Long-time-scale revivals in ion traps

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In this paper we investigate the interaction of a single ion in a trap with laser beams. Our approach, based on unitary transformating the Hamiltonian, allows its exact diagonalization without performing the Lamb-Dicke approximation. We obtain a transformed Jaynes-Cummings-type Hamiltonian, and we demonstrate the existence of super-revivals in that system. [S1050-2947(99)06203-4]

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There has been recently a great deal of interest in single trapped ions interacting with laser beams. This system allows the preparation of nonclassical states of the vibrational motion of the ion [1,2]. In fact, the generation of Fock, coherent, squeezed [3], and Schrödinger-cat states [4] has been already accomplished. Besides, there are potential practical possibilities in several fields, such as precision spectroscopy [5] and quantum computation [6], for instance.

We would like to remark that the theoretical treatment of the interaction of a trapped ion with one or several laser beams constitutes a complicated problem. Normally the problem is solved in limiting situations, e.g., in the Lamb-Dicke regime, in which the ion is confined in a region much smaller than the laser wavelength. Other limiting situations concern the laser intensity; if the effective Rabi frequency Ω of the ion-laser interaction is such that $\Omega \ll \nu$, where ν is the trap frequency [7,8], we have the low-excitation regime. Otherwise, if $\Omega \gg \nu$, this corresponds to the strong-excitation regime [9].

Here we adopt a new approach to this problem. We depart from the full ion-laser Hamiltonian, and perform a unitary transformation that allows us to obtain a Jaynes-Cummings-like Hamiltonian without the rotating-wave approximation (RWA). We therefore are able to analytically solve the problem, by applying the RWA, which corresponds to the regime $\Omega \approx \nu$. We should stress that our method makes possible to close the gap between the different intensity regimes, and moreover, with no restrictions on the Lamb-Dicke parameter.

We consider a single trapped ion interacting with two laser plane waves (frequencies ω_1 and ω_2), in a Raman-type configuration. Both laser beams, as usual, will be treated as classical fields propagating along the x axis, so that we have a one-dimensional problem. The corresponding scheme of levels is shown in Fig. 1. The lasers effectively drive the electric-dipole forbidden transition $|g\rangle \leftrightarrow |e\rangle$ (frequency ω_0), and we may have a detuning $\delta = \omega_0 - \omega_L$, where $\omega_L = \omega_1 - \omega_2$. For a sufficiently large detuning Δ, the third level $|r\rangle$ may be adiabatically eliminated [10], and we end up with an effective two-level system. This situation is described by the following Hamiltonian [9]:

$$\hat{H} = \hbar \nu \hat{a}^\dagger \hat{a} + \hbar \frac{\delta}{2} \sigma_z + \hbar \frac{\Omega}{2} (\hat{\sigma}_- e^{-i\eta(\hat{a}+\hat{a}^\dagger)} + \hat{\sigma}_+ e^{i\eta(\hat{a}+\hat{a}^\dagger)}),$$

(1)

η being the Lamb-Dicke parameter, $\hat{\sigma}_+ = |e\rangle\langle g|, (\hat{\sigma}_- = |g\rangle\langle e|)$ the usual electronic raising (lowering) operator, and $\hat{a}^\dagger (\hat{a})$ the ion's vibrational creation (annihilation) operator.

By applying the unitary transformation

$$\hat{T} = \frac{1}{\sqrt{2}} \left\{ \frac{1}{2} [\hat{D}^\dagger(\beta) + \hat{D}(\beta)]\hat{I} + \frac{1}{2}[\hat{D}^\dagger(\beta) - \hat{D}(\beta)]\hat{\sigma}_z \right.$$
$$\left. + \hat{D}(\beta)\hat{\sigma}_+ - \hat{D}^\dagger(\beta)\hat{\sigma}_- \right\},$$

(2)

to the Hamiltonian in Eq. (1), where $\hat{D}(\beta) = \exp(\beta \hat{a}^\dagger - \beta^* \hat{a})$ is Glauber's displacement operator, with $\beta = i\eta/2$, we obtain the following transformed Hamiltonian

$$\hat{\mathcal{H}} \equiv \hat{T}\hat{H}\hat{T}^\dagger = \hbar \nu \hat{n} + \hbar \Omega \hat{\sigma}_z$$
$$- i\hbar \frac{\eta \nu}{2} \left[(\hat{a}^\dagger - \hat{a}) - i\frac{\delta}{\eta \nu} \right](\hat{\sigma}_- + \hat{\sigma}_+) + \hbar \nu \frac{\eta^2}{4}.$$

(3)

This result holds for any value of the Lamb-Dicke parameter η. Our Hamiltonian in Eq. (1) becomes a Jaynes-Cummings-type Hamiltonian, and therefore its diagonalization is allowed provided we perform the rotating-wave approximation (RWA). This is possible in the regime $\nu = 2\Omega$, i.e., when the Rabi frequency of the coupling between the laser and the ion (Ω), is of the order of the vibrational frequency of the ion in the trap (ν).

