Phase conjugation of multicomponent Bose-Einstein condensates

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We consider a trapped multicomponent atomic Bose-Einstein condensate, concentrating specifically on condensates in the hyperfine ground state $F=1$, where spin exchange collisions result in a transfer of population between $m=0$ and $m=\pm 1$ internal states. Drawing an analogy with the optical situation, we show that this system can be regarded as a matter-wave analog of optical multiwave mixing. This opens up the way to realize matter-wave phase conjugation, whereby an incident atomic beam can be ''time reversed.'' In addition, matter-wave phase conjugation also offers novel diagnostic tools to study the coherence properties of condensates, as well as to measure the relative scattering lengths of hyperfine sublevels. [S1050-2947(99)03602-1]

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I. INTRODUCTION

The experimental observation of Bose-Einstein condensation in low density atomic vapors [1] has triggered a flurry of theoretical activity [2]. Theoretical predictions on the condensate dynamics, ground-state population, and spectrum of elementary excitations have been made and are in excellent agreement with experiments.

At the same time, further experimental advances have led to the realization of multicomponent condensates, both in ^{87}Rb and in ^{23}Na. In the first case, sympathetic cooling, together with a fortuitous coincidence in the scattering lengths of the spin states $|F=1,m=1\rangle$, $|F=2,m=2\rangle$ led to the coexistence of both components in a magnetic trap [3]. A multicomponent condensate was also achieved with the three hyperfine ground-state components of sodium in a far-off-resonant dipole trap [4]. These results, along with further experiments involving condensates in double well potentials [5], have now led to considerable theoretical work on the static and dynamic properties of multicomponent condensates, including studies of their true ground state [6], analyses of the elementary excitations' spectrum, and the determination of their instability regions [7]. In addition, multicomponent condensates open up the way to novel schemes to launch vortices and permanent currents [8] and to the study of novel phenomena such as vector and quadrupole spin wave modes, the topological and energetic instabilities of doubly quantized singular vortices [9], etc. The two-body interactions characteristic of spin-1 condensates can lead to intercomponent coupling via two-body collisions.

In the zero-temperature limit, a q-component Bose-Einstein condensate can be thought of as a q-mode system, whereby the various modes are coupled by two-body (and possibly higher-order) collisions which result in the exchange of particles between these modes. As such, they correspond to a situation quite similar to that of multiwave mixing in nonlinear optics. A main difference is of course that in the case of matter waves, the coupling is due to the collisions, which find their origin in the electromagnetic interaction between the atoms. Collisions can then be thought of as the effective atom-atom interaction resulting from the elimination of the electromagnetic field from the system dynamics. This is to be contrasted with the optical case, where multimode mixing relies on the interaction of the electromagnetic field with a common atomic sample whose dynamics is traced over. In that case, it is the elimination of the material dynamics that results in an effective field-field coupling. This observation is the origin of nonlinear atom optics, which is the matter-wave equivalent of nonlinear optics.

A close analogy can easily be established between the dynamics of a spin-1 condensate as realized in sodium experiments and the situation of degenerate four-wave mixing in optics, as we demonstrate explicitly in this paper. In particular, for situations where the $m=0$ state is macroscopically populated while the $m=\pm 1$ states are weakly excited, one can think of the first state as a ''pump'' or ''central'' mode, while $m=\pm 1$ form side modes, which are coupled via the pump, leading to the familiar effects of degenerate four-wave mixing, including phase conjugation. This is the effect that we study in detail in this work. We then show how matter-wave phase conjugation can be used as a diagnostic tool to study the coherence properties of Schrödinger fields, as well as the relative scattering lengths of the states involved.

Matter-wave phase conjugation has previously been studied, but in a situation where the coupling between the partial matter waves was induced by the near-resonant electric dipole-dipole interaction [10]. As such, it relied explicitly on having a substantial population of electronically excited atoms, and the incoherent effects of spontaneous emission rapidly destroyed the coherent wave coupling responsible for phase conjugation. In contrast, the situation with a condensate of dipole-trapped sodium atoms does not suffer from this drawback: since we are considering ground-state atoms in a far-off-resonant trap with hyