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Cooperative effects in the light and dark periods of two dipole-interacting atoms

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If an atom is able to exhibit macroscopic dark periods, or electron shelving, then a driven system of two atoms has three types of fluorescence periods (dark, single, and double intensity). We propose to use the average durations of these fluorescence types as a simple and easily accessible indicator of cooperative effects. As an example, we study two dipole-interacting V systems by simulation techniques. We show that the durations of the two types of light periods exhibit marked separation-dependent oscillations and that they vary in phase with the real part of the dipole-dipole coupling constant. [S1050-2947(99)03903-7]

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I. INTRODUCTION

Cooperative effects in the radiative behavior of atoms stored in a trap may arise from their mutual dipole-dipole interaction if the atoms are close enough to each other. This is particularly interesting for two or three atoms, and has attracted considerable interest in the literature [1–25]. Recently, the present authors investigated in detail the transition from antibunching to bunching for two two-level systems with decreasing atomic distance [26].

For a single multilevel system with a metastable state one has the striking phenomenon of macroscopic dark periods, or electron shelving, in which the electron is essentially shelved for seconds or even minutes in a metastable state without photon emissions [27–35]. For two such systems their fluorescence behavior would, without cooperative effects, be just the sum of the separate photon emissions, with dark periods of both atoms, and light periods of a single atom and two atoms. In Ref. [36] the fluorescence intensity of two and three such ions in a Paul trap was measured, and a large fraction of almost simultaneous jumps by two and even all three ions was recorded. This fraction was orders of magnitudes larger than that expected for independent ions. A quantitative explanation of such a large cooperative effect has been found to be difficult [13,37–40], and we will briefly discuss this question again in Sec. V. Other experiments at larger distances and with different ions showed no cooperative effects [41,42]. A numerical approach to the study of double jumps faces the problem that for good statistics one needs very long simulation times.

As a simpler indicator of cooperative effects for systems with light and dark periods, here we propose to use the mean durations T_0, T_1, and T_2 of the three different types of fluorescence periods, i.e. dark, single-intensity, and double-intensity periods, respectively. The running time can be much shorter than required for double jumps, making these quantities easily accessible, both experimentally and in simulations.

In this paper we therefore present a study of cooperative effects on the mean durations of the three types of fluorescence periods of two three-level V systems with a metastable state, as a function of their distance r. The level scheme of a single V system is depicted in Fig. 1 [43]. Our simulations show that the mean durations of the single- and double-intensity periods, T_1 and T_2, depend sensitively on the dipole-dipole interaction and thus on the atomic distance r. They exhibit marked oscillations which decrease in amplitude when r increases. These oscillations seem to continue up to a distance of well over five wavelengths of the strong transition, where we have stopped our simulation. The real part of the dipole-dipole coupling constant of the two V systems also exhibits damped oscillations. As a remarkable fact we find that these oscillations are in phase with those of T_1 and T_2. This correspondence is easy to understand intuitively, since the real part of the dipole-dipole coupling constant enters the decay rates of the excited states of the combined two-atom system.

In Sec. II we explain the methods employed, which are based on the quantum jump approach [44–48] (equivalent to the Monte Carlo wave-function approach [49] and to quantum trajectories [50]). This approach is here adapted to two dipole-interacting V systems.

In Sec. III we define in more detail the three types of fluorescence periods of zero, single, and double intensity. This involves an averaging procedure, both experimentally and theoretically. We then present the results of our simulations.

In Sec. IV the numerical results are interpreted, and it is shown that one can associate three distinct subspaces of states of the coupled system with the three types of fluorescence periods. During each period the coupled system remains in the corresponding subspace. In Sec. V we discuss and summarize our results.

II. QUANTUM JUMP APPROACH

In the quantum jump approach [44–48], the time development of an atomic system is described by a conditional

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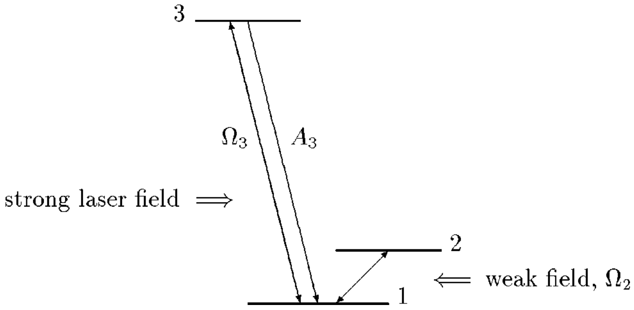

FIG. 1. V system with metastable level 2 and Einstein coefficient A_3 for level 3. Ω_2 and Ω_3 are the Rabi frequencies of the two lasers driving the weak 1-2 transition and the strong 1-3 transition, respectively.

non-Hermitian Hamiltonian H_{cond}, which gives the time development between photon emissions, and by a reset operation which gives the state or density matrix right after an emission. For a general N-level system these have been derived in Refs. [46,47]. The derivation is adapted here for a system consisting of two atoms. Slight modifications arise here since the field operator appears with different position arguments.

We consider two atoms, each a V configuration as shown in Fig. 1, with the levels $|1\rangle_i$, $|2\rangle_i$, and $|3\rangle_i$ ($i=1$ and 2), and fixed at positions $\mathbf{r}_i$. We define operators $S_{ij}^{\pm}$ ($i=1,2$; $j=2$ and 3) in the two-atom Hilbert space by $S_{ij}^{+}=|j\rangle_{ii}\langle 1|$ and $S_{ij}^{-}=|1\rangle_{ii}\langle j|$. For simplicity we consider the case where the dipole moments of two atoms are the same, i.e., ${}_1\langle 1|\mathbf{X}_1|j\rangle_1={}_2\langle 1|\mathbf{X}_2|j\rangle_2\equiv\mathbf{D}_{1j}$. If $\mathbf{D}_{1j}$ is real, then the angle it forms with the line connecting the atoms is denoted by ϑ_j. In general ϑ_j is defined through $\cos^2\vartheta_j=|(\mathbf{D}_{1j},\mathbf{r})|^2/r^2D_{1j}^2$, where $\mathbf{r}=\mathbf{r}_2-\mathbf{r}_1$. We assume the laser radiation normal to this line so that the lasers are in phase for both atoms. The two lasers are denoted by L_2 and L_3. We take zero detuning and $\mathbf{E}_{Lj}(\mathbf{r},t)=\text{Re}[\mathbf{E}_{0j}\exp\{-i(\omega_{j1}t-\mathbf{k}\cdot\mathbf{r})\}]$, where $\hbar\omega_{j1}$ is the energy difference between level 1 and level j. Making the usual rotating-wave approximation and going over to the interaction picture the interaction Hamiltonian becomes

$$H_I=\sum_{i=1}^{2}\sum_{j=2}^{3}\sum_{\mathbf{k},s}\hbar[g_{j\,\mathbf{k},s}\,a_{\mathbf{k},s}\,e^{i(\omega_{j1}-\omega_k)t}\,e^{i\mathbf{k}\cdot\mathbf{r}_i}S_{ij}^{+}+\text{H.c.}]+H_L, \tag{1}$$

with the coupling constants

$$g_{j\,\mathbf{k},s}=ie\left(\frac{\omega_k}{2\epsilon_0\hbar L^3}\right)^{1/2}(\mathbf{D}_{1j},\boldsymbol{\epsilon}_{\mathbf{k},s}), \tag{2}$$

and laser part

$$H_L=\frac{\hbar}{2}\sum_{i=1}^{2}\sum_{j=2}^{3}\Omega_j\{S_{ij}^{+}+S_{ij}^{-}\}. \tag{3}$$

The Rabi frequencies of the lasers are $\Omega_j=(e/\hbar)\,\mathbf{D}_{1j}\cdot\mathbf{E}_{0j}$ for $j=2$ and 3, and they are the same for both atoms. The operator H_I implicitly contains the dipole-dipole interaction of the two atoms, as seen from the conditional Hamiltonian H_{cond} further below. In the Power-Zienau formulation, which we have used above, this interaction is due to photon exchange [1].

Conditional Hamiltonian and waiting times. As explained in Refs. [44–48], H_{cond} is obtained (in the interaction picture) from the short-time development under the condition of no emission, i.e., from the relation

$$\mathbf{1}-\frac{i}{\hbar}H_{\text{cond}}\Delta t=\langle 0_{ph}|U_I(\Delta t,0)|0_{ph}\rangle, \tag{4}$$

where the right-hand side is evaluated in second-order perturbation theory for Δt intermediate between inverse optical frequencies and atomic decay times. In a similar way as for a single atom [44–47], for the system of two three-level atoms one obtains [51]

$$H_{\text{cond}}=\frac{\hbar}{2i}\left[\sum_{j=2}^{3}A_j(S_{1j}^{+}S_{1j}^{-}+S_{2j}^{+}S_{2j}^{-})+C_j(S_{1j}^{+}S_{2j}^{-}+S_{2j}^{+}S_{1j}^{-})\right]+H_L, \tag{5}$$

with the r-dependent coupling constants

$$C_j=\frac{3A_j}{2}e^{ik_{j1}r}\left[\frac{1}{ik_{j1}r}(1-\cos^2\vartheta_j)+\left(\frac{1}{(k_{j1}r)^2}-\frac{1}{i(k_{j1}r)^3}\right)(1-3\cos^2\vartheta_j)\right], \tag{6}$$

which contain the dipole-dipole interaction between the atoms. The dependence of C_j on r is maximal for $\vartheta_j=\pi/2$ (see Fig. 2). In the following, for the Einstein coefficients and Rabi frequencies we will assume the relations

$$\Omega_2\ll\Omega_3,\quad \Omega_2\ll\Omega_3^2/A_3\quad\text{and}\quad A_2\approx 0. \tag{7}$$

Then A_2 and $\text{Re}\,C_2$ can be neglected in H_{cond}; we will also neglect $\text{Im}\,C_2$, which is allowed if r is not small compared to λ_{21}, as seen from Fig. 2.

Let $|g\rangle$, $|e_2\rangle$, and $|e_3\rangle$ denote the states where both atoms are in the ground state and the excited states $|2\rangle$ and $|3\rangle$, respectively, and let $|s_{jk}\rangle$ be the symmetric and $i|a_{jk}\rangle$ be the antisymmetric combinations of $|j\rangle|k\rangle$ and $|k\rangle|j\rangle$. Then Eq. (5) becomes

$$H_{\text{cond}}=\frac{\hbar}{2i}\Bigg[A_3\,(|s_{23}\rangle\langle s_{23}|+|a_{23}\rangle\langle a_{23}|)+(A_3+C_3)|s_{13}\rangle\langle s_{13}|+(A_3-C_3)|a_{13}\rangle\langle a_{13}|+2A_3\,|e_3\rangle\langle e_3| \\ +\left\{\sum_{j=2}^{3}\sqrt{2}i\Omega_j(|g\rangle\langle s_{1j}|+|s_{1j}\rangle\langle e_j|)+i\Omega_2(|s_{13}\rangle\langle s_{23}|+|a_{13}\rangle\langle a_{23}|)+i\Omega_3(|s_{12}\rangle\langle s_{23}|-|a_{12}\rangle\langle a_{23}|)+\text{H.c.}\right\}\Bigg]. \tag{8}$$

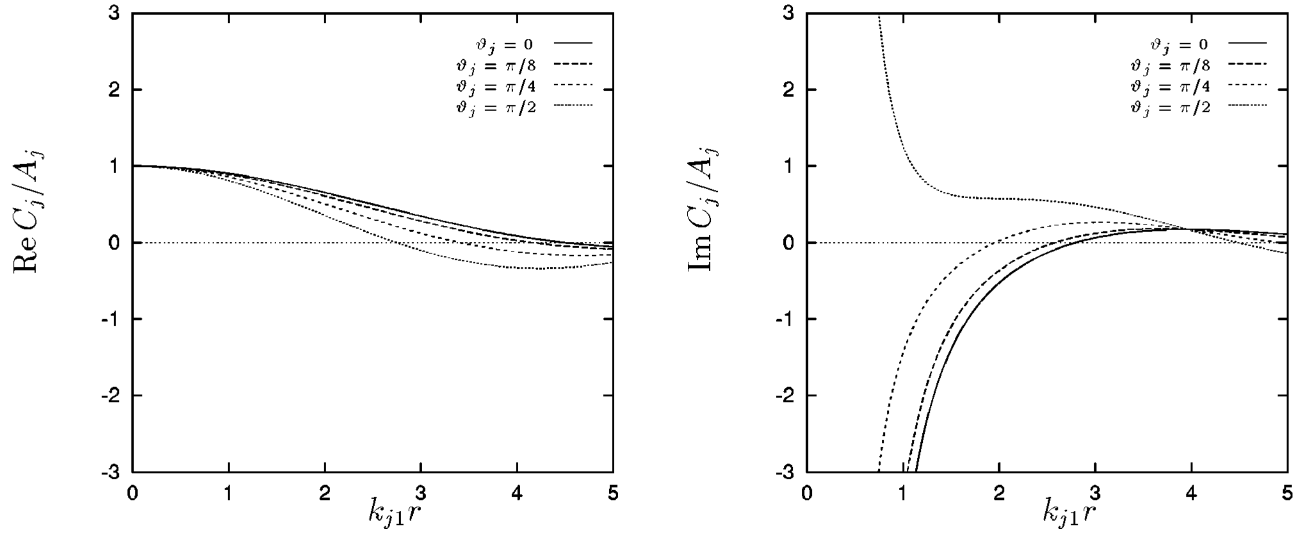

FIG. 2. Dependence of C_j/A_j ($j=2,3$) on r.

Without lasers the conditional Hamiltonian is diagonal in this basis.

Between emissions the atomic time development is given by $U_{\mathrm{cond}}(t,0)=\exp\{-iH_{\mathrm{cond}}\,t/\hbar\}$, which is nonunitary since H_{cond} is non-Hermitian. The corresponding decrease in the norm of a vector is connected to the waiting time distribution [32] for emission of a (next) photon. If at $t=0$ the initial atomic state is $|\psi\rangle$, then the probability P_0 to observe *no* photon by a broadband detector (over all space) is given by [44–47]

$$P_0(t;|\psi\rangle)=\|U_{\mathrm{cond}}(t,0)|\psi\rangle\|^2, \tag{9}$$

and the probability density w_1 of finding the first photon at time t is

$$w_1(t;|\psi\rangle)=-\frac{d}{dt}P_0(t;|\psi\rangle). \tag{10}$$

For an initial density matrix instead of $|\psi\rangle$ the expressions are analogous, with a trace instead of a norm squared in Eq. (9).

According to Eqs. (9) and (10), $A_3 \pm \mathrm{Re}\, C_3$ describe the decay rates of $|s_{13}\rangle$ and $|a_{13}\rangle$, respectively. From this the well-known fact follows that two atoms with dipole interaction can decay faster or slower than two independent atoms (superradiance and subradiance [23]). $\mathrm{Im}\, C_3$ corresponds to a level shift of $|s_{13}\rangle$, and $-\mathrm{Im}\, C_3$ to a level shift of $|a_{13}\rangle$, caused by the interaction between the atoms.

Reset matrix. Now we determine the reset operation which gives the state or density matrix right after a photon detection. Let the state of the combined system, atoms plus quantized radiation field, be given at time t by $|0_{\mathrm{ph}}\rangle\,\rho\,\langle 0_{\mathrm{ph}}|$, i.e., the atomic system is described by the density matrix ρ, and there are no photons (recall that the laser field is treated classically). If at time $t+\Delta t$ a photon is found by a non absorptive measurement, the combined system is in the state

$$\mathbb{P}_{>}U_I(t+\Delta t,t)|0_{\mathrm{ph}}\rangle\,\rho\,\langle 0_{\mathrm{ph}}|U_I^{\dagger}(t+\Delta t,t)\mathbb{P}_{>}\,, \tag{11}$$

where $\mathbb{P}_{>}=\mathbf{1}-|0_{\mathrm{ph}}\rangle\mathbf{1}_A\langle 0_{\mathrm{ph}}|$ is the projector onto the one or more photon space (since Δt is in the above range, and thus small one could directly take the projector onto the one-photon space). The probability for this event is the trace over Eq. (11). For the state of the atomic system it is irrelevant whether the detected photon is absorbed or not (intuitively the photon travels away and no longer interacts with the atomic system). Hence after a photon detection at time $t+\Delta t$, the non-normalized state of the atomic system alone, denoted by $\mathcal{R}(\rho)\Delta t$, is given by the partial trace over the photon space,

$$\begin{aligned}\mathcal{R}(\rho)\Delta t = {} & \mathrm{tr}_{\mathrm{ph}}(\mathbb{P}_{>}U_I(t+\Delta t,t)\\ & \times|0_{ph}\rangle\rho\,\langle 0_{ph}|U_I^{\dagger}(t+\Delta t,t)\mathbb{P}_{>}).\end{aligned} \tag{12}$$

We call $\mathcal{R}(\rho)$ the non-normalized reset state [46]. Proceeding as in Refs. [46,47] and using perturbation theory, one obtains [51]

$$\begin{aligned}\mathcal{R}(\rho) = {} & \mathrm{Re}\, C_3(S_{13}^{-}\rho S_{23}^{+}+S_{23}^{-}\rho S_{13}^{+})\\ & +A_3(S_{13}^{-}\rho S_{13}^{+}+S_{23}^{-}\rho S_{23}^{+}).\end{aligned} \tag{13}$$

The normalized reset state is $\hat{\mathcal{R}}(\rho)\equiv\mathcal{R}(\rho)/\mathrm{tr}\mathcal{R}(\rho)$. By Eq. (11) the normalization of $\mathcal{R}(\rho)$ is such that $\mathrm{tr}_A\mathcal{R}(\rho)\Delta t$ is the probability for a photon detection at time $t+\Delta t$ when the (normalized) state of the atomic system at time t is ρ. The laser field does not appear in the reset state, just as in the case of a single atom [46,47], since its effect during the short time Δt is negligible.

By a simple calculation one checks that Eq. (13) can be written as

$$\mathcal{R}(\rho)=(A_3+\mathrm{Re}\, C_3)R_{+}\rho R_{+}^{\dagger}+(A_3-\mathrm{Re}\, C_3)R_{-}\rho R_{-}^{\dagger} \tag{14}$$

where

$$\begin{aligned}R_{+} = {} & (S_{13}^{-}+S_{23}^{-})/\sqrt{2}=|g\rangle\langle s_{13}|+|s_{13}\rangle\langle e_3|\\ & +(|s_{12}\rangle\langle s_{23}|-|a_{12}\rangle\langle a_{23}|)/\sqrt{2},\\ R_{-} = {} & (S_{13}^{-}-S_{23}^{-})/\sqrt{2}=|g\rangle\langle a_{13}|+|a_{13}\rangle\langle e_3|\\ & +(|s_{12}\rangle\langle a_{23}|+|a_{12}\rangle\langle s_{23}|)/\sqrt{2}.\end{aligned} \tag{15}$$

If ρ is a pure state, $\rho=|\psi\rangle\langle\psi|$ say, then $R_{\pm}\rho R_{\pm}^{\dagger}$ are also pure states. This decomposition of $\mathcal{R}(\rho)$ is advantageous for simulations of trajectories. As pointed out above, $A_3 \pm \mathrm{Re}\, C_3$ describe the decay rates of $|s_{13}\rangle$ and $|a_{13}\rangle$ to $|g\rangle$. The state $|e_3\rangle$ can decay both to $|s_{13}\rangle$ and $|a_{13}\rangle$, with respective decay rates $A_3 \pm \mathrm{Re}\, C_3$. The decay rate of the states $|s_{23}\rangle$ and $|a_{23}\rangle$ is A_3, and is the same as in the case of two independent atoms.