Rewriting $\hat{\mathcal{H}}$ in the interaction representation,

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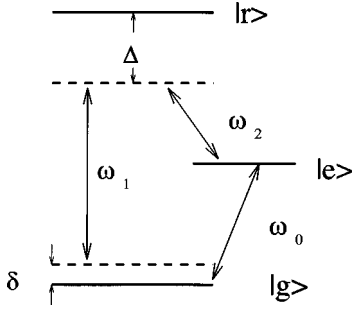
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FIG. 1. Configuration of levels of the trapped ion interacting with two laser beams, of frequencies ω_1 and ω_2.

$$\hat{\mathcal{H}} = -i\hbar \frac{\eta\nu}{2}\Bigg[(\hat{a}^\dagger \hat{\sigma}_- e^{i(\nu-2\Omega)t} - \text{H.c.})$$

$$+ (\hat{a}^\dagger \hat{\sigma}_+ e^{i(\nu+2\Omega)t} - \text{H.c.}) - i\frac{\delta}{\eta\nu}(\hat{\sigma}_+ e^{2i\Omega t} + \text{H.c.})\Bigg], \tag{4}$$

we see that we may neglect the last two rapidly oscillating terms on the right-hand side of Eq. (4), which corresponds to the RWA. We then obtain

$$\hat{\mathcal{H}} \approx \hat{\mathcal{H}}_{RWA} = \hbar\nu\hat{n} + \hbar\frac{\nu}{2}\hat{\sigma}_z - i\hbar g(\hat{a}^\dagger\hat{\sigma}_- - \hat{a}\hat{\sigma}_+), \tag{5}$$

which coincides with the Jaynes-Cummings Hamiltonian. The effective coupling constant is $g=\eta\nu/2$, and the term $\hbar\nu\eta^2/4$ has not been taken into account because it just represents an overall phase. We note that our final result in Eq. (5) is valid even for nonzero detunings δ.

The time evolution of the state vector, for an initial state $|\psi(0)\rangle$ is

$$|\psi(t)\rangle = \hat{T}^\dagger \hat{U}_I(t)\hat{T}|\psi(0)\rangle, \tag{6}$$

where $\hat{U}_I(t)$ is the Jaynes-Cummings evolution operator [11] in the interaction picture

$$\hat{U}_I(t) = \frac{1}{\sqrt{2}}\Bigg\{\frac{1}{2}[\hat{C}_{n+1}+\hat{C}_n]\hat{I} + \frac{1}{2}[\hat{C}_{n+1}-\hat{C}_n]\hat{\sigma}_z$$

$$+ \hat{S}_{n+1}\hat{a}\hat{\sigma}_+ - \hat{a}^\dagger\hat{S}_{n+1}\hat{\sigma}_-\Bigg\}, \tag{7}$$

with $\hat{C}_{n+1} = \cos(gt\sqrt{\hat{a}\hat{a}^\dagger})$ and $\hat{S}_{n+1} = \sin(gt\sqrt{\hat{a}\hat{a}^\dagger})/\sqrt{\hat{a}\hat{a}^\dagger}$.

For an initial state $|\psi(0)\rangle = |e\rangle|\alpha\rangle$, or the atom in the excited state [see Fig. (1)] and the ion in a coherent (vibrational) state, the mean number of vibrational quanta as a function of the (dimensionless) scaled time $\tau=gt$, or $\langle\hat{n}\rangle = \langle\psi(\tau)|\hat{n}|\psi(\tau)\rangle$ is given by

$$\langle\hat{n}\rangle = \frac{1}{2}\sum_{n=0}^{\infty}(|c_{n,e}(t)|^2 + |c_{n,g}(t)|^2)\left(n+\frac{\eta^2}{4}\right)$$

$$+ \left(\frac{i\eta}{4}\right)\sum_{n=0}^{\infty}\sqrt{n+1}[c_{n+1,e}^*(t)c_{n,g}(t)$$

