Multichannel communication using an infinite dimensional spatiotemporal chaotic system

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Chaotic spatiotemporal dynamics generated by a nonlinear set of coupled partial differential equations are synchronized via a scalar complex variable. Our results show that synchronization is robust to noise, and we demonstrate chaotic communication in a high-dimensional system. We have also discovered that the added dimensionality creates a new quasisynchronous state, enabling the transmission of multiple messages through a single scalar complex channel. The feasibility of these novel ideas is demonstrated on a realistic numerical model of chaotic semiconductor lasers. [S1050-2947(99)05503-1]

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In 1993 it was shown that small perturbations could be used to encode a symbol sequence on a chaotic system [1] and this was demonstrated using coupled electronic circuits [2]. Since then much interest has focused on using synchronized chaotic systems to send and receive messages. The majority of work has concentrated on the case where there are two identical systems synchronized through a weak control signal [3]. To send a message, one of the parameters in the transmitter system is changed slightly and the resulting desynchronization is measured at the receiver. For this scheme to work, the synchronization must be fairly robust so that the system can recover synchronization after the message is sent.

It was quickly realized that it might be possible to synchronize higher dimensional chaotic systems with more than a single positive Liapunov exponent [4]. Further work showed that it was also possible to synchronize large numbers of coupled chaotic systems using a single complex signal [5]. Synchronization of spatiotemporal chaos using a vector coupling was demonstrated numerically very recently using both coupled reaction-diffusion and complex Ginsburg-Landau partial differential equations (PDES) [6]. An important observation of [5] was that as the dimension of the synchronization manifold increases, its basin of attraction shrinks and the system becomes sensitive to noise. Bursts of desynchronization are observed to interrupt the laminar phases of the synchronized dynamics, consistent with the observation that a small amount of noise in a communications channel can adversely affect the synchronization of globally coupled oscillators [7]. Thus it remains an open question whether synchronization of high dimensional chaotic systems, coupled nonlinear PDES in particular, is robust to noise and/or error signals representing transmitted messages. In other words, if one were to cause a loss of synchronization (e.g., to encode a message), the system might not regain a synchronized state.

Simultaneously in the optics community an effort was underway to apply these ideas to laser systems. In 1994 two solid state lasers were synchronized [8] and communication schemes were proposed [9]. Very recently, communication has been demonstrated using coupled erbium-doped fiber ring lasers [10] and with semiconductor lasers using electro-optical feedback [11]. Synchronization and message encoding has been discussed in a simplified model of two coupled ring laser cavities [12]. Additional theoretical work was done on semiconductor lasers using single mode models [13].

In this paper we report on the successful multiplexing of random bit sequences between transmitter/receiver semiconductor lasers modeled by coupled nonlinear PDES. Multiplexing is naturally considered through the higher dimensionality introduced with the inclusion of a spatial variable, in this case the longitudinal modes. Additionally we report synchronization of spatiotemporal chaos through a single scalar complex variable (the total electric field) where synchronization is robust to noise in the laser, noise in the communications channel, and strong transient perturbations. The separate messages are encoded into individual channels (modes) by injecting weak signals at the relevant mode frequencies. This results in a third, hitherto unknown quasisynchronous state where some spatial modes are synchronized while other modes are unsynchronized.

The use of the full PDE model is essential to capturing the relevant physics of the problem. For example, a nominally single-longitudinal mode semiconductor laser, when subjected to a weak external delayed feedback, begins to run chaotically on several of its previously suppressed longitudinal modes. This observation, for which there is very recent experimental evidence, is critical to the multiplexing scheme that we propose here. Of course, the complexity of the problem restricts us to numerical simulation.

Our model of the semiconductor laser includes counter-propagating traveling waves coupled to the total carrier density. While the full gain bandwidth can accommodate hundreds of longitudinal modes, the external optical feedback causes only 15–20 modes to oscillate and these are located near the gain peak. Thus we can assume that the gain has a parabolic shape locally. The parabolic gain assumption has been shown to be adequate through comparison with a microscopic model [14], where the contribution which leads to the linewidth enhancement factor of the semiconductor laser is lumped into an effective background term. The present model has been successful in describing the experimental observation of longitudinal mode hopping in a chaotic semiconductor laser, subject to external optical injection [15]. The forward (E^+) and backward (E^-) propagating fields in the cavity and the total carrier density are fully resolved. The parabolic gain dispersion is provided by introducing a second-order derivative in z. The transmitter and receiver

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system are nearly identical and are differentiated by a subscripted T or R. Coupling of the two systems is achieved through a small injection term in the boundary conditions,

$$\frac{\partial E^{\pm}_{T,R}}{\partial t} \pm \frac{\partial E^{\pm}_{T,R}}{\partial z} = \kappa\left((N_{T,R}-1)\left(1+G_d \frac{\partial^2}{\partial z^2}\right) - iRN_{T,R} - \alpha \right) E^{\pm}_{T,R}, \tag{1}$$

$$\frac{\partial N_{T,R}}{\partial t} = J_{T,R}(t) - \gamma N_{T,R} - (N_{T,R}-1)[|E^{+}_{T,R}|^2 + |E^{-}_{T,R}|^2].$$

The boundary conditions are

$$E^{+}_{T,R}(z=0) = \sqrt{R_1} E^{-}_{T,R}(z=0), \tag{2}$$

$$E^{-}_{T,R}(z=1) = \sqrt{R_2} E^{+}_{T,R}(z=1) + (1-R_2)\, \eta E^{+}_{T}(t-\tau_{fb,cc}),$$

where κ is the damping rate of the electric field in the cavity, R is the linewidth enhancement factor (which describes the phase amplitude coupling), G_d is a diffusion constant which describes the gain curvature, α represents internal nonradiative laser losses, γ is the carrier damping rate, J is the pumping parameter, $R_{1,2}$ are the facet reflectivities, η is the feedback/injection strength, τ_{fb} is the external cavity round trip time, and τ_{cc} is the delay in the communication channel.

In the absence of feedback the laser operates in a single mode. Once the feedback is introduced, there is a pronounced lifting of the solitary laser longitudinal mode spectrum [16]. Figure 1 contrasts the single mode situation with the multimode spectrum in the presence of optical feedback. The solitary laser is running at about 5% above threshold. Notice that the multimode spectral shape mimics the actual parabolic gain dispersion. The individual spectral peaks in (b) are broadened due to the chaotic motion but this broadening is not evident on the scale of the figure which encompasses a 10 THz spectral window.

Each longitudinal mode acts as a replica of the single mode system, with weak coupling to every other mode through spatial gratings induced in the carriers. Because of the feedback, the single longitudinal mode equations are formally infinite dimensional, however the correlation dimension [17] is calculated to be 1.6 implying that the actual dynamics is constrained to a low dimensional attractor. There also exists a single Liapunov exponent for the single mode system. Thus, while the single mode system with feedback is formally infinite dimensional, in practice it has much in common with simpler systems which have been extensively studied. Only through the weak coupling present in the multimode system does the actual dimension of the dynamics grow.

How this additional dimensionality affects the overall dynamics of the system is still being debated. Two key features are emerging. First is the behavior which affects all the modes simultaneously (an example is low frequency fluctuations [16]). This requires that the modes are in synch, forcing

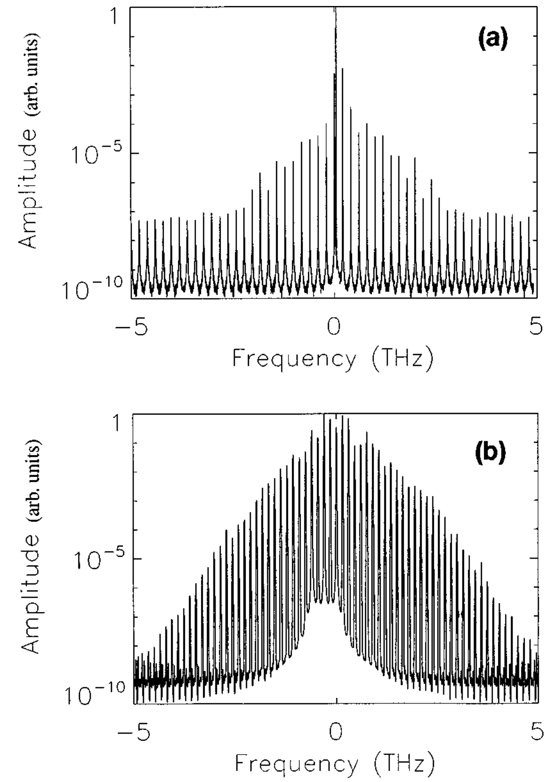

FIG. 1. Log plot of the solitary laser cavity mode spectrum for (a) the solitary laser and (b) the laser with feedback. All but one of the laser modes are suppressed in the solitary laser while the mode spectrum of the laser with feedback follows the parabolic shape of the gain dispersion.

the simultaneous existence of several spatial patterns in the laser cavity. A second behavior is that of antiphase dynamics between the modes [18]. At any one time most of the laser energy is in a single mode, however the energy hops rapidly (subnanosecond scale) from one mode to the next. So while at any one time there is a dominant spatial mode present in the cavity, this pattern is not stationary and changes very quickly. It is the coexistence of several spatial patterns which we hope to utilize to achieve multiplexing.

The total electric field output of the transmitter laser drives the receiver system into synchrony with the transmitter [19] and the messages are extracted optically by transmitter/receiver signal subtraction and spectral decomposition of the difference signal. All of the processing can be done optically in real time and offers the potential for high bandwidth multiplexed message transmission. Numerical simulations show that the infinite dimensional system synchronizes as well as the single mode system. With the addition of noise or parameter mismatch (as high as 1%) the system still synchronizes. Also we have studied how the synchronization of our system recovers from large changes in the fields and carrier density. A large, random, instantaneous error was added to the receiver laser and the system rapidly recovered. In most foreseeable instances that could be encountered in a real laser we observed synchronization. This indicates that the synchronization is not only very robust but that the basin of synchronization is quite large.

A second scheme has been proposed in the literature for single mode laser systems [13], which involves a weak dif-

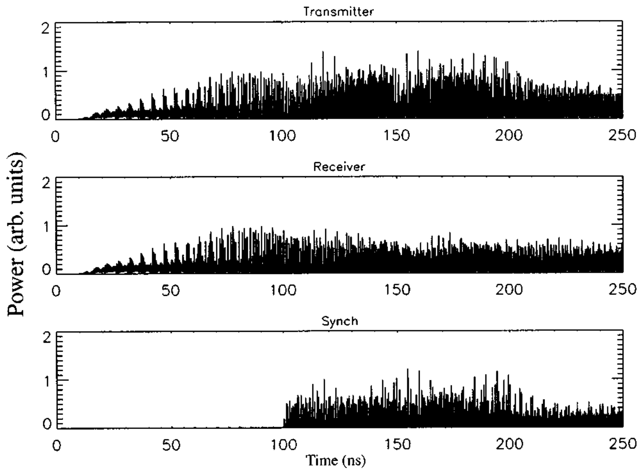

FIG. 2. When both laser systems have external feedback they initially synchronize. After 100 ns the two systems are made to lose synchrony by changing the current of the transmitter laser. After 200 ns both systems are brought back to identical states, however they remain unsynchronized.

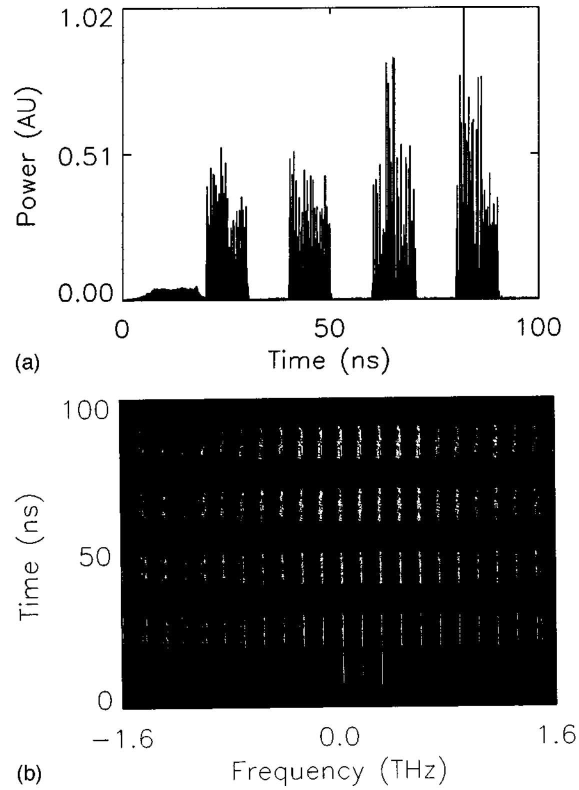

FIG. 3. (a) Difference of the transmitter and receiver signal with a message encoded on the transmitter. A ''0'' bit corresponds to a synchronized state; a ''1'' bit corresponds to an unsynchronized state. The message sent is a repeating ''0''-''1'' sequence of bits. (b) Fourier transform of the difference signal. When a message is sent the laser gains and loses synchronization across all of the laser cavity modes.

ference coupling between transmitter and receiver. In this system the PDE boundary condition for the receiver becomes

$$E_R^-(z=1)=\sqrt{R_2}E_R^+(z=1)+(1-R_2)\,\eta E_R^+(t-\tau_{fb})+\zeta(E_T^+(t)-E_R^+(t)). \quad (3)$$

Studies using single mode models have shown this method to be extremely robust to noise and parameter mismatch. At issue is whether a single synchronization channel is sufficient to synchronize the many spatial patterns coexisting in the multimode laser cavity. This is equivalent to studying the problem where many ordinary differential equations are coupled, each longitudinal mode corresponding to a separate ODE. With each single mode system contributing a single Liapunov exponent, the Liapunov dimension grows depending on the number of excited modes. If the two systems are initialized so that the longitudinal modes in both cavities have roughly comparable energy distribution, the system synchronizes quickly (see Fig. 2). If the two systems are brought out of synchrony temporarily, as most communication methods require, we find that they cannot regain synchrony as shown in Fig. 2. We interpret this as similar to the results reported by Tsimring *et al.* [5]. As the effective Liapunov dimension increases, the basin of synchronization decreases and we find that different initial conditions, added noise, or parameter mismatch (as small as 10^{-6}) cause this system to quickly lose synchronization and never recover.

Robust synchronization is necessary for communication since a message is sent by changing a parameter in the transmitter system. The second scheme [with the receiver boundary conditions as in Eq. (3)] is therefore unsuitable for communication and we restrict our discussion to the first scheme [Eqs. (1) and (2)]. For a first attempt we modulate the laser pumping current, affecting all of the cavity modes equally. Figure 3(a) shows the results of a repeating ''0'' - ''1'' bit sequence that is modulated every 10 ns. To send the message, the transmitter current pumping was increased by 10% for a ''1'' bit. At the receiver end we subtract the transmitter intensity from the receiver intensity. If the bit is a ''1,'' the transmitter and receiver are no longer synchronized and a difference signal is recorded. For a ''0'' the two signals are synchronized and the difference signal is zero. After a brief initial transient due to switch-on, the two systems begin to synchronize and there is a very clean message showing the repeated ''0''-''1'' bit sequence.

The difference signal is Fourier transformed to resolve the many laser cavity modes present. Figure 3(b) shows that the message is encoded across several of the laser cavity modes. The initial switch-on transient is confined to only three of the cavity modes. When the two lasers lose synchrony (corresponding to a ''1'' bit) all of the laser cavity modes become unsynchronized. The two lasers are next brought back into synchrony (corresponding to a ''0'' bit) and synchronization is rapidly seen in every mode. It is a remarkable observation that all of the modes above the background noise level synchronize.

Next we demonstrate a new, quasisynchronous state. As discussed earlier, each longitudinal mode has a corresponding spatial pattern inside the laser cavity. In some regimes the spatial patterns are coexistent, in other regimes they rapidly hop from one pattern to the other. In both instances what we call total synchronization is when the spatial dynamics of both transmitter and receiver are slaved. If one can selectively synchronize the spatial patterns in the receiver so that

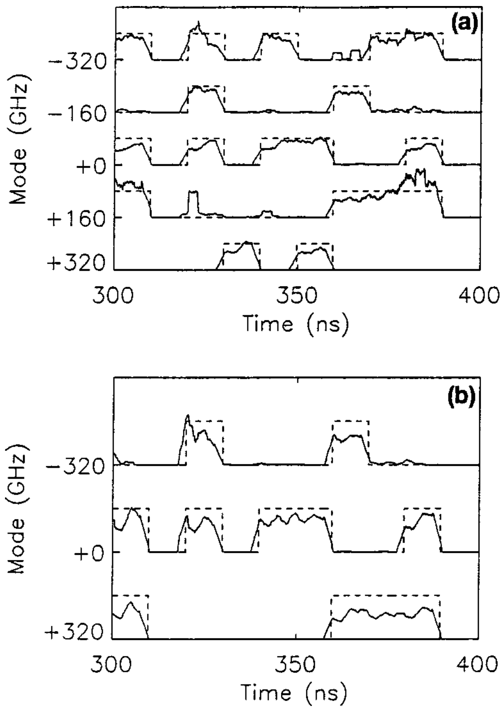

FIG. 4. Random bit sequences sent over laser cavity modes. Solid line is the recovered message while the dashed line is the transmitted message. (a) Adjacent laser modes are used as channels, resulting in substantial cross talk between the modes. (b) Alternating modes are used as channels resulting in a significant decrease in crosstalk. The recovered message has been averaged over 1 ns for clarity.

one pattern is slaved to the transmitter while the other patterns are not, we have a quasisynchronous state where the system is only partially synchronized. To reach this state one can either selectively filter the synchronization signal or selectively change the transmitter. We investigate the latter technique.

By changing the transmitter in such a way that individual modes are affected, we create a situation in which the two systems are not fully synchronized yet individual modes are. Messages can be resolved at the receiver by taking the Fourier transform of the difference signal. The challenge lies in encoding the message on individual laser cavity modes at the transmitter. The semiconductor laser offers a nice physical method. Since each laser cavity mode corresponds to a different frequency of light, a message can be encoded in a single laser cavity mode by means of external injection at the corresponding optical frequency. To add this message capability to the model we include a new injection term $F_T^{\mathrm{inj}} = \Sigma_n \zeta_n(t) E e^{\iota \omega_n t}$, where ω_n corresponds to the nth laser cavity mode optical frequency. The individual messages are encoded by varying $\zeta_n(t)$, which represents the weak signal of the nth message-encoding injection laser.

Figure 4(a) shows the recovered signal where five random bit sequences have been sent across five neighboring laser cavity modes. The encoded signal at the transmitter is superimposed as a square wave dashed pattern. The fast dynamics are averaged out of the recovered signal for clarity of presentation. Mostly the encoded messages are clearly evident in the extracted error signal, however there does exist some cross talk between the modes. When a higher mode has an unsynchronized state simultaneous with the synchronized state of a lower mode there is a small signal in the lower mode. This can be clearly seen at about 360 ns in the mode corresponding to -320 Ghz. This cross talk is easily understood as there exists an instability where strong fields are coupled to lower laser cavity modes. When the semiconductor laser is chaotic it exhibits strong pulsing. The externally injected message signal increases the strength of the pulsations causing some of the energy to couple into lower modes. This affects the synchronization signal in the lower mode. However, this coupling is very small and so has a nearly negligible affect on the recovered message.

There do exist some sequences in the recovered message that are irrecoverably garbled. The most striking instance is seen between 320 and 330 ns in the $+160$ Ghz laser cavity mode. This error is a result of cross talk between the laser cavity modes, possibly inherent to the quasisynchronous state. When alternating modes are used as channels [Fig. 4(b)], the cross talk completely disappears. Obviously, cross talk can be minimized by using a shorter laser cavity and increasing the spacing between individual modes.

The semiconductor laser offers unique challenges and opportunities in this respect. Increasing the channel capacity is a high priority. Recent progress in computing the semiconductor optical response from the full many-band microscopic physics has resulted in the first quantitative agreement between experimentally measured and theoretically computed gain spectra [20]. It should now be feasible to *a priori* design a semiconductor gain shape that departs from the locally parabolic shape evident from Fig. 1 and thereby promote the simultaneous oscillation of tens of longitudinal modes.

In summary, we have demonstrated the first robust multiplexing of independent message streams by utilizing the synchronization of multimode chaotic semiconductor lasers. This, to our knowledge, is the first direct evidence of the robustness of synchronization in high dimensional nonlinear dynamical systems. What is remarkable is that a pointwise application (at $z=1$ in the receiver laser) of a complex scalar signal is sufficient to synchronize the entire spatiotemporal evolution of two infinite dimensional nonlinear PDES. Both the chaos and multilongitudinal mode dynamics are simultaneously promoted by the weak external optical feedback loop in the transmitter laser. In addition to the noise intrinsic to the semiconductor laser itself, we have also added noise in the communications channel between transmitter and receiver laser systems. A new, quasisynchronous state allows for multiplexed messages encoded through the separate optical injection of random bit sequences into individual longitudinal modes of the solitary laser, which have been successfully decoded from the spectrally resolved multimode output of the driven receiver laser. In the present study, cross talk between chaotic communication channels could be eliminated by sending messages over alternate channels.

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