Simulation of a single trajectory. Starting at $t=0$ with a pure state, the state develops according to U_{cond} until the first emission at some time t_1, determined from w_1 in Eq. (10). Then the state is reset according to Eq. (13) to a new density matrix (which has to be normalized), and so on.

The decomposition of $\mathcal{R}(\rho)$ in Eq. (14) allows one, however, to work solely with pure states, which is numerically much more efficient. One can start with a pure state $|\psi\rangle$, develop it with U_{cond} until t_1 to the (non-normalized) $|\psi(t_1)\rangle$, reset to one of the pure states $R_\pm|\psi(t_1)\rangle/\|\cdot\|$ with relative probabilities given by the factors $A_3 \pm \mathrm{Re}\, C_3$ appearing in Eq. (14), and so on. The waiting time distributions are not changed by this procedure.

III. FLUORESCENCE JUMPS FOR TWO ATOMS

For a single atom in a V configuration, the existence of dark periods is due to two widely different time scales in the times between two subsequent photon emissions (cf., e.g., Refs. [32,44,45,35]). The smaller time scale T' is of the order of A_3^{-1}, while the larger time scale T'' is the inverse of the smallest eigenvalue of H_{cond}/i for a single atom. One can pick a time, $\tilde{T}$, say, with $T' \ll \tilde{T} \ll T''$, and if the time between two subsequent photon emissions is longer than $\tilde{T}$ one may then define this as a dark period. The mean duration of such dark periods is essentially independent of $\tilde{T}$, if chosen as above, and is given by T''. When the waiting time between two photons is less than $\tilde{T}$, the atom is said to be in a light period. The average intensity in a not too short light period is that of a driven two-level system, i.e., in our case the 1 and 3 levels.

If one has two independent, *noninteracting*, atoms, the combined fluorescence is just the sum of the individual contributions. When both atoms are in a dark period one has a dark period of the combined system. If only one atom is in a dark period, one observes a fluorescence period with intensity of that of a single two-level atom (single-intensity period), and if both atoms are radiating one observes a double-intensity period. However, due to fluctuations in the emission times the latter two periods are not sharply defined if the atoms are so close to each other that one cannot determine from which atom a particular photon came. To distinguish the periods therefore, one has to use an average photon intensity, obtained by means of an averaging time ΔT. This ΔT has to be large enough so that the photon intensity doubles when both atoms are not in a dark period. If, on the other hand, ΔT is chosen too large, one may overlook short fluorescence periods, and one will see more seemingly direct transitions between dark periods and double intensity periods (double jumps). The analytic treatment of fluorescence of two independent atoms is easily obtained from the single-atom case [36].

For two *dipole-interacting* atoms which are sufficiently close to each other, the photons cannot be attributed to a particular atom either, and one has to consider the two interacting atoms as a jointly radiating system. To be able to differentiate between different fluorescence phases one again has to average the photon numbers over a time interval ΔT to arrive at an intensity, and in the following we present the results of numerical simulations which have been obtained by the methods explained in Sec. II.

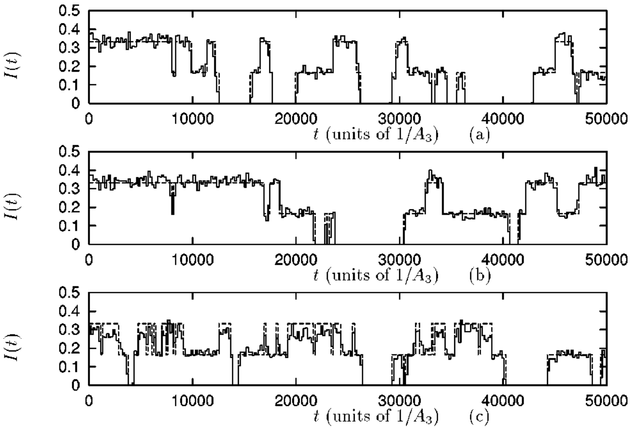

FIG. 3. Number of photons, $I(t)$, per time A_3^{-1} emitted by two dipole-interacting atoms, averaged over $\Delta T=190/A_3$, for $\Omega_2 = 0{,}01\, A_3$, $\Omega_3=0{,}5\, A_3$, and $\vartheta_3=\pi/2$. (a) $k_{31}r=10$, (b) $k_{31}r=5$, and (c) $k_{31}r=2$. The dashed curve indicates the fluorescence type (zero, single, and double intensity).

Atomic distances of a few wavelengths. Figure 3 shows the number of emitted photons per time A_3^{-1}, averaged over a time interval $\Delta T=190/A_3$. If the atomic distance is larger than a third of a wavelength of the fast transition, $r>\frac{1}{3}\lambda_{31}$, i.e., $k_{31}r>2$, one can clearly distinguish three types of fluorescence periods: dark periods (0), single-intensity periods (1), and double-intensity periods (2). For two noninteracting atoms these would correspond to radiation of no atom, one atom, and two atoms, respectively. However, in the case of interaction and small distance the system of two atoms radiates as a whole, and in general one cannot attribute periods 1 and 2 to radiation of individual atoms as in the noninteracting case.

