A}]+\mathbf{d}_0\cdot\mathbf{E}_0.\tag{13}$$

B. The Hamiltonian and the quantum phase

By means of the expressions of the canonical momenta (5) and (8), the Hamiltonian H may be derived from the Lagrangian (4). For time-independent potentials, the corresponding Schrödinger equation $H\Psi=E\Psi$ reads

$$\left(\frac{\left(\mathbf{P}-\frac{q}{c}(\mathbf{r}'\cdot\boldsymbol{\nabla})\mathbf{A}\right)^2}{2m}+\frac{\left(\mathbf{P}'-\frac{q}{c}\mathbf{A}\right)^2}{2m_r}+q\mathbf{r}'\cdot\boldsymbol{\nabla}\Phi+U(r')\right)\Psi=E\Psi.\tag{14}$$

Using the commutation relations, one finds, after lengthy but straightforward calculations, that the laws of motion of the mean values of the quantum observables $\mathbf{P}-(q/c)(\mathbf{r}'\cdot\boldsymbol{\nabla})\mathbf{A}$, $\mathbf{P}'-(q/c)\mathbf{A}$, and $\mathbf{r}'\times[\mathbf{P}'-(q/c)\mathbf{A}]$ obey equations of motions formally identical to Eq. (7), (9), and (10) of the corresponding classical quantities. This result is not surprising since it may be deduced from Ehrenfest's theorem applied to a time-independent Hamiltonian of this type [10]. It follows that the quantum behavior of the dipole maintains a certain analogy with its classical behavior. The mean value $\langle F\rangle$ of a quantum variable F generally corresponds to the average $\langle F\rangle_{\vartheta}$ over the constant phase ϑ of the corresponding classical variable [10].

The main difference between classical and quantum behavior is due to the existence of the quantum phase ϕ of the wave function which, through the process of interference, may lead to an observable phase shift $\Delta\phi$.

As can be shown by direct substitution or following the procedure of Baym [11], the solution of the Schrödinger equation (14) has the form

$$\Psi=e^{(i/\hbar)\int\mathbf{Q}\cdot d\mathbf{x}}\Psi_0=e^{(i/\hbar c)\int(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x}}\Psi_0,$$

where $\mathbf{Q}=(1/c)(q\mathbf{r}'\cdot\boldsymbol{\nabla})\mathbf{A}$ from Eq. (5) and Ψ_0 solves the equation with $\mathbf{A}=\mathbf{0}$. Thus, the quantum phase ϕ coincides with Eq. (3). This result has general validity, since the only approximation made so far on the potentials is that they are time independent.

With $\Psi=\exp(i\phi)\Psi_0$, the Schrödinger equation $H\Psi=E\Psi$ reduces to $H_0\Psi_0=E\Psi_0$, i.e.,

$$\left(\frac{\mathbf{P}^2}{2m}+\frac{\mathbf{P}'^2}{2m_r}+q\mathbf{r}'\cdot\boldsymbol{\nabla}\Phi+U(r')\right)\Psi_0=E\Psi_0, \qquad (15)$$

where H_0 is obtainable also by the unitary transformation $H_0=THT^{\dagger}$ with $T=e^{-i\phi}$.

Equation (15) may be used to obtain $\mathbf{d}$ and calculate the phase (3). However, since $\Psi_0=\Psi_0(\mathbf{x},\mathbf{r}')$, the center of mass and relative variables are still coupled, and it is difficult to find a meaningful solution and a manageable expression for the phase unless the behavior of the dipole is given or some approximations are made.

We consider here only the special case of a uniform electric field $\mathbf{E}_0=-\boldsymbol{\nabla}\Phi=$const, which will be used in Sec. V to find a quantum effect for electric dipoles. In this case, the variables $\mathbf{x}$ and $\mathbf{r}'$ in Eq. (15) may be decoupled by separation of variables setting $\Psi_0(\mathbf{x},\mathbf{r}')=\psi(\mathbf{x})\Psi'(\mathbf{r}')$, where $\Psi'(\mathbf{r}')$ represents the solution of

$$\left(\frac{\mathbf{P}'^2}{2m_r}-q\mathbf{r}'\cdot\mathbf{E}_0+U(r')\right)\Psi'=E'\Psi', \qquad (16)$$

and $\psi(\mathbf{x})=\exp[(i\mathbf{P}_0\cdot\mathbf{x})/\hbar]$ solves the equation $(\mathbf{P}^2/2m)\psi(\mathbf{x})=(E-E')\psi(\mathbf{x})$.

