Linewidth of four-level microcavity lasers

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We use standard quantum laser theory to examine the linewidth of lasers with a four-level atomic gain medium as a function of the spontaneous emission rate into the lasing mode and the decay rates from the upper and lower lasing levels. We find that the laser linewidth ceases to exhibit the customary decrease with increased photon number when a critical photon number, proportional to the ratio of the decay rate from the lower lasing level to the rate of spontaneous emission into the lasing mode, is reached. This critical photon number would decrease as the size of the optical cavity is decreased. In addition we show that, in general, the laser linewidth depends critically on the ratio of the overall decay rates from the upper and lower lasing levels. [S1050-2947(99)00903-8]

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I. INTRODUCTION

It has long been understood that spontaneous decay rates are not immutable properties of atoms but rather can be influenced by the nature of the environment surrounding the atoms [1,2]. The cavity surrounding a gain medium in a laser system not only serves as an output coupler and feedback system for the electromagnetic radiation but also influences the spontaneous decay rates from the lasing levels [3,4]. The influence that the cavity structure has on the spontaneous decay rates is most pronounced when the cavity dimensions approach the wavelength of the lasing light. We term such a cavity a microcavity and so refer to microcavity lasers. Efforts to develop microcavities have focused on the use of semiconductor multilayer structures [5], whispering-gallery-mode semiconductor lasers [6], and photonic band-gap materials [7]. Initial interest in microcavity lasers was spurred by the prediction that if all of the spontaneous emission from the upper lasing level entered the lasing cavity mode, then there would be no laser threshold discernible from an examination of a plot of output power as a function of pump power [8,9]. Every pump event, in this situation, exits the gain medium as either stimulated or spontaneous emission into the lasing cavity mode and so there is no characteristic change in the derivative, signifying the onset of lasing, of the output power as a function of pump power. To date the experimental realizations of microcavity lasers have used semiconductor gain media [5,6,10]. The four-level atomic theory presented here is not intended as a detailed model of these semiconductor systems but rather to illustrate that interesting and unusual behavior results when significant spontaneous emission occurs into the lasing mode.

In this paper we examine the photon number fluctuations and laser linewidth in a laser with a four-level atomic gain medium as a function of the spontaneous emission rate into the lasing cavity mode and the decay rates from the upper and lower atomic levels. We employ standard Scully-Lamb laser theory [11,12] in the good-cavity limit. In particular, we find that the diagonal elements of the reduced density matrix for the field are exactly Poissonian for a four-level laser that decays from the upper lasing level only via spontaneous emission into the lasing cavity mode. Such a laser is often termed a $\beta=1$ laser, with β being the ratio of the spontaneous decay rate from the upper lasing level into the lasing mode to the overall decay rate from the upper lasing level. Rice and Carmichael found this result previously by a somewhat different method [13]. We predict the existence of a significant ''linewidth floor'' (i.e., that the linewidth ceases to decrease with increased photon number) in a laser in which the rate of spontaneous emission into the lasing mode is a significant fraction of the decay rate from the lower level of the lasing transition. The existence of the linewidth floor has not been noted in the traditional treatments of the laser linewidth [11,12,14–17] largely because it is significant in a regime that was not of practical interest at the time these treatments were developed. Finally, we show that the contribution of the linear gain coefficient to the laser linewidth depends critically on the decay rate from the upper lasing level to the decay rate from the lower lasing level.

II. EVOLUTION OF THE DENSITY MATRIX

We consider a model consisting of a four-level gain medium placed inside an optical cavity. The atomic level structure is depicted in Fig. 1. The gain medium is pumped from the ground level to a pump level. Atoms in the pump level are assumed to decay rapidly to the upper lasing level a, effectively pumping this level at a rate r. An undepleted pump approximation is made in obtaining the results of this paper. Level a is coupled to the lower lasing level b by the interaction with the optical field. The cavity decay rate is taken to be much smaller than the rates describing the atomic evolution. This is the so-called good-cavity limit. We take the field mode to be resonant with the atomic transition. Our model is exactly that used by Scully and Lamb [11,12] and Eq. (1) below, save for a change of notation and the assumption of a resonant atom-field interaction, is a transcription of Eq. (85) of Ref. [11]. The main contribution of this paper is an examination of the consequences of this relation in the regime where the spontaneous emission into the cavity mode is a significant fraction of the either or both of the overall decay rates of the atomic laser levels. In the number representation, the elements of the reduced density matrix describing the intracavity field satisfy the evolution equation

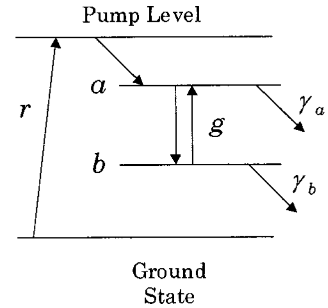

FIG. 1. Four-level atomic gain medium. The atomic levels a and b are coupled by the electromagnetic field of the cavity mode at a rate proportional to the coupling constant g. The upper lasing level is pumped at an effective rate r via rapid decay from a pump level. The decay rates from the atomic levels are denoted γ_a and γ_b.

$$\dot{\rho}_{nm}=\frac{-r(\gamma_C/2\gamma_b)[(n+1+m+1)+(\gamma_C/2\gamma_b)(n-m)^2]}{\gamma_a/\gamma_b+(\gamma_C/2\gamma_b)(1+\gamma_a/\gamma_b)(n+1+m+1)+(\gamma_C/2\gamma_b)^2(n-m)^2}\rho_{nm}$$
$$+\frac{r(\gamma_C/\gamma_b)\sqrt{nm}}{\gamma_a/\gamma_b+(\gamma_C/2\gamma_b)(1+\gamma_a/\gamma_b)(n+m)+(\gamma_C+2\gamma_b)^2(n-m)^2}\rho_{n-1,m-1}$$
$$+C\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}-\frac{1}{2}C(n+m)\rho_{nm}. \tag{1}$$

Here r is the pump rate, C is the cavity loss rate, γ_a is the atomic decay rate of the upper lasing level excluding spontaneous decay into the lasing mode, and γ_b is the decay rate of the lower lasing level. The parameter $\gamma_C=2g^2/\gamma$ can be interpreted as the spontaneous decay rate from the upper lasing level into the lasing cavity mode and arises in the formalism from the coupling of the atomic system to the cavity mode in the same way that the stimulated emission rate $\gamma_C\langle n\rangle$ does. Here the rate of decay of the off-diagonal elements of the atomic density matrix (i.e., the decay rate of the atomic coherence) is γ and the parameter g is the usual atom-field coupling parameter. A parameter commonly used to characterize microcavity lasers is β, the fraction of the total spontaneous emission from the upper lasing level that occurs into the laser mode. In our notation, this parameter would have the form

$$\beta=\frac{\gamma_C}{\gamma_C+\gamma_a}. \tag{2}$$

III. PHOTON STATISTICS OF A $\beta=1$ LASER

In some sense, as we show below, the ideal laser state can be reached by letting β approach unity. This possibility could be achieved by eliminating spontaneous emission out of the sides of the laser medium, thereby channeling all of the spontaneous emission into the laser mode. In our notation this is implemented by setting $\gamma_a=0$. The photon statistics are determined by the diagonal elements ρ_{nn} of the reduced density matrix for the field. These ρ_{nn} represent the probability that the intracavity field contains n photons. For a laser with $\beta=1$ the diagonal elements of the reduced density matrix for the field have the same form, for all values of the pump, that would be found in a coherent state. The off-diagonal elements, discussed in Sec. IV, are emphatically not those of a coherent state. The evolution of the diagonal elements of the reduced density matrix for a laser with $\beta=1$ is described by the particularly simple relation

$$\dot{\rho}_{nn}=-r\rho_{nn}-C(n)\rho_{nn}+r\rho_{n-1,n-1}+C(n+1)\rho_{n+1,n+1}. \tag{3}$$

The steady-state solution to Eq. (3) is

$$\rho_{nn}=\frac{1}{n!}\left(\frac{r}{C}\right)^n\rho_{00}. \tag{4}$$

This is of course a Poissonian distribution.

For this case, the average photon number and variance in average photon number are

$$\langle n\rangle=\sum_{n=0} n\rho_{nn}=r/C \tag{5}$$

and

$$\Delta n^2=\sum_{n=0}^{\infty}(n-\langle n\rangle)^2\rho_{nn}=r/C=\langle n\rangle. \tag{6}$$

Two important features of these relations should be noted. First, the average photon number is a linear function of the pump and so exhibits no characteristic turn-on at threshold. Second, the variance is equal to the mean for all values of the

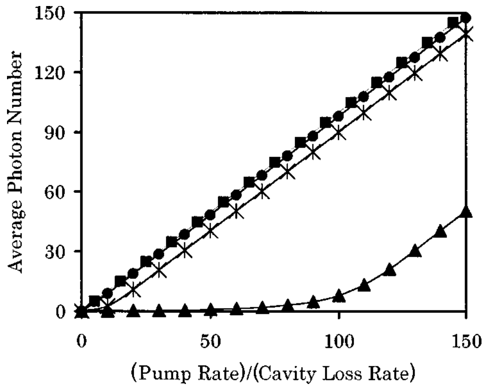

FIG. 2. Average photon number as a function of the pump rate scaled by the cavity loss rate for $\beta=1$ (rectangles), 0.5 (circles), 0.1 (asterisks), and 0.01 (triangles). Note that the laser ''threshold'' occurs roughly at the inverse of β. These plots (except the $\beta=1$ plot) were done using $\gamma_a/\gamma_b=0.01$.

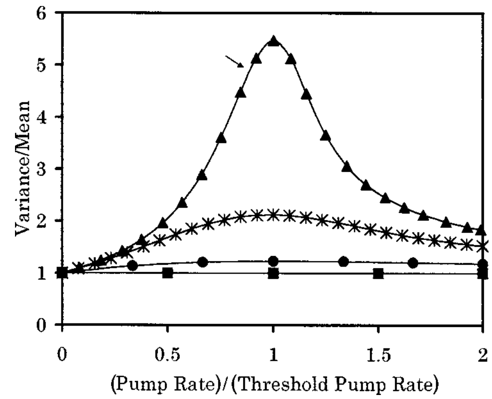

FIG. 3. Ratio of the variance in the photon number to the mean photon number as a function of the pump rate normalized to the threshold pump rate for $\beta=1$ (squares), 0.5 (circles), 0.1 (asterisks), and 0.01 (triangles). The fluctuation peak defines the threshold pump rate for the purposes of this plot. These plots (except the $\beta=1$ plot) were done using $\gamma_a/\gamma_b=0.01$.

pump. In an ordinary low-β laser the variance is greater than the mean and approaches the mean only far above threshold. In addition, the variance over the mean is quite large in a low-β laser in the threshold region. It is for these reasons that a $\beta=1$ laser is often called a thresholdless laser. We note that, even for $\beta=1$, in a three-level laser with the ground state being the lower lasing level, there remains a peak in the variance over the mean near threshold and there is a turn-on in the average photon number versus pump rate [10].

IV. PHOTON STATISTICS FOR ARBITRARY β

In this section we examine the photon statistics of a four-level micro-cavity laser as a function of β. Setting $n=m$ in Eq. (1) and using Eq. (2) to eliminate γ_C leads to the relation

$$\dot{\rho}_{nn}=\frac{-r(n+1)}{1/\beta+\gamma_a/\gamma_b+n(1+\gamma_a/\gamma_b)}\rho_{nn}-C(n)\rho_{nn}+\frac{rn}{1/\beta+\gamma_a/\gamma_b+(n-1)(1+\gamma_a/\gamma_b)}\rho_{n-1,n-1}+C(n+1)\rho_{n+1,n+1}. \tag{7}$$

Equation (7) can be solved in a steady state with the result

$$\rho_{nn}=\frac{\left(\frac{r/C}{1+\gamma_a/\gamma_b}\right)^n\left(\frac{1/\beta+\gamma_a/\gamma_b}{1+\gamma_a/\gamma_b}-1\right)!}{\left(\frac{1/\beta+\gamma_a/\gamma_b}{1+\gamma_a/\gamma_b}+n-1\right)!}\rho_{00}. \tag{8}$$

The average photon number and variance in average photon number have the forms

$$\langle n\rangle=\sum_{n=0}n\rho_{nn}=\frac{r}{C(1+\gamma_a/\gamma_b)}-\left[\frac{1/\beta+\gamma_a/\gamma_b}{1+\gamma_a/\gamma_b}-1\right](1-\rho_{00}) \tag{9}$$

and

$$\Delta n^2=\sum_{n=0}^{\infty}(n-\langle n\rangle)^2\rho_{nn}=\frac{r}{C(1+\gamma_a/\gamma_b)}-\left(\frac{1/\beta+\gamma_a/\gamma_b}{1+\gamma_a/\gamma_b}-1\right)\langle n\rangle\rho_{00}. \tag{10}$$

Plots of the average photon number and the variance in average photon number vs the scaled pump rate are displayed in Figs. 2 and 3 for a variety of values of β. Note that the low-threshold character of these lasers persists even for relatively small β on the order of 0.1 or so. That is, they are not artifacts of taking β to be exactly unity.

V. LASER LINEWIDTH

The laser linewidth can be determined by noting that the expectation value of the electric field in the lasing mode takes the form

$$\langle E(t)\rangle=\tfrac{1}{2}E_0\sum_{n=0}^{\infty}\rho_{n,n+1}\sqrt{n+1}e^{-i\nu t}+\text{c.c.} \tag{11}$$

where ν is the frequency of the laser field. The off-diagonal elements required to evaluate Eq. (11) are solutions to Eq. (1) with $m=n+1$. That is, they are solutions to the relation

$$\dot{\rho}_{n,n+1}=\frac{-r(\gamma_C/2\gamma_b)[(2n+3)+\gamma_C/2\gamma_b]}{\gamma_a/\gamma_b+(\gamma_C/2\gamma_b)(2n+3)+(\gamma_C/2\gamma_b)^2}\rho_{n,n+1}-\frac{1}{2}C(2n+1)\rho_{n,n+1}+\frac{r(\gamma_C/\gamma_b)\sqrt{n(n+1)}}{\gamma_a/\gamma_b+(\gamma_C/2\gamma_b)(1)(2n+1)+(\gamma_C/2\gamma_b)^2}\rho_{n-1,n}+C\sqrt{(n+1)(n+2)}\rho_{n+1,n+2}. \tag{12}$$

We assume a trial solution of the form

$$\rho_{n,n+1}=\sqrt{\rho_{nn}\rho_{n+1,n+1}}e^{-Dt/2}. \tag{13}$$

With this form, the expectation value of the electric field becomes

$$\langle E(t)\rangle=\langle E(0)\rangle\cos(\nu t)e^{-Dt/2}. \tag{14}$$

Thus D plays the role of the laser linewidth. The procedure outlined in this section is that developed by Scully and Lamb [11,12,18]. Below we employ this procedure and examine the results in parameter regimes not previously explored.

A. Linewidth of a $\beta=1$ laser

As noted above in Sec. III, a four-level laser with spontaneous emission only into the lasing mode produces a photon number probability distribution identical to that of a coherent state for all values of the pump. However, even in this case, a mixed-case density matrix describes the laser field and so the off-diagonal elements of the laser-field density matrix must differ from those of the pure-case density matrix of the coherent state. The coherent state of course exhibits no phase diffusion and so would formally be described by a relation such as Eq. (13) with $D=0$.

Using Eqs. (4) and (13) in Eq. (12) and setting $\gamma_a=0$ leads to the following simple expression for the linewidth of a $\beta=1$ laser:

$$D=C\frac{1+\gamma_C/2\gamma_b(2n+1)}{2n+1+\gamma_C/2\gamma_b}. \tag{15}$$

Note that in arriving at Eq. (15) it was implicitly assumed that D was independent of n. This inconsistency is typically resolved by replacing n by its average value. Such a replacement is approximately valid as long as the diagonal density matrix elements in Eq. (13) are sharply peaked about the average value. In the present case this condition holds when $\langle n\rangle\gg 1$. Making this replacement gives

$$D\approx C\frac{1+\gamma_C/2\gamma_b(2\langle n\rangle+1)}{2\langle n\rangle+1+\gamma_C/2\gamma_b}. \tag{16}$$

This result is valid for all values of the pump as long as the photon number distribution is sufficiently narrowly peaked about the average photon number. Some insight into the nature of the lasing linewidth can be garnered by examining Eq. (16) in several different limits.

In the limit that $(\gamma_C/\gamma_b)\langle n\rangle\ll 1$ the linewidth takes on the familiar form

$$D\approx\frac{C}{2\langle n\rangle}. \tag{17}$$

This limit corresponds to what is sometimes called third-order laser theory (but with $\beta=1$).

In the interesting regime $\gamma_C/\gamma_b\ll 1$ but $\gamma_c\langle n\rangle/\gamma_b$ not small compared to unity, Eq. (16) takes the approximate form

$$D\approx\frac{C}{2\langle n\rangle}(1+\gamma_C/\gamma_b\langle n\rangle). \tag{18}$$

The factor $1+(\gamma_C/\gamma_b)\langle n\rangle$ is roughly the number of times that a given pump event leads to emission from the upper lasing level into the cavity mode before the energy associated with that pump event exits the lasing system via decay from the lower lasing level. This interpretation follows from the fact that $\gamma_C\langle n\rangle$ is the stimulated absorption rate. A spontaneous emission event that places the atom in the lower lasing level is followed either by decay from the lower lasing level or by stimulated absorption followed by another emission event. Each reemission evidently introduces an additive broadening of amount $C/2\langle n\rangle$. The limiting result given as Eq. (18) agrees with results given (after translating to our notation) earlier in the literature [16,17] in the form

$$D=\frac{P_a}{P_a-P_b}\frac{C}{2\langle n\rangle} \tag{19}$$

only when γ_C/γ_b is negligible compared to unity. Here P_a and P_b are, respectively, the steady-state probabilities of finding an atom in the upper and lower lasing levels. Equation (19) contains the linewidth enhancement factor since the populations of the upper and lower lasing levels become closer in value as the field in the cavity grows. However, the earlier treatments did not, understandably, examine the case in which the spontaneous emission rate into the lasing mode becomes a significant fraction of the decay from the lower lasing level. Further, the linewidth given as Eq. (19) does not agree with the results given in Sec. V B for arbitrary decay rate from the upper lasing level in the case that the laser is significantly above threshold.

In the more extreme limit $\langle n\rangle\gg\gamma_b/\gamma_C$, the linewidth ceases to decrease with increased average photon number (recall that for the $\beta=1$ case the average photon number is simply the ratio of the pump rate to the cavity loss rate). In fact, in this limit, the linewidth tends to the constant value

$$D=C\frac{\gamma_C}{2\gamma_b}=\frac{\gamma_C}{2}\frac{C}{\gamma_b}. \tag{20}$$

We refer to this constant value as a linewidth floor. The linewidth floor is approached for photon numbers exceeding γ_b/γ_C. Recall that γ_C represents the rate at which energy enters the lasing mode due to spontaneous emission. The ratio C/γ_b is the fraction of that energy that exits the system as laser output. (The remainder exits the system via decay from the lower lasing level.) Enhancing spontaneous emission into the lasing mode (i.e., increasing γ_C) evidently raises the linewidth floor and reduces the average photon number for which the floor becomes important. The existence of a linewidth floor is essentially a reflection of the fact that the number of spontaneous emission events associated with a single pump event increases as the average photon number increases. This effect is illustrated in Figure 4.

To conclude this discussion we point out a rather remarkable formal feature of the linewidth of $\beta=1$ lasers. The only approximations involved in the procedure used in this section to determine the linewidth stem from the replacement of the number state quantum number n with the average photon number in going from Eq. (15) to Eq. (16). An examination of Eq. (15) shows that the linewidth is independent of n if

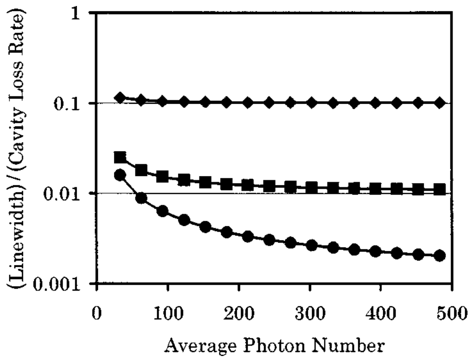

FIG. 4. Linewidth, scaled by the cavity loss rate, of a $\beta=1$ laser for $\gamma_C/2\gamma_b=0.1$ (diamonds), 0.01 (squares), and 0.001 (circles). Not that for larger values of $\gamma_C/2\gamma_b$ the linewidth floor is higher and the linewidth reaches this floor for smaller photon numbers.

$\gamma_c=2\gamma_b$. In this case the linewidth reduces to the cavity loss rate C. This is an exact result. That is,

$$\rho_{n,n+1}=\sqrt{\rho_{nn}\rho_{n+1,n+1}}e^{-Ct/2} = \frac{1}{\sqrt{n!(n+1)!}}\left(\frac{r}{C}\right)^{n+1/2}\rho_{00}e^{-Ct/2} \tag{21}$$

is an exact solution to the density matrix equation of motion for the case that $\beta=1$ and $\gamma_C=2\gamma_b$. That the linewidth takes on the value of the cavity decay rate when the coupling of the atoms to the field becomes comparable to the lower state decay rate is sensible. In this case there is considerable (randomly phased) spontaneous emission into the cavity mode and the cavity simply filters the very broad laser line. These features of the laser linewidth that emerge primarily for high-β lasers have not heretofore been addressed in the literature to our knowledge.

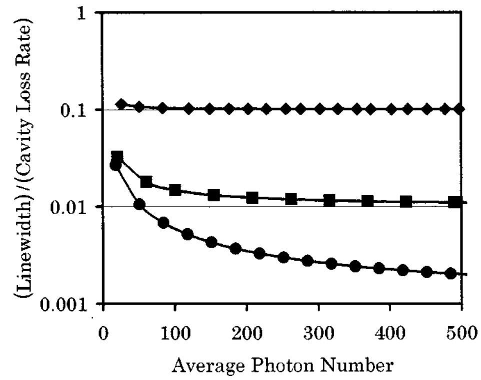

FIG. 5. Linewidth, scaled by the cavity loss rate, of a $\beta=0.01$ laser for $\gamma_C/2\gamma_b=0.1$ (diamonds), 0.01 (squares), and 0.001 (circles). Note that the curves in this figure are nearly identical to those displayed in Fig. 4 for a $\beta=1$ laser. The linewidth is much more sensitive to γ_C/γ_b and concomitantly to γ_a/γ_b than it is to β.