The transition between the periods occurs rapidly but not instantaneously. The duration of the periods is long compared to the atomic time scale. From a sufficiently long trajectory one can obtain the average lengths of the periods, denoted by T_0, T_1, and T_2. The results are shown in Fig. 4. The Rabi frequency Ω_2 of laser 2, which drives the weak atomic 1-2 transitions, has been chosen in such a way that for independent atoms one has $T_0=2000/A_3$. As seen in Fig. 4, T_0 is essentially independent of the atomic distance and thus of the dipole-dipole interaction. In contrast to this, the two light periods are strongly distance dependent. In Fig. 4(a), T_2 varies between $1200/A_3$, and $2600/A_3$, and a similar behavior is also seen in Fig. 4(b). The curve for T_1 resembles that for T_2, except for smaller relative variation.

There is an interesting correspondence between the r dependence of T_1 and T_2 with that of $\mathrm{Re}\, C_3$. As seen from a comparison of Figs. 4 and 5, the variations with r of all three quantities seem to be in phase. For $\mathrm{Im}\, C_3$ the variation is out of phase, as seen from Eq. (6). Since $\mathrm{Re}\, C_3$ influences the decay rates of the two-atom systems, this in-phase behavior suggests that the variation in the lengths of periods 1 and 2

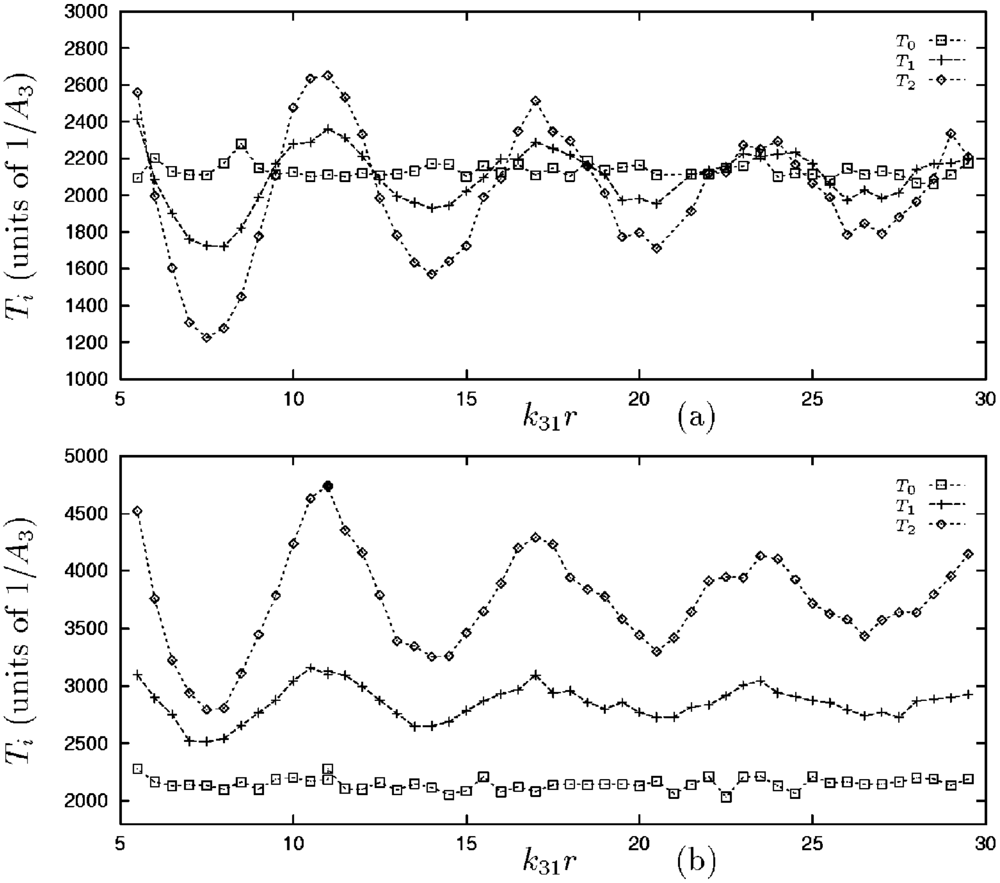

FIG. 4. T_0, T_1, and T_2 as a function of r for $\vartheta_3 = \pi/2$ and $\Delta T = 250/A_3$. (a) $\Omega_3 = 0.3 A_3$ and (b) $\Omega_3 = 0.6 A_3$. Ω_2 has been chosen such that for two noninteracting atoms one has $T_0 = 2000/A_3$.

are due to an r dependence of the decay rates.

Small atomic distances. Our simulations have shown, furthermore, that for $r < \frac{1}{4}\lambda_{31}$ the intensity in period 2 decreases and no longer reaches that of two simultaneously radiating independent atoms. For very small distance only periods 0 and 1 remain. The reason for this will be discussed below.

IV. INTERPRETATION OF RESULTS

For a single atom (in a V configuration as in Fig. 1) one can associate light and dark periods with certain atomic states and density matrices. During a dark period the atomic state rapidly approaches the eigenstate of $H_{\rm cond}$ with the smallest imaginary part of the eigenvalue [44,35,52]. This eigenstate is very close to $|2\rangle$, up to terms of order $\Omega_2 A_3/\Omega_3^2$ and Ω_2/Ω_3 [52]. Thus in a dark period, the atom can be regarded to be approximately in the state $|2\rangle$. During a light period the atom can be regarded to be in the equilibrium state (density matrix) of the 1-3 subsystem driven by Ω_3, again up to terms of the above orders [52]. A jump from one fluorescence period to the other corresponds to a transition between these atomic states, and such a transition is caused by laser 2. With $\Omega_2 = 0$ and $A_2 = 0$ there would be no transitions. This correspondence clearly carries over to the three fluorescence periods of two independent atoms.

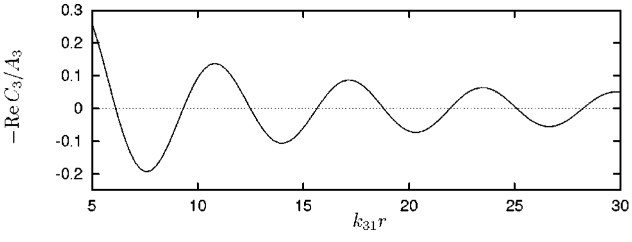

FIG. 5. $\mathrm{Re}\,C_3/A_3$ as a function of r for $\vartheta_3 = \pi/2$.

We are now going to suggest a similar correspondence between fluorescence periods and states for two dipole-interacting atoms. Figure 6 depicts the Dicke states of the two-atom system [see Eq. (8)]. Dashed arrows indicate the weak driving by laser 2, solid arrows indicate strong driving by laser 3 and decay, respectively. Now, for $\Omega_2 = 0$, i.e., no dashed arrows, the states in Fig. 6 decompose into three nonconnected subspaces, namely, one spanned by $|e_2\rangle$ and the two others spanned by the four inner and outer states, respectively:

$$\text{subspace } 0: \quad |e_2\rangle,$$

$$\text{subspace } 1: \quad |s_{12}\rangle, |a_{12}\rangle, |s_{23}\rangle, |a_{23}\rangle,$$

$$\text{subspace } 2: \quad |g\rangle, |s_{13}\rangle, |a_{13}\rangle, |e_3\rangle.$$

If the two atoms are in state $|e_2\rangle$, then each of them is in its dark state, and thus no photon can be emitted. For $\Omega_2 = 0$, the time development in subspace 1 is exactly the same as that for a system of two noninteracting atoms in the same subspace of states. This can be seen directly from Eq. (8),

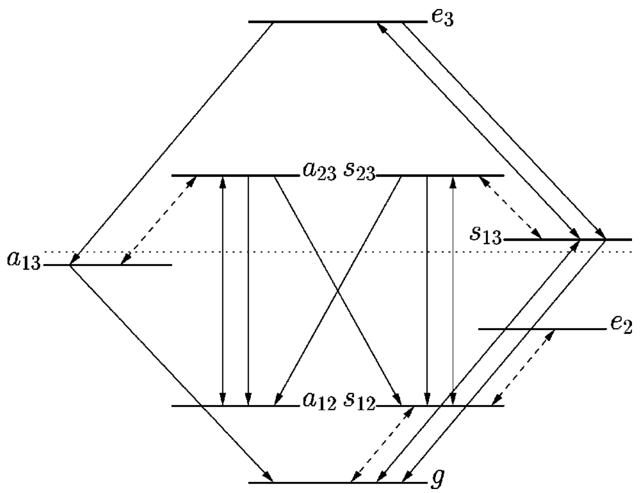

FIG. 6. Dicke states. The dashed and solid double arrows denote weak and strong driving, respectively. Simple arrows denote decays.

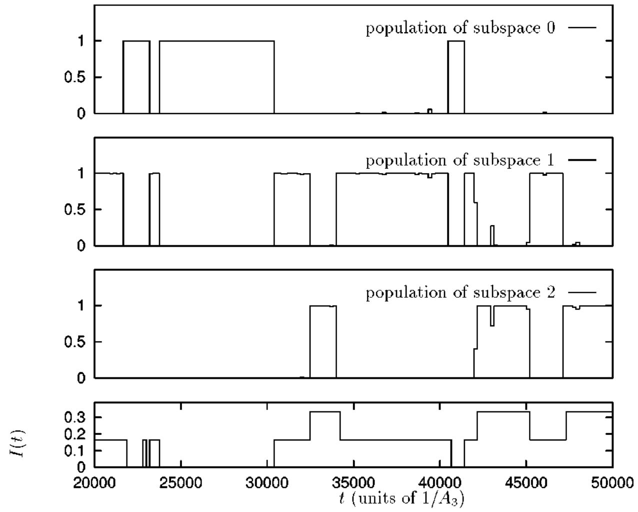

FIG. 7. Correspondence between fluorescence types and subspaces 0, 1, and 2 for $\Omega_2=0{,}01A_3$, $\Omega_3=0{,}5A_3$, and $\vartheta_3=\pi/2$. The atomic distance is chosen as $5\lambda_{31}/2\pi$.

