COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on ''Geometric phase in coupled neutron interference loops''

Apoorva G. Wagh*
Bhabha Atomic Research Centre, 86 Dhruva, Mumbai 400085, India

(Received 4 December 1998)

Two interferometrically split and recombined subbeams which are in the same quantum state are treated as mutually orthogonal states by Hasegawa *et al.* [Phys. Rev. A **53**, 2486 (1996)]. This conceptual error has created the illusion of a geometric phase in U(1) evolutions. [S1050-2947(99)04602-8]

PACS number(s): 03.75.Dg, 03.65.Bz, 07.60.Ly, 61.12.Ld

Hasegawa *et al.* performed a two-loop interference experiment [1] with unpolarized neutrons using a four-plate perfect crystal interferometer. Employing U(1) Hamiltonians in the form of a scalar phase shifter and a partial absorber, they observed the expected interference effects for the U(1) evolution. Using an ''analogy'' with the up-down-spin superposition, Hasegawa *et al.* interpreted this experiment as an observation of a ''geometric phase.'' In this Comment, I show that their interpretation is not correct.

To elucidate the proposed theme of this experiment, let us examine the evolution of the normalized wave function

$$|\Psi\rangle=\frac{e^{i\chi_I}|\uparrow\rangle+\sqrt{\mathrm{T}}e^{i\chi_{\mathrm{II}}}|\downarrow\rangle}{\sqrt{1+T}},$$

obtained by superposing the up- and down-spin states of a spin-1/2 particle. As the phases of the two constituent states are varied for a fixed intensity attenuation $T=\tan^2\alpha/2$, say, of the down state, the spin traces an arc spanning an azimuthal angle $\theta=\Delta\chi_{\mathrm{II}}-\Delta\chi_{\mathrm{I}}$ on a cone of polar angle α on the unit sphere of directions. The phase acquired by the initial state $|\Psi(0)\rangle$ in this evolution is prescribed by the Pancharatnam connection [2–5] as

$$\phi=\arg\langle\Psi(0)|\Psi\rangle=\frac{\Delta\chi_I+\Delta\chi_{\mathrm{II}}}{2}-\arctan\left(\tan\frac{\theta}{2}\cos\alpha\right),$$

comprising a dynamical component [6,7]

$$\Phi_D=-i\int\langle\Psi|\dot{\Psi}\rangle dt=\frac{\Delta\chi_{\mathrm{I}}+T\Delta\chi_{\mathrm{II}}}{1+T}=\frac{\Delta\chi_{\mathrm{I}}+\Delta\chi_{\mathrm{II}}}{2}-\frac{\theta}{2}\cos\alpha$$

and a geometric component [2,4,8–10]

$$\beta=\phi-\Phi_D=\frac{\theta}{2}\cos\alpha-\arctan\left(\tan\frac{\theta}{2}\cos\alpha\right)=-\frac{\Omega}{2}.$$

Here Ω denotes the solid angle spanned by the closed curve obtained by joining the ends of the arc traced on the unit spin sphere with the shorter geodesic [4,9,11], i.e., the great circle arc. Thus even when the evolution is noncyclic, i.e., the states $|\Psi(0)\rangle$ and $|\Psi\rangle$ are distinct, the total phase ϕ as well as its dynamical and geometric components are well delineated. We note that the $|\uparrow\rangle$ and $|\downarrow\rangle$ states being mutually orthogonal, add only in intensity so that the superposed beam intensity remains proportional to $1+T$ regardless of their phases. With a variation in their relative phase, the spin of their superposed state evolves on the unit sphere producing the solid angle-dependent geometric phase.

For the unpolarized incident beam in the experiment [1] of Hasegawa *et al.*, however, the split subbeams on paths I and II differ only in momentum and U(1) phase. Hence over the region of superposition, they produce *spatial* interference fringes. This cosinusoidal intensity variation has the argument $[(\mathbf{k}_{\mathrm{I}}-\mathbf{k}_{\mathrm{II}})\cdot\mathbf{r}+\chi_{\mathrm{I}}'-\chi_{\mathrm{II}}']$, defined completely by the differences between the wave vectors and phases. A change in the phase difference merely *shifts* the fringe pattern. The analyzer slab of the interferometer combines the subbeams to produce the superposed (O and H) beams [9]. The two subbeam states constituting each superposed beam are identical in all respects, differing only in phase. In the forward diffracted (O) beam studied in Ref. [1], both the constituent subbeam states $|\Psi_{\mathrm{I}}\rangle$ and $|\Psi_{\mathrm{II}}\rangle$ are identical to the incident state $|\Psi_0\rangle$ in momentum, energy and spin. In contrast to the mutually orthogonal subbeam rays ($\langle\uparrow|\downarrow\rangle=0$) in up-down-spin superposition, the two normalized subbeam rays in the experiment [1] are identical ($\langle\psi_{\mathrm{I}}|\psi_{\mathrm{II}}\rangle=1$). Hence the two subbeams add coherently in *amplitude* and the superposed beam intensity varies as $1+T+2\sqrt{T}\cos(\chi_{\mathrm{II}}'-\chi_{\mathrm{I}}')$, as observed in a multitude of experiments (see, e.g., Ref. [12]). A variation of the phases χ_{I}' and χ_{II}' therefore leaves the superposed normalized wave function

$$|\Psi\rangle=\frac{e^{i\chi_{\mathrm{I}}'}+\sqrt{T}e^{i\chi_{\mathrm{II}}'}}{\sqrt{1+T+2\sqrt{T}\cos(\chi_{\mathrm{II}}'-\chi_{\mathrm{I}}')}}|\Psi_0\rangle=e^{i\phi'}|\Psi(0)\rangle,$$

*Electronic address: nintsspd@magnum.barc.ernet.in latched to the incident state, changing only its phase ϕ'. That the superposed state remains stationary is also evident from the full interference contrast it produced [1] with the reference subbeam from another path, which should belong to ''another subspace'' according to Hasegawa *et al.* The phase

$$\phi' = \frac{\Delta\chi_{\mathrm{I}}' + \Delta\chi_{\mathrm{II}}'}{2} - \arctan\frac{\cos\alpha\,\sin[(\Delta\chi_{\mathrm{II}}' - \Delta\chi_{\mathrm{I}}')/2]}{\cos[(\Delta\chi_{\mathrm{II}}' - \Delta\chi_{\mathrm{I}}')/2] + \sin\alpha\,\cos[(\chi_{\mathrm{II}}' - \chi_{\mathrm{I}}' + \chi_{\mathrm{II}}'^{0} - \chi_{\mathrm{I}}'^{0})/2]},$$

differs from ϕ above in the sinα term which arises from the cross term $\sqrt{T}\{\exp[i(\chi_{\mathrm{II}}' - \chi_{\mathrm{I}}'^{0})] + \exp[i(\chi_{\mathrm{I}}' - \chi_{\mathrm{II}}'^{0})]\}$ between the complex conjugate of the initial amplitude of either subbeam and the final amplitude of the other subbeam. In the up-down-spin superposition, this cross term vanishes since the two subbeams are mutually orthogonal. Hasegawa *et al.* [1] observed the phase for the trivial case, termed the ''cyclicity condition,'' $\Delta\chi_{\mathrm{II}}' - \Delta\chi_{\mathrm{I}}' = 2\pi$ wherein the phases ϕ' and ϕ above happen to be equal. The geometric phase does not require a cyclic evolution [4,5]. To establish the claimed ''evolution'' [1] of the superposed state, the experiment ought to be performed by varying the phase difference $\Delta\chi_{\mathrm{II}}' - \Delta\chi_{\mathrm{I}}'$ continuously. According to Hasegawa *et al.*, the evolution would then be ''noncyclic'' and a phase varying as ϕ above should be observed with an appropriately reduced interference contrast due to the noncyclicity [5,13]. From the neutron interferometric data produced in the 1970's by the Vienna-Dortmund group and other groups (see, e.g., Ref. [9]), however, it is clear that such an experiment shall observe the phase ϕ' above with full interference contrast. The ''analogy'' with the up-down-spin superposition professed by Hasegawa *et al.* is therefore not appropriate.

Any Poincare spherelike ray space has the fundamental property that every pair of diametrically opposite points therein represents mutually orthogonal states. This implies that when superposed, such states must add only in intensity, since their cross term vanishes. It is only for two orthogonal states that the superposed state evolves on a cone about their axis in the ray space when their relative phase is varied. Since the states $|\Psi_{\mathrm{I}}\rangle$ and $|\Psi_{\mathrm{II}}\rangle$ in the experiment are not mutually orthogonal but identical, placing them at opposite poles on the two-sphere ray space (Fig. 2 in Ref. [1]) amounts to a conceptual error.

For the superposed beam in the experiment [1], the state and hence the density operator remain stationary in the ray space throughout the evolution. The evolution thus being U(1), is cyclic all through and the phase produced is scalar and hence wholly dynamical [14], $\phi' \equiv \Phi_D'$. Geometric phase which requires a ray-space evolution through noncommuting density operators [9,11,15] vanishes identically here. Hasegawa *et al.* mistake $|\Psi_{\mathrm{I}}\rangle$ and $|\Psi_{\mathrm{II}}\rangle$ for orthogonal states (Fig. 2 in Ref. [1]) and misinterpret the observed purely dynamical phase as geometric phase.

Hasegawa *et al.* [1] appear to have incorrectly cited the neutron polarimetric measurement [16] of Weinfurter and Badurek as an experimental demonstration of geometric phase for noncyclic evolutions. For polarized neutrons in a uniformly rotating magnetic field, Weinfurter and Badurek mistook the rotation angle of the field for the noncyclic geometric phase and claimed to have measured [16] this ''phase.'' Rakhecha and I [17] delineated the correct noncyclic phase for these evolutions, propounded a polarimetric method for observing noncyclic phases and showed that the polarimetric experiment [16] did not constitute a phase measurement. The noncyclic phase for neutrons was observed in an interferometric experiment [13] whose results are in close agreement with theory [5].

To conclude, the phase observed in the coupled loops of a four-plate neutron interferometer [1] is U(1), i.e., wholly dynamical. Contrary to the claim made by Hasegawa *et al.*, the experiment did not show a geometric phase.

[1] Y. Hasegawa, M. Zawisky, H. Rauch, and A.I. Ioffe, Phys. Rev. A **53**, 2486 (1996).
[2] S. Pancharatnam, Proc.-Indian Acad. Sci., Sect. A **44**, 247 (1956).
[3] M. V. Berry, J. Mod. Opt. **34**, 1401 (1987).
[4] J. Samuel and R. Bhandari, Phys. Rev. Lett. **60**, 2339 (1988).
[5] A. G. Wagh and V. C. Rakhecha, Phys. Lett. A **197**, 107 (1995).
[6] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).
[7] A. G. Wagh, Phys. Lett. A **146**, 369 (1990); A. G. Wagh and V. C. Rakhecha, *ibid.* **148**, 17 (1990).
[8] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[9] A. G. Wagh and V. C. Rakhecha, Prog. Part. Nucl. Phys. **37**, 485 (1996).
[10] R. Bhandari, Phys. Rep. **281**, 1 (1997).
[11] V. C. Rakhecha and A. G. Wagh, Pramana, J. Phys. **46**, 315 (1996).
[12] H. Rauch and J. Summhammer, Phys. Lett. **104A**, 44 (1984).
[13] A. G. Wagh, V. C. Rakhecha, P. Fischer, and A. Ioffe, Phys. Rev. Lett. **81**, 1992 (1998).
[14] A. G. Wagh and V. C. Rakhecha, Phys. Rev. Lett. **78**, 1399 (1997).
[15] A. G. Wagh, V. C. Rakhecha, J. Summhammer, G. Badurek, H. Weinfurter, B. E. Allman, H. Kaiser, K. Hamacher, D. L. Jacobson, and S. A. Werner, Phys. Rev. Lett. **78**, 755 (1997).
[16] H. Weinfurter and G. Badurek, Phys. Rev. Lett. **64**, 1318 (1990).
[17] A. G. Wagh and V. C. Rakhecha, Phys. Lett. A **197**, 112 (1995).