B. Linewidth of arbitrary β lasers

When the spontaneous decay rate γ_a from the upper laser level to all but the lasing cavity mode is nonzero, the form of the linewidth obtained by the procedure of the preceding subsection is decidedly more cumbersome. We find

$$D=\frac{r\frac{\gamma_C}{\gamma_b}\left(2n+3+\frac{\gamma_C}{2\gamma_b}\right)}{\frac{\gamma_a}{\gamma_b}+\frac{\gamma_C}{2\gamma_b}\left(1+\frac{\gamma_a}{\gamma_b}\right)(2n+3)+\left(\frac{\gamma_C}{2\gamma_b}\right)^2}-2r\left(\frac{(n+1)(n+2)}{\left[\frac{\gamma_a}{\gamma_C}+\left(1+\frac{\gamma_a}{\gamma_b}\right)(n+2)\right]\left[\frac{\gamma_a}{\gamma_C}+\left(1+\frac{\gamma_a}{\gamma_b}\right)(n+1)\right]}\right)^{1/2}+C(2n+1)$$
$$-\frac{2C\left(\frac{\gamma_C}{\gamma_b}\right)\left\{n(n+1)\left[\frac{\gamma_a}{\gamma_C}+\left(1+\frac{\gamma_a}{\gamma_b}\right)(n+1)\right]\left[\frac{\gamma_a}{\gamma_C}+\left(1+\frac{\gamma_a}{\gamma_b}\right)n\right]\right\}^{1/2}}{\left[\frac{\gamma_a}{\gamma_b}+\frac{\gamma_C}{2\gamma_b}\left(1+\frac{\gamma_a}{\gamma_b}\right)(2n+1)+\left(\frac{\gamma_C}{2\gamma_b}\right)^2\right]}. \tag{22}$$

The validity of this formal result depends upon the photon number distribution being sharply peaked so that the replacement $n\rightarrow\langle n\rangle$ can be made in Eq. (22) with negligible error.

We show in Fig. 5 that the significant linewidth floor discussed in Sec. V A still exists as β is reduced significantly. The plots for the same value of β differ in the ratio γ_C/γ_b. To maintain constant β as γ_C is decreased γ_a must also be decreased. The curves in Fig. 5 are for $\beta=0.01$ and are nearly identical to the curves shown in Fig. 4 for $\beta=1$. This similarity shows that the linewidth depends more critically on the ratios of γ_C/γ_b and γ_a/γ_b than it does on $\beta=\gamma_C/(\gamma_a+\gamma_C)$. Of course the pump rate required to obtain a given average photon number is much higher for the parameters used in Fig. 5 than for those used in Fig. 4.

Finally, we compare the linewidth found as Eq. (22) above to commonly used expressions for the linewidth found in the literature [11,12,14–19]. The linewidth is often quoted as $D=C/2\langle n\rangle$ or $(A+C)/4\langle n\rangle$. Here A is the linear gain coefficient given in this model as $r\beta/(1+\beta\gamma_a/\gamma_b)$. The latter commonly used expression for the linewidth also exhibits a linewidth floor since the linear gain rate and the average photon number are both linear functions of the pump rate far above threshold. The two linewidth relations have nearly the same value in the so-called third-order regime in which $(A-C)/C$ is small compared to unity. We find that the actual linewidth can be approximated by either of these expressions near threshold (in the third-order regime). However, significantly above threshold, the linewidth is not, in general, given