and it is also physically obvious since two atoms can only interact via photon exchange if none of them is in the dark state $|2\rangle$. The photon rate for subspace 1 is therefore that of a single two-level atom with levels 1 and 3. Subspace 2 corresponds to the level scheme of a system of two dipole-dipole-interacting two-level atoms (with levels 1 and 3), as, for example, recently discussed in Ref. [26]. The photon rate of this system is, in good approximation, twice that of a single two-level atom, provided $r > \frac{1}{4}\lambda_{31}$. For smaller atomic distance the photon rate rapidly decreases to zero, due to the increasing level shifts.

If the two-atom system has been in one of the subspaces 0, 1, or 2, it will quickly approach the corresponding equilibrium state for driving Ω_3 and distance r. If $\Omega_2 \neq 0$ the additional weak driving will, from time to time and in analogy to a single V system, cause transitions between the three subspaces 0, 1, and 2, and each transition will correspond to a jump in the fluorescence.

Thus the fluorescence periods 0, 1, and 2 should correspond to (the equilibrium states of) the subspaces 0, 1, and 2, respectively. This is verified by the numerical evaluation in Fig. 7. In the lower part of Fig. 7 a particular realization of an intensity trajectory with alternating periods of fluorescence is plotted. The three upper curves show the populations of the three subspaces corresponding to this realization, obtained by the conditional Hamiltonian and reset matrix of Sec. II. The agreement between fluorescence periods 0, 1, and 2, and subspaces 0, 1, and 2 is striking. During dark periods the two-atom system is in the subspace 0, and similarly for periods 1 and 2.

This correspondence, however, depends to some extent on how large ΔT is chosen for the averaging of photon counts. If ΔT is chosen too large one can overlook some jumps between subspaces and some very short fluorescence periods. If ΔT is chosen too small there may be large intensity fluctuations, resulting in an incorrect determination of the different periods.

If the atomic distance decreases below $\frac{1}{4}\lambda_{31}$ the level shifts of $|s_{13}\rangle$ and $|a_{13}\rangle$ by $\mathrm{Im}\, C_3$ increase rapidly. This renders the driving by laser 3 within the subspace 2 much less efficient. Hence for very small atomic distances the driving is essentially restricted to subspace 1. This explains the vanishing of double-intensity periods for very small atomic distances.

V. DISCUSSION AND SUMMARY

Cooperative effects of two driven three-level atoms have been studied, where each individual atom can exhibit light and dark periods. The atoms were considered to be a fixed distance r apart. If r is of the order of a few wavelengths of the fluorescent light the individual photons are no longer attributable to a particular atom, and the two-atom system radiates as a whole, due to the dipole-dipole interaction. In addition to dark periods the system shows two types of light periods, one with fluorescence intensity as if only a single atom were radiating, and the other with double intensity.

We have proposed to study the mean durations T_1 and T_2 of the two types of light periods as a quantity sensitively depending on the dipole-dipole interaction and thus on the atomic distance. Experimentally and numerically these quantities are easily accessible.

We have performed fluorescence simulations for atomic separations of up to five wavelengths and have found oscillations in T_1 and T_2 of up to 40% in amplitude. The amplitude decreases with the atomic separation but the oscillations seem to continue for separations larger than five wavelengths. By simulations we have shown that the r dependence of T_1 and T_2 is in phase with that of the real part of the dipole-dipole constant. This is eminently reasonable since $\mathrm{Re}\, C_3$ directly influences the decay rates of the excited states of the two-atom system.

We have associated the three types of fluorescence periods with certain subspaces of states for the two atoms and to equilibrium states in these subspaces. The equilibrium states depend on the driving of the strong transition and on the distance. The weak driving then causes transitions between the subspaces. The transition rates depend not only on the weak driving, but also on the form of the respective equilibrium states and thus on the strong driving and on the atomic distance. In contrast to the mean durations of the light periods the mean duration of the dark periods is practically independent of r. This is intuitively quite clear, since in the dark state there is essentially no photon exchange and thus no induced dipole-dipole interaction.

To define fluorescence periods, one has to average the number of photon emissions over a time interval ΔT of a some finite length. Hence very short fluorescence periods are washed out and not observed, and this can lead to apparent direct transitions between double-intensity periods and dark periods, or, vice versa, so-called double jumps. Experimentally these were seen in Ref. [27]. The cooperative effects of up to 40% found by us in the duration of the single-intensity and double-intensity periods are noticeable. Therefore, we expect that the frequency for the appearance of double jumps is also modified by the dipole-dipole interaction, but we cannot predict whether the changes are of the two orders of magnitude reported in Ref. [27]. In addition, the system in Ref. [27] differs from the one considered here, and it is not obvious if and how our results would carry over to that system.

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