Self-channeling and pulse shortening of femtosecond pulses in multiphoton-ionized dispersive dielectric solids

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The self-focusing of femtosecond optical pulses in dielectric solids is studied under the influence of the combined effects of diffraction, Kerr nonlinearity, normal group velocity dispersion, and self-induced multiphoton ionization. The numerical simulations show a significant pulse shortening with a compression factor of 5.5 and self-channeling of the beam when the critical power is exceeded. [S1050-2947(99)09703-6]

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Recently there has been considerable interest in the investigation of self-focusing of femtosecond optical pulses [1–7]. In a Kerr medium with a positive nonlinear refractive index change a laser beam in the long pulse regime self-focuses if the power P in the beam is greater than the critical power of self-focusing P_{cr}. After a sufficiently long interaction length, transverse instabilities arise which result in the filamentation of the beam. For femtosecond pulses the effect of material dispersion modifies drastically the spatiotemporal propagation. In the positive group velocity dispersion (GVD) regime the dispersion increases the self-focusing threshold and can lead to temporal splitting of the pulse [1–3]. In the pulse spectra supercontinuum generation and conical emission occur at the pulse splitting threshold as a result of the combination of diffraction, Kerr nonlinearity, and dispersion [3]. Lately a striking observation has been reported on the propagation of intense femtosecond pulses in air with powers in the gigawatt range [5,6]. In these experiments after an initial narrowing owing to self-focusing, self-channeling of the pulse beam has been observed over distances of several tens of meters. The self-trapping over distances larger than the Rayleigh length has been interpreted as a counterbalance of the self-focusing due to the Kerr effect and the defocusing effect due to the formation of a low-density plasma [5,7]. Under such conditions the beam breakup into filaments owing to self-focusing can be prevented and under certain conditions a stabilized self-trapped [or $(2+1)$-dimensional optical solitonlike] beam can exist. Theoretically the evolution of an optical high-power beam in gases under the influence of the Kerr nonlinearity and the nonlinear index change by multiphoton ionization was numerically studied in Ref. [5] and by the use of a variational approach in Ref. [7].

In this Brief Report we present a numerical study of the spatiotemporal evolution of femtosecond pulses in dielectric solids [8]. In solids the role of the GVD is more dominant and the characteristic parameters for the optical Kerr effect as well as for the multiphoton ionization differ by some orders of magnitude in comparison to those in gases. Despite these distinct physical conditions, the results of our numerical investigation show the possibility of self-channeling of high-peak-power femtosecond pulses with powers somewhat above the critical power P_{cr} and beam diameter in the order of ten μm. The refraction index change by the Kerr effect and the optical field-induced ionization can be described by the relation $\Delta n = n_2|A|^2 - 0.5N_e/(n_0 N_{\mathrm{cr}})$, where A is the slowly varying amplitude of the electric field strength, n_0 is the linear refractive index, n_2 is the nonlinear coefficient due to the Kerr effect, $N_{\mathrm{cr}} = m\omega_0^2\epsilon_0(1+\delta^2)/e^2$ is the critical plasma density, and N_e is the number density of free plasma electrons. δ stands for $\delta = (\omega_0\tau_c)^{-1}$ with τ_c as the electron-phonon transport (momentum) collision time. In the following we consider the case in which multiphoton ionization dominates compared with the tunneling ionization and where the avalanche ionization plays no significant role because of the shortness of the pulses. The electron density therefore can be calculated by the Keldysh expression for the probability of multiphoton ionization in condensed media given by [9]

$$\frac{\partial N_e}{\partial t} = \frac{2}{9\pi}\omega_0\left(\frac{m'\omega_0}{\hbar}\right)^{3/2}\left(\frac{e^2}{16m' I_i \omega_0^2}\right)^n \exp(2n)\Phi(s)|A|^{2n} = B|A|^{2n}, \tag{1}$$

where I_i is the ionization energy corresponding to the transition energy across the gap between the valence band to the conduction band, m' is the reduced exciton mass with $(m')^{-1} = (m_e)^{-1} + (m_h)^{-1}$, m_e and m_h are the electron and the hole mass, respectively, ω_0 is the optical frequency, n is the number of quanta necessary for ionization, $n = \mathrm{mod}[I_i/(\hbar\omega_0)+1]$, $\Phi(s) = \exp(-s^2)\int_0^s \exp(s'^2)ds'$ represents Dawson's integral, and $s = [2n - 2I_i/(\hbar\omega_0)]^{1/2}$.

The reduced Maxwell equation for the slowly varying amplitude can then be written in the dimensionless form

$$i\frac{\partial \Psi}{\partial z} = -\frac{1}{2}\alpha\frac{\partial^2\Psi}{\partial \eta^2} + \frac{1}{2}\left(\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2}\right) + |\Psi|^2\Psi - (1+i\delta)\Gamma\Psi \times \int_{-\infty}^{\eta}|\Psi(t)|^{2n}dt - i\gamma|\Psi|^{2n-2}\Psi \tag{2}$$

with $\Psi = (k^2 n_2 w_0^2/n_0)^{1/2}A$, $\eta = (\tilde{t} - \tilde{z}/v_g)/\tau_0$, $z = \tilde{z}/(kw_0^2)$, $x = \tilde{x}/w_0$, $y = \tilde{y}/w_0$, $\alpha = kk'' w_0^2/\tau_0^2$, $\Gamma = 0.5[k^2\tau_0/(n_0^2 N_{\mathrm{cr}} w_0^{2n-2})]B[n_0/(k^2 n_2)]^n$, and $\gamma = n\hbar\omega_0 B k w_0^2 n_0^{n-1}/(k^2 n_2 w_0^2)^{n-1}$.

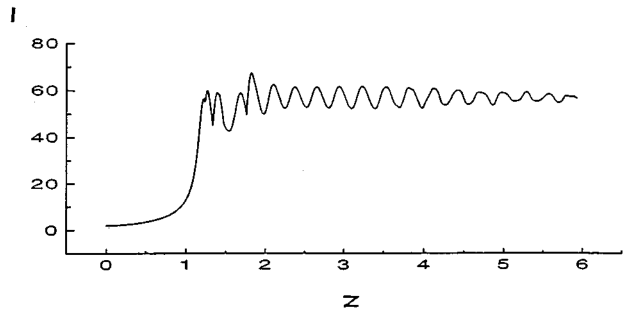

FIG. 1. Normalized axial peak intensity in dependence on the normalized propagation distance z.

Here $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are the physical coordinates and $\tilde{t}$ is the time. w_0 is the beam radius ($2w_0$ is the FWHM), τ_0 is the pulse duration, $k''=d^2k/d^2\omega$ is the GVD parameter, v_g is the group velocity, and $k=2\pi n_0/\lambda$ is the wave number. The constant B has been defined in relation (1). γ is the nonlinear coefficient for n-photon absorption and Γ describes the reaction of the plasma to the optical field.

Equation (2) is numerically solved with the help of a beam progation method with the initial condition of the form $\Psi=\Psi(x,y,z=0,\eta)=\Psi_0\exp[-(x^2+y^2)\ln 2/2]/\cosh(1.76\eta)$. The following dimensionless parameters were chosen: $\alpha=0.016$, $n=5$, $\Gamma=1.15\times10^{-7}$, $\gamma=4\times10^{-7}$, and $\Psi_0=\sqrt{2\ln 2P_0/P_{\mathrm{cr}}}=1.44$. These parameters correspond, for example, to typical experimental parameters in fused silica with $I_i=7.8\,\mathrm{eV}$, $k''=3.4\times10^{-26}\,\mathrm{s}^2/\mathrm{m}$, $n_0=1.4$, $n_2=6.12\times10^{-23}\,\mathrm{m}^2/\mathrm{V}^2$, and $\lambda=790\,\mathrm{nm}$, $\tau_0=50\,\mathrm{fs}$, $w_0=10.3\,\mu\mathrm{m}$, and $P_0/P_{\mathrm{cr}}=1.5$ or $P_0=3.23\,\mathrm{MW}$, where $P_{\mathrm{cr}}=\pi[n_0^2/(k^2n_2)]\epsilon_0 c$ is the critical power of self-focusing. The electron-phonon collision time τ_c depends on the electron energy with values for fused silica in the range from 10^{-14} s up to 10^{-16} s (Ref. [10]). The averaged kinetic energy of the electrons can be roughly estimated by the above-threshold-ionization energy of the free electrons which yields with the parameters in Ref. [10] the estimate $\delta<0.1$. Multiplying Eq. (2) with Ψ^* and integrating over the transverse coordinates and the time we find the conditions $0.18\gamma z|\Psi_{\max}|^8<1$ and $0.15\delta\Gamma z|\Psi_{\max}|^{10}<1$, where the nonlinear loss is small. In agreement with experimental observations (compare, e.g., Refs. [12,13]), from these estimations it follows that for the conditions considered here the nonlinear loss associated with the multiphoton absorption and electron-ion scatterings influences only weakly the pulse propagation but it restricts the maximum length for a self-trapped regime.

In Fig. 1 the evolution of the normalized peak axial intensity $I=|\Psi(0,0,z,\eta_{\max})|^2$ is depicted versus the propagation distance z. As expected, in the initial stage self-focusing leads to an increase of the intensity and a beam narrowing until the intensity of the pulse is high enough to generate a low-density plasma in the dielectric solid. The counteracting self-defocusing caused by the plasma leads to a self-channeled beam which is stabilized against further self-focusing, and spatial instabilities by the pure Kerr effect disappear. The self-channeled regime remains stable over distances much larger than the Rayleigh length. The maximum intensity level is mainly determined by the condition that the nonlinear refractive index changes induced by the Kerr effect and by the plasma cancel each other. This yields the condition $a\Gamma|\Psi_{\max}|^{2n-2}\approx1$, where a is a pulse shape and beam factor in the order of unity. In our exact numerical calculations we have found that this condition is fulfilled in Fig. 1 as well as for rather different parameters. The oscillations of the intensity in Fig. 1 after the beam reached the self-trapped region are typical for $(2+1)$-dimensional beam propagation in media with a saturable nonlinear refraction index change $\Delta n(|A|^2)$, which can also be found for a refractive index of the form $\Delta n=n_2|A|^2/(1+|A|^2/I_{\mathrm{sat}})$ (see [11]). Such oscillations are long-lived over large distances and do not converge to the spatial soliton solution even without the influence of the GVD, however such a stable soliton solution characterized by constant beam parameters exists for definite input parameters and input shape of the beam for the above-mentioned types of nonlinearity (see Refs. [7] and [11]).

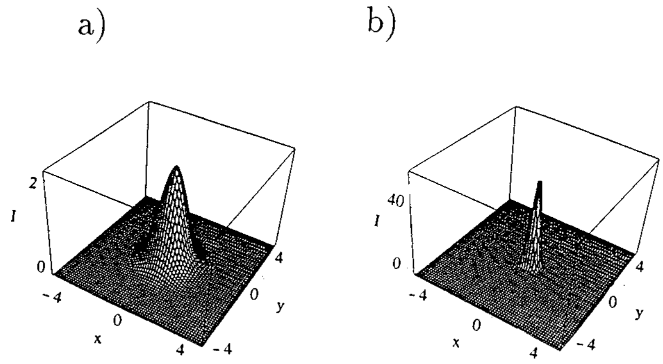

FIG. 2. Beam shape (a) at the input and (b) after propagation at $z=5.66$.

In Fig. 2 the initial beam shape (a) and the beam shape after a propagation length of $z=5.66$ [Fig. 2(b)] are depicted where $z=5.66$ corresponds to a physical length of $\tilde{z}=1.18\,\mathrm{mm}$. As seen after propagation, the self-channeled beam is narrower by a factor of 2.3. In Fig. 3 the temporal shape of the pulse at the input (dotted line) and after propagation (solid line) is shown. Take note of the appearance of small side maxima in the pulse shape of Fig. 3 which are related to the corresponding spectral continuum generation. Clearly, the nonlinear evolution leads to a large amount of compression whereby the compression factor is about 5.5 and the peak intensity is 27.5 times larger than the input intensity. In physical units the peak intensity is $I=1.84\times10^{13}\,\mathrm{W/cm}^2$ and the pulse duration is $\tau_0=9.1$ fs. By comparison, in a fiber in the normal-dispersion regime the combination of self-phase-modulation

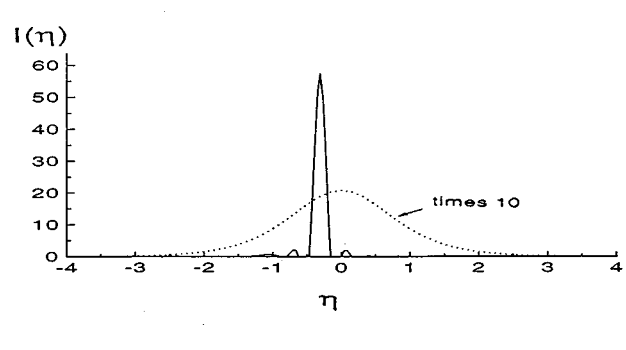

FIG. 3. Temporal shape at the input (dotted line) and after propagation at $z=5.66$ (solid line). The input intensity is multiplied by a factor of 10.

(SPM) and GVD broadens invariably the pulse as it propagates. The physical reason for the pulse shortening predicted here is connected with the space-time coupling. The higher intensity in the pulse maximum experiences a higher focusing and the wings with lower intensity are suppressed. Previously it was shown that the inclusion of the diffraction term in planar wave guides can lead to a pulse compression with a compression factor smaller than 2 followed by pulse splitting [4].

In contrast to the self-channeling described here, in Ref. [14] pulse splitting into two pulses has been observed in bulk BK7 glass for pulses with 1 mJ and 85 fs duration. These observations confirm previous theoretical studies in Ref. [2] where Eq. (2) was solved without the terms associated with multiphoton ionization ($\Gamma=\gamma=0$) and a pulse splitting due to the influences of GVD was predicted. However, this completely different behavior in self-focusing of ultrashort pulses in Refs. [2, 14] compared with our results is no contradiction but can be explained by the different beam parameters studied in Refs. [2, 14] and here. Note that self-trapping as described here can only occur in a rather restricted range of input and material parameters because of the trade-off between the different competing effects described in Eq. (2). The experiments and the theoretical simulation in Ref. [14] were done for similar parameters as considered here but for about a five times larger beam radius or corresponding 10^{-6} times smaller Γ and 80 times larger parameter α. Under such conditions the ionization does not play a significant role in beam propagation. Self-channeling can only occur if the critical length z_{ch}—where the channel is formed—is smaller compared with the distance at the onset of pulse splitting z_{sp}. The self-channeling distance z_{ch} is only slightly smaller than the critical self-focusing length z_{SF} which in our notation is given by $z_{\mathrm{ch}}<z_{\mathrm{SF}}=0.53[(p_0^{1/2}-0.852)^2-0.0719]^{-1/2}$ with $p_0=P_0/P_{\mathrm{cr}}$. Thus on the one hand for $p_0=1.5$ we obtain $z_{\mathrm{SF}}=1.5$ while from Fig. 1 a self-channeling distance $z_{\mathrm{ch}}=1.3$ can be obtained. On the other hand, the splitting distance can be estimated by the following estimation. The combined effects of SPM and GVD lead to a broadening of the pulse and reduce the peak power from its initial value p_0 to the critical power $p(z)=1$ if the dispersion plays a dominant role. The numerical calculation of Rothenberg [2] has directly shown that pulse splitting occurs at a distance z_{sp} where the power is reduced to the critical power: $p(z_{\mathrm{sp}})=1$. The pulse broadening owing to SPM and normal GVD can be described by an analytical expression derived in the variational approach with a Gaussian trial pulse shape function [15]:

$$z=\frac{0.357}{\alpha}\left[\frac{(y-1)}{\sqrt{1+\xi}}\left(y+\frac{1}{1+\xi}\right)^{1/2}+\frac{1}{2}\,\frac{\xi}{(1+\xi)^{3/2}}\ln X\right] \quad (3)$$

with $X=\{2(1+\xi)(y-1)^{1/2}[y-(1/1+\xi)]^{1/2}+2y(1+\xi)-\xi\}(2+\xi)^{-1}$, $y=\tau_L/\tau_{L0}$, and $\xi=p_0/\sqrt{2}\alpha$. By using the conservation law $p(z)\tau_L=p_0\tau_{L0}$ and setting $p(z_{\mathrm{sp}})=1$ we obtain from Eq. (3) the splitting distance $z_{\mathrm{sp}}=z(y=p_0)$. For the above considered parameters $\alpha=0.016$ and $p_0=1.5$ we obtain $z_{\mathrm{sp}}=4.1$, i.e., the ionization dominates, and self-channeling prevents pulse splitting. In contrast to parameters as considered in Ref. [1] with $p_0=3$ and $\alpha=5$, the splitting distance from Eq. (3) is $z_{\mathrm{sp}}\approx 0.2$, in good agreement with the numerical results. In that case with $z_{\mathrm{sp}}<z_{\mathrm{ch}}$ ionization does not play a role and pulse splitting occurs preventing a possible self-channeling. More generally, the condition for self-channeling can be expressed by a relation for the parameter α with $\alpha<\alpha_{\mathrm{cr}}$. The critical parameter α_{cr} is determined by the relation $z_{\mathrm{sp}}(p_0,\alpha=\alpha_{\mathrm{cr}})=z_{\mathrm{SF}}(p_0)$. By using Eq. (3) and the relation for z_{SF}, the critical parameter $\alpha_{\mathrm{cr}}(p_0)$ can be found which yields for $p_0=1.5$ the value $\alpha_{\mathrm{cr}}=0.07$.

The narrow set of parameters required to achieve a self-channeled regime is determined by the following conditions. The peak power should not exceed P_{cr} by more than a factor of 3 ($P_{\mathrm{cr}}<P<3P_{\mathrm{cr}}$), otherwise beam filamentation occurs. On the one hand, the input beam radius has to be small enough in order that ionization comes into play before beam filamentation or damage occurs ($\Gamma>10^{-9}$) and large enough to avoid damage [$P\tau_L/(\pi w^2)<J_{\mathrm{dam}}\approx 5\ \mathrm{J/cm^2}$]. Note the sensitive dependence of the parameter Γ on w_0 and n_2 ($\Gamma\sim w_0^{-2n+2}n_2^{-n}$ with $n=5$). On the other hand, pulse duration and beam radius are limited by the requirement $\alpha<\alpha_{\mathrm{cr}}$, where α_{cr} determines the dominance of the dispersion leading to pulse splitting.

Finally let us remark that recently experimental observations of self-channeled fs-pulse propagation in dielectric solids have been reported [12,13]. The presented study gives an insight and understanding of these observations. Our results agree rather well with these measurements concerning the spatial self-trapping behavior whereas the pulse shape of the channeled beam has not been resolved.

In conclusion we have presented a study of self-focusing of femtosecond pulses in normally dispersive dielectric solids. It has been shown that the pulses can be self-channeled due to the influence of multiphoton-ionization while simultaneously a significant pulse shortening appears.

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