$$- c_{n,e}^*(t)c_{n+1,g}(t) - \text{c.c.}], \tag{8}$$

where

$$c_{n,e}(t) = \exp(-|\tilde{\alpha}|^2/2)\Bigg[\frac{\tilde{\alpha}^n}{\sqrt{n!}}\cos(gt\sqrt{n+1})$$

$$+ \frac{\tilde{\alpha}^{n+1}}{\sqrt{(n+1)!}}\sin(gt\sqrt{n+1})\Bigg], \tag{9}$$

$$c_{n,g}(t) = \exp(-|\tilde{\alpha}|^2/2)\Bigg[\frac{\tilde{\alpha}^{n-1}}{\sqrt{(n-1)!}}\sin(gt\sqrt{n})$$

$$- \frac{\tilde{\alpha}^n}{\sqrt{n!}}\cos(gt\sqrt{n})\Bigg], \tag{10}$$

and $\tilde{\alpha} = \alpha - i\eta/2$.

The structure of the equation above is similar to the one obtained for $\langle\hat{n}\rangle$ in the driven-Jaynes-Cummings model (DJCM) [12]. Therefore, we expect the phenomenon of super-revivals to be present in the ion system, as an analogy to the DJCM. Super-revivals are long-time-scale revivals arising in the atom-field dynamics. This peculiar behavior is
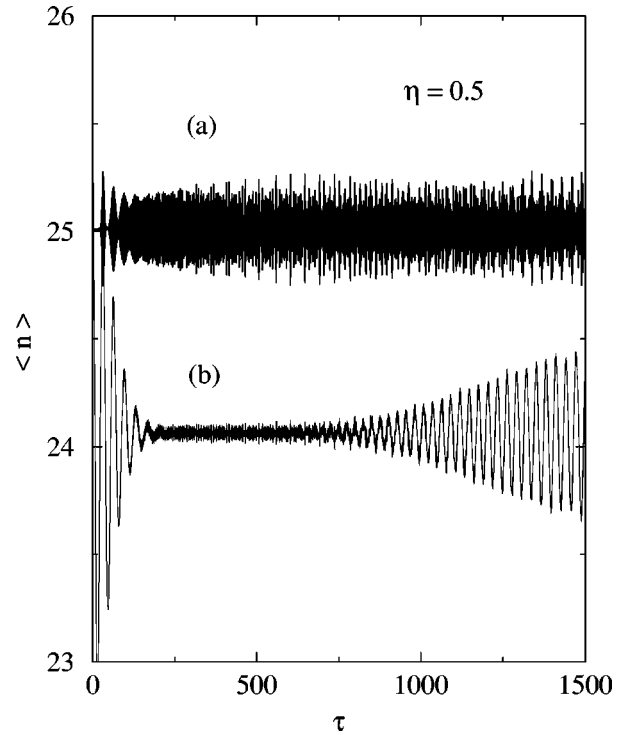

FIG. 2. Plot of the mean excitation number $\langle\hat{a}^\dagger\hat{a}\rangle$ as a function of the (dimensionless) scaled time $\tau=gt$. (a) Ordinary revivals occurring for $\alpha=(5.0,0.5)$, and (b) super-revivals ocurring for $\alpha=(0.5,5.0)$. In both cases the Lamb-Dicke parameter is $\eta=0.5$.

illustrated in Fig. 2. The super-revivals in $\langle \hat{n} \rangle$ occur for a time $\tau_{sr} \approx 4|\alpha|^2 \tau_r$ ($\tau_r = 2\pi\sqrt{\bar{n}}/\Omega$ is the revival time). We note that the existence or not of super-revivals is narrowly connected to the preparation of the initial vibrational state. For instance, if we have $\alpha = (5.0, 0.5)$ and $\eta = 0.5$, the super-revivals do not occur [see Fig. 2(a)], and we have ordinary revivals only. However, for appropriate values of the parameters $\alpha = (0.5, 5.0)$ and $\eta = 0.5$, super-revivals take place in that system [see Fig. 2(b)]. We also would like to point out that because dissipation in ion traps is much smaller than in cavities, it might be allowed the experimental observation of super-revivals in that system.

In conclusion, we have presented an approach for studying the dynamics of the trapped ions driven by laser beams. We have succeeded in linearizing the Hamiltonian through the application of a unitary transformation. Then we were able to treat the problem in a specific intensity regime ($\Omega \approx \nu$), without the need of performing the Lamb-Dicke approximation. As a result we have predicted the existence of long-time-scale revivals (super-revivals) for the excitation number in trapped ions.

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