The wave function assumes the final form

$$\Psi=e^{i\phi}e^{i(\mathbf{P}_0\cdot\mathbf{x})/\hbar}\ \Psi'(\mathbf{r}').$$

Since in Eq. (16) the electric field $\mathbf{E}_0$ is uniform, the expectation value

$$\langle\mathbf{d}\rangle=\int\Psi^*(\mathbf{d})\Psi\ d^3x=\langle\Psi|\mathbf{d}|\Psi\rangle=\langle\Psi'|\mathbf{d}|\Psi'\rangle=\mathbf{d}_0=\text{const}$$

does not depend on $\mathbf{x}$.

Using the Lagrangian L_0 of Eq. (13), valid for uniform fields, one obtains directly the result

$$\phi=(1/\hbar c)\int(\mathbf{d}_0\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x}.$$

In fact, the canonical momentum due to interaction of the Lagrangian L_0 is $\mathbf{Q}_0=1/c(\mathbf{d}_0\cdot\boldsymbol{\nabla})\mathbf{A}$. Thus, the wave function of the corresponding Hamiltonian possesses a phase $\phi=(1/\hbar)\int\mathbf{Q}_0\cdot d\mathbf{x}=(1/\hbar c)\int(\mathbf{d}_0\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x}$, coincident with that given by Eq. (3) with $\mathbf{d}$ replaced by $\mathbf{d}_0$.

IV. OBSERVABLE PHASE SHIFT

In the interference experiments with particles possessing an electric dipole moment, the observable quantity is the phase shift $(1/\hbar c)\oint\mathbf{Q}\cdot d\mathbf{x}=(1/\hbar c)\oint(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x}$. Since

$$\langle\Psi|\oint(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x}|\Psi\rangle=\oint(\langle\Psi'|\mathbf{d}|\Psi'\rangle\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x}$$
$$=\oint(\mathbf{d}_0\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x},$$

its expectation value reads

$$\Delta\phi=\frac{1}{\hbar}\oint\mathbf{Q}_0\cdot d\mathbf{x}=\frac{1}{\hbar c}\oint(\mathbf{d}_0\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{x}$$
$$=\frac{1}{\hbar c}\oint[\mathbf{B}\times\mathbf{d}_0+\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}. \qquad (17)$$

By using vector identities, we have made explicit in Eq. (17) the relevant term $\mathbf{B}\times\mathbf{d}_0$ known as the Röntgen interaction [12].

It can be shown that the phase shift $\Delta\phi$ is gauge independent. In a gauge transformation one lets $\mathbf{A}'=\mathbf{A}+\boldsymbol{\nabla}\chi$, where χ is a scalar function. Using expression (3) in order to point out some properties of the AB phase shift, we write the contribution to the phase shift due to the gauge transformation as

$$\delta\phi=\frac{1}{\hbar c}\oint(\mathbf{d}_0\cdot\boldsymbol{\nabla})\boldsymbol{\nabla}\chi\cdot d\mathbf{x}$$
$$=\frac{q}{\hbar c}\oint(\boldsymbol{\nabla}_1\chi)\cdot d\mathbf{x}_1-\frac{q}{\hbar c}\oint(\boldsymbol{\nabla}_2\chi)\cdot d\mathbf{x}_2.$$

The gauge independence of the AB phase shift is based on the fact that the scalar function χ is a monovalued function for which $\oint(\boldsymbol{\nabla}\chi)\cdot d\mathbf{x}=0$. Thus, one obtains immediately $\delta\phi=0$.