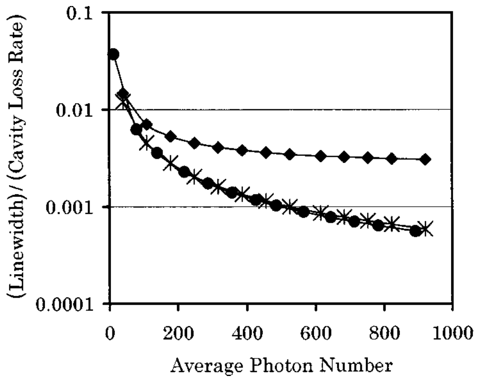

FIG. 6. Linewidths given by Eq. (22) (curve with asterisks), $C/2\langle n\rangle$ (circles), and $(A+C)/4\langle n\rangle$ (diamonds), as a function of the average photon number, compared for γ_a/γ_b small. In this case the linewidth is well approximated by the simple expression $C/2\langle n\rangle$ until the linewidth floor is reached. Note that the curve representing Eq. (22) begins to deviate from that representing $C/2\langle n\rangle$ for large photon numbers. This is an indication of the existence of the linewidth floor in Eq. (22).

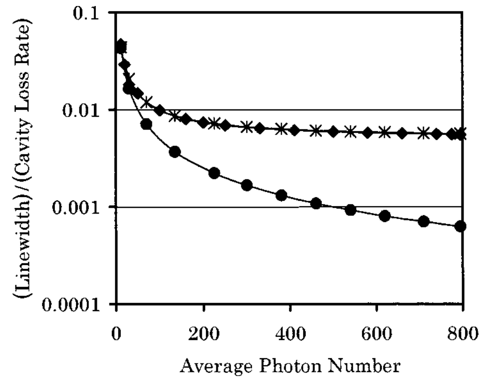

FIG. 7. Linewidths given by Eq. (22) (curve with asterisks), $C/2\langle n\rangle$ (circles), and $(A+C)/4\langle n\rangle$ (diamonds), as a function of the average photon number, compared for $\gamma_a/\gamma_b=1$. Note that in this case the linewidth resulting from Eq. (22) and the simple expression $(A+C)/4\langle n\rangle$ are indistinguishable for the range and resolution of the plot.

by either of the two results but rather takes on a form critically dependent on ratio of the upper and lower level decay rates. We find that for γ_a/γ_b much less than unity, the linewidth is well approximated by $C/2\langle n\rangle$ until the linewidth floor is reached. For $\gamma_a/\gamma_b=1$ (the choice made in the series of papers by Scully and Lamb) the linewidth is, as pointed out by Scully and Zubairy [19], rather well approximated by $(A+C)/4\langle n\rangle$ for all values of the photon number above threshold. These results are illustrated in Figs. 6 and 7. Intermediate choices for the ratio γ_a/γ_b give rise to behavior intermediate between the commonly used linewidth expressions. We find this result intriguing. The extent to which the linear gain contributes to the linewidth is governed by the ratio of the decay rates from the upper and lower lasing levels. If there is no decay from the upper lasing level then the contribution from the linear gain term is reduced to that of the loss reservoir. Apparently, the dephasing of the atomic dipole introduced by random emission events from the upper lasing level adds frequency noise to the laser field.

VI. CONCLUSIONS

We have shown that the laser linewidth shows significant departures from the $1/\langle n\rangle$ dependence if the rate of spontaneous emission γ_C into the lasing mode is appreciable compared to the decay rate γ_b from the lower lasing level. In fact, the linewidth ceases to decrease for values of $\langle n\rangle$ on the order of γ_b/γ_C. Lasers with small cavities tend to have large coupling between the lasing transition and the dominant lasing mode and consequently can have larger values of γ_C than macroscopic lasers. Most microcavity lasers either built or being considered are semiconductor lasers with features not adequately modeled by the four-level atomic structure considered in this paper. However, we would expect that some features similar to the predictions of this paper would be found in the semiconductor systems.

We find that the linewidth cannot, in general, be approximated by $(A+C)/4\langle n\rangle$. The linewidth is well modeled by this relationship only when the ratio γ_a/γ_b is unity. As γ_a/γ_b decreases the linewidth tends towards the significantly smaller (when the laser is well above threshold) value $C/2\langle n\rangle$ until the linewidth floor is reached.

ACKNOWLEDGMENT

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