Our result (17) for the phase shift of an electric dipole differs from that proposed by other authors ([6] and [7]) for the presence of the extra term $(1/\hbar c)\oint[\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}$. Before dealing with this aspect, we recall that the properties of the AB effect allow us to write the corresponding phase shift as

$$\Delta\phi_{\mathrm{AB}}=\frac{q}{\hbar c}\oint\mathbf{A}\cdot d\mathbf{x}=\frac{q\Phi}{2\pi\hbar c}\oint(\boldsymbol{\nabla}\theta)\cdot d\mathbf{x}=\frac{q\Phi}{2\pi\hbar c}\oint d\theta$$
$$=\frac{q\Phi}{\hbar c}\,\delta n,$$

where Φ is the magnetic flux and, for a solenoid along the z axis, $\boldsymbol{\nabla}\theta=\boldsymbol{\nabla}\tan^{-1}(x/y)=(-\hat{\mathbf{i}}y+\hat{\mathbf{j}}x)/(x^2+y^2)$ and δn is the difference between the topological winding numbers n of the Feynman paths encircling the singularity (here, the solenoid).

Thus, one can see that the mentioned difference is not trivial because, if the quantity $\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})$ turns out to be proportional to the gradient of the multivalued function $\theta(\mathbf{x})$, the integral $\oint\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})\cdot d\mathbf{x}\propto\oint\boldsymbol{\nabla}\theta\cdot d\mathbf{x}=\oint d\theta$ does not vanish. A physical example where $\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})\cdot d\mathbf{x}\propto\boldsymbol{\nabla}\theta$ is mentioned in Ref. [5] and is obtained with the field configuration proposed by Wilkens [6]. In order to show that with Wilkens' configuration $\oint\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})\cdot d\mathbf{x}\neq 0$, we expand here the relevant argument.

Wilkens considers a distribution of currents that does not depend on z and that generates a magnetic field with cylindrical symmetry on the closed path of the particle encircling the singularity. On the path of the particles there are no sources and the fields satisfy Maxwell's equations $\boldsymbol{\nabla}\cdot\mathbf{B}=0$ and $\boldsymbol{\nabla}\times\mathbf{B}=0$. The magnetic field obtained by Wilkens reads

$$\mathbf{B}=\frac{g}{2\pi}\frac{\mathbf{e}_r}{r}=\frac{g}{2\pi}\frac{(\hat{\mathbf{i}}x+\hat{\mathbf{j}}y)}{r^2},$$

where g is a constant and $\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}(x,y)=\hat{\mathbf{i}}\partial_y A_z-\hat{\mathbf{j}}\partial_x A_z$. With a dipole $\mathbf{d}_0=\hat{\mathbf{k}}d_0$ oriented in the z direction and moving in the x-y plane, the phase shift obtained by Wilkens,

$$\Delta\phi_W\propto\oint[\mathbf{B}\times\mathbf{d}_0]\cdot d\mathbf{x}\propto\oint d\mathbf{x}\cdot\frac{(-\hat{\mathbf{i}}y+\hat{\mathbf{j}}x)}{r^2}=\oint d\mathbf{x}\cdot\boldsymbol{\nabla}\theta$$
$$\neq 0, \tag{18}$$

has topological properties analogous to those of the AB phase shift.

However, the quantity $\oint[\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}$ does not vanish in this case. In fact, since there is no z dependence,

$$\oint[\boldsymbol{\nabla}(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}=\oint[d_0(\hat{\mathbf{i}}\partial_x A_z+\hat{\mathbf{j}}\partial_y A_z)]\cdot d\mathbf{x}$$
$$=-\oint(\mathbf{B}\times\mathbf{d}_0)\cdot d\mathbf{x}=\propto-\oint d\theta\neq 0$$

and in Eq. (17) $\Delta\phi\propto\oint d_0\partial_z\mathbf{A}(x,y)\cdot d\mathbf{x}=0$.

Another way to realize, intuitively, that the phase shift (17) is zero for Wilkens' configuration consists in relating $\Delta\phi$ to the sum of the AB phase shifts of the charges of the dipole. According to Eq. (2), $\Delta\phi_{\mathrm{AB}}=(q/c\hbar)\oint\mathbf{A}\cdot d\mathbf{x}=(q/c\hbar)\Phi$, where Φ is the magnetic flux through the loop of area S formed by the path of the particle moving in the x-y plane and encircling the singularity. In this case, the normal to the area of the loop is $\mathbf{n}=\hat{\mathbf{k}}$, and $\Phi=\mathbf{B}\cdot\mathbf{n}S=\Delta\phi_{\mathrm{AB}}=0$. Thus, also for the dipole $\Delta\phi=\Delta\phi^{+}_{\mathrm{AB}}+\Delta\phi^{-}_{\mathrm{AB}}\propto(\Phi_{+}-\Phi_{-})=0$.

In conclusion, the phase shift given by Eq. (17) derived here is physically different from that of Eq. (18) proposed by Wilkens and by Wei *et al.* For the experiment devised by Wilkens [6], result (17) predicts the null result $\Delta\phi=0$.

V. QUANTUM EFFECT FOR AN ELECTRIC DIPOLE

Let us now apply result (17) to dipoles moving in the presence of a vector potential $\mathbf{A}$ and the uniform field $\mathbf{E}_0$.

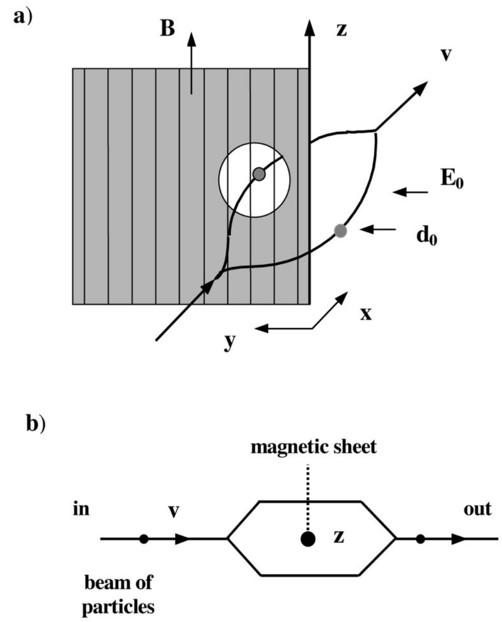

FIG. 1. (a) A magnetic sheet covers the y-z semiplane (from $y=0$ to $y=\infty$). Its magnetic field $\mathbf{B}$ is oriented in the z direction and is confined within the sheet. The edge of the sheet coincident with the z axis corresponds to a singularity for the interaction momentum $\mathbf{Q}\propto(\mathbf{d}_0\cdot\boldsymbol{\nabla})\mathbf{A}$ contributing to the phase. Particles have their electric moment $\mathbf{d}_0$ oriented along the y direction and move on opposite sides of the singularity. For induced dipoles, the uniform electric field $\mathbf{E}_0$ must be present. (b) Interferometric path of particles possessing the electric dipole moment $\mathbf{d}_0$. The incoming beam of particles is split on the plane of motion before reaching the magnetic sheet. The path encircles the singularity z and one of the arms of the interferometric path goes through a hole in the magnetic sheet. Particles moving on opposite sides of the singularity acquire opposite phases and the phase of the outcoming beam is shifted by the observable amount $\Delta\phi$.

We look for a current distribution which generates a phase shift $\Delta\phi$ with a topology equivalent to that of the AB effect. The restriction to a uniform electric field $\mathbf{E}_0$ made to obtain Eq. (17) rules out also the configuration of fields proposed by Wei *et al.* [7] because it implies the use of dipoles induced by a nonuniform field $\mathbf{E}(\mathbf{x})$.

For our purpose, we extend the line of magnetic dipoles (or solenoid) producing the AB effect to form a sheet covering a semiplane perpendicular to the direction of motion of the particle, as shown in Fig. 1(a). Choosing the velocity of the particle along the x axis, the sheet will be covering the y-z semiplane (from $y=0$ to $y=\infty$) with the lines of magnetic dipoles oriented in the z direction. If the interferometric path of the particle has to encircle the z axis, a segment of the path will have to intersect the magnetic sheet and a hole in the sheet must be left through which the particles may travel undisturbed (a small hole leaves potential and field practically unchanged).

If m is the magnetic dipole per unit of volume and τ is the thickness of the sheet, then the magnetic field inside the sheet $(0\leqslant x\leqslant\tau)$ is $\mathbf{B}=\hat{\mathbf{k}}4\pi m$ and the corresponding vector potential is $\mathbf{A}_B=\hat{\mathbf{j}}4\pi m(-\tau/2+x)$. Outside the sheet, the magnetic field is zero. However, to obtain the total vector potential $\mathbf{A}(\mathbf{x})$ we have to add to $\mathbf{A}_B$ the contribution due to the lines of magnetic dipoles, $\mathbf{A}_m(\mathbf{x})=\int_V d^3x_m[\mathbf{m}\times(\mathbf{x}-\mathbf{x}_m)]/|\mathbf{x}-\mathbf{x}_m|^3$, that can be calculated integrating the elementary expression for a magnetic dipole over the volume V of the magnetic sheet. Actually, what one needs are only the derivatives $\partial\mathbf{A}_m/\partial x$ and $\partial\mathbf{A}_m/\partial y$ which turn out to be

$$\frac{\partial\mathbf{A}_m}{\partial x}=-2m\tau\left(\frac{\hat{\mathbf{i}}x+\hat{\mathbf{j}}y}{r^2}\right),\quad \frac{\partial\mathbf{A}_m}{\partial y}=2m\tau\left(\frac{-\hat{\mathbf{i}}y+\hat{\mathbf{j}}x}{r^2}\right), \tag{19}$$

where $\nabla\times\mathbf{A}_m=\mathbf{0}$ and $\nabla\times\mathbf{A}=\nabla\times(\mathbf{A}_B+\mathbf{A}_m)=\mathbf{B}$.

At the intersection with the magnetic sheet the dipole crosses a region with $\mathbf{B}\neq\mathbf{0}$. However, from Eq. (17) it is apparent that the topology of our effect depends on the singular behavior of $\mathbf{Q}_0=c^{-1}(\mathbf{d}_0\cdot\nabla)\mathbf{A}$ regardless of the distribution of fields and sources of potentials. Taking $\mathbf{d}_0$ parallel to the $\mathbf{m}\times\mathbf{v}$ direction (here the y direction), one finds $(\mathbf{d}_0\cdot\nabla)\mathbf{A}_B=d_0\partial_y\mathbf{A}_B=0$ and

$$\mathbf{Q}_0=c^{-1}(\mathbf{d}_0\cdot\nabla)\mathbf{A}_m=c^{-1}2d_0m\tau(-\hat{\mathbf{i}}y+\hat{\mathbf{j}}x)/r^2$$
$$=c^{-1}2d_0m\tau\nabla\theta.$$

Thus, our phase shift has a topology equivalent to that of the AB effect, and from Eq. (17)

$$\Delta\phi=\frac{1}{\hbar}\oint\mathbf{Q}_0\cdot d\mathbf{x}=\frac{1}{\hbar c}\oint(\mathbf{d}_0\cdot\nabla)\mathbf{A}\cdot d\mathbf{x}=\frac{2d_0m\tau}{\hbar c}\oint d\theta$$
$$=\frac{(\delta n)}{\hbar c}d_0B\tau, \tag{20}$$

where, in most experimental conditions, we may take $\delta n=1$. A phase shift twice that of Eq. (20) is achieved by adding another magnetic sheet (with opposite magnetic moment) in the other y-z semiplane.

Note that the components d_{0x} and d_{0z} [if they do not vanish, the term d_0 in Eq. (20) reads d_{0y}], do not contribute to the phase shift because they lead to terms of the integrand which are either perpendicular to the path of the particle or are of the type $\oint[\nabla(1/r)]\cdot d\mathbf{x}=0$.

We derive now result (20) using the equivalent expression

$$\Delta\phi=\frac{1}{\hbar c}\oint[\mathbf{B}\times\mathbf{d}_0+\nabla(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}$$

on the right-hand side of Eq. (17). First, we show that for our configuration $\oint\nabla(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}=0$. Since the contribution of both $\mathbf{A}_m$ and $\mathbf{A}_B$ have to be taken into account, $\nabla(\mathbf{d}_0\cdot\mathbf{A})=\hat{\mathbf{i}}d_{0y}\partial_xA_B+\nabla(\mathbf{d}_0\cdot\mathbf{A}_{my})$. Using Eq. (19), $\nabla(\mathbf{d}_0\cdot\mathbf{A}_{my})=2m\tau[d_{0x}\nabla(1/r)+d_{0y}\nabla\theta]$, so that

$$\oint[\nabla(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}=-4\pi d_0m\tau+2d_0m\tau\oint\nabla\theta\cdot d\mathbf{x}=0.$$

Thus, the phase shift is given by

$$\Delta\phi=\frac{1}{\hbar c}\oint(\mathbf{B}\times\mathbf{d}_0)\cdot d\mathbf{x}=\frac{1}{\hbar c}\oint(-\hat{\mathbf{i}}d_{0y}B+\hat{\mathbf{j}}d_{0x}B)\cdot d\mathbf{x}$$
$$=\frac{1}{\hbar c}d_{0y}B\tau$$

as in Eq. (20).

To interpret intuitively the physical result (20), we relate it once more to the sum of the AB phase shifts of the charges of the dipole. According to Eq. (2), $\Delta\phi_{\mathrm{AB}}=(q/c\hbar)\oint\mathbf{A}\cdot d\mathbf{x}=(q/c\hbar)\Phi$, where $\Phi=\mathbf{B}\cdot\hat{\mathbf{n}}S$ is the magnetic flux through the loop formed by the path of the particle encircling the singularity. Referring to Fig. 1(a), one can see that the magnetic field is localized inside the sheet in the region $y\geqslant0$ and, thus, for a dipole moment $d_0=d_{0y}$, the path of the outer charge encircles a greater flux. Since $\mathbf{d}_0=q\mathbf{r}'$, while crossing the uniform field $\mathbf{B}$ at the intersection with the sheet, the dipole sweeps the area difference $(S_+-S_-)\hat{\mathbf{n}}\cdot\hat{\mathbf{B}}=(\mathbf{r}'\times d\mathbf{x})\cdot\hat{\mathbf{B}}=r'_xdy-r'_ydx$, and the quantity $(\mathbf{B}\times\mathbf{r}')\cdot d\mathbf{x}=(\mathbf{r}'\times d\mathbf{x})\cdot\mathbf{B}=\Delta S\hat{\mathbf{n}}\cdot\mathbf{B}$ represents the elementary magnetic flux through the area swept by the dipole. Thus, with $dx=\tau$,

$$\Delta\phi=\frac{q}{\hbar c}\Phi_+-\frac{q}{\hbar c}\Phi_-=\frac{q}{\hbar c}\mathbf{B}\cdot\hat{\mathbf{n}}(S_+-S_-)=\frac{q}{\hbar c}B\tau r'$$
$$=\frac{1}{\hbar c}d_0B\tau,$$

in agreement with Eq. (20).

VI. ANALYSIS OF POSSIBLE EXPERIMENTAL VERIFICATION OF THE PHASE SHIFT

To discuss the experimental verification of the phase shift, it is convenient to express Eq. (20) as

$$\Delta\phi\sim\ 4.0\frac{d_0}{(ea_o)}\frac{\tau}{(mm)}\frac{B}{(kG)}\ \sim\ \frac{2\alpha E_0\tau B}{\hbar}.$$

These two equivalent expressions for $\Delta\phi$ are given in physical and mks units, respectively, and the last term has a form suitable for induced dipoles.

To test the quantum phase, one needs first to prepare a beam of dipoles in the state $\langle\mathbf{d}\rangle=\mathbf{d}_0\neq\mathbf{0}$, moving with uniform velocity and with $\mathbf{d}_0$ in the direction of the field $\mathbf{E}_0+c^{-1}\mathbf{v}\times\mathbf{B}$. If the vector $\mathbf{d}_0$ is not completely parallel to $\mathbf{E}_0+c^{-1}\mathbf{v}\times\mathbf{B}$, the quantity d_0 in Eq. (20) stands for the component of $\langle\mathbf{d}\rangle$ parallel to the field. For induced dipoles with $\mathbf{d}_0=\alpha\mathbf{E}_0$, the arrangement needed to prepare the mentioned beam is relatively simpler because the orientation is determined by the uniform, external electric field $\mathbf{E}_0$.

Note that, for the average quantity $\mathbf{d}_0$, the condition (12) obtained in the discussion of the classical behavior of the dipole reads $\mathbf{d}_0\times\mathbf{E}'=\mathbf{d}_0\times(\mathbf{E}_0+c^{-1}\mathbf{v}\times\mathbf{B})=\mathbf{0}$. Furthermore, $(\mathbf{d}_0\cdot\nabla)\mathbf{E}_0=0$ and also $\nabla\times(\mathbf{B}\times\mathbf{d}_0)=-\hat{\mathbf{j}}d_0\partial_zB=\mathbf{0}$ because $\mathbf{B}$ depends on x only. Thus, the equation of motion (7) implies $d\mathbf{v}/dt=0$, so that the particle moves with the uniform velocity $\mathbf{v}=\mathbf{v}_0$ even when the em interaction is switched on [13].

To measure the phase, one may employ interferometers in which the incoming beam of particles of Fig. 1(b) is split into two coherent beams that pass on opposite sides of the singularity and then recombine. When the em interaction is switched on, particles on opposite sides of the interferometric path acquire opposite phases and the outcoming beam is phase shifted by the amount $\Delta\phi$ to be measured by the interferometer.

Some atom interferometers can detect phases of 0.1 rad [14], and atomic beam splitters may reach the supermillimeter range [15]. Thus, the thickness τ may be of the order of 1 mm and both molecular ($d_0 \sim 4ea_0$) or atomic ($d_0 \leqslant ea_0$) interferometers may be used. If only moderate magnetic fields are achievable, it is convenient to use polar molecules with a large permanent dipole moment, since for single atoms and nonpolar molecules the induced dipoles are rather small even when a strong electric field is applied.

For our configuration, the B field may reach relatively high values because the solenoids extend from $z=-\infty$ to $z=\infty$ and, as in the AB effect with a toroid, need not be open. By using material with high permeability or superconducting magnetic sheets, the field strength may be well above the kG range. For alkali-metal atoms $\alpha \sim 10\times 10^{40}$ Fm^2, and with $B\sim 1T$ and $E_0 \sim 10^6$ V/cm it is possible to achieve a phase shift greater than $\pi/2$. Thus, the verification of the proposed quantum effect for an electric dipole is feasible.

However, other field configurations may turn out to be suitable for the purpose of observation. For example, for the experimental verification of the AC effect, different configurations have been proposed [16] and, by analogy, can be extended to the present effect. A suitable configuration consists of a uniform **B** field confined within a tiny toroid with its axis of symmetry in the **v** direction and physically cut into two halves by the plane of motion. If the outer radius of the toroid is R, the two cuts are placed at $y=\pm R$ in correspondence to the split beams of Fig. 1(b). The two resulting horseshoe-shaped magnets facing one another can be slightly separated to allow the beam of particles to pass through. The field in the gap between the magnets penetrates the plane of motion and has opposite signs at the position of the cuts. With τ being now the inner diameter of the toroid, particles passing through the gaps travel a distance τ in the presence of the magnetic field of strength B and generate the shift (20).

An even simpler configuration would arise if it were possible to prepare a beam of particles formed by two coherent beams, not spatially separated, possessing *opposite* electric moments and made to pass through the *same* magnetic field. In order to experimentally detect the AC phase, Sangster *et al.* [17] have developed such an arrangement for *magnetic* dipoles, which has proved effective for determining the independence of the quantum phase from velocity and its proportionality to the electric field.

With the last two configurations both τ and B may be increased, leading to a value of the phase shift greater than that of the magnetic sheet configuration.

VII. CONCLUSIONS

We have considered a simple model of an electric dipole and analyzed its behavior. Although the quantum behavior of the mean values of the relevant observables is analogous to that of the corresponding classical variables, a quantum dipole is characterized by the phase given by expression (3) that is an application of the AB phase.

In order to observe a quantum effect for electric dipoles, we have devised the current distribution corresponding to the magnetic sheet shown in Fig. 1(a). This configuration provides a phase shift which is path-independent, a property that recalls the topological features of the AB effect. However, the present effect is not a nonlocal topological effect of the AB type because the Röntgen interaction is effective when the particles cross the magnetic sheet and are in the presence of the field **B**, while in the AB effect the particles move in a field-free region.

In conclusion, this quantum effect for electric dipoles may be observed in atom or molecular interferometry and its verification is within reach of present experimental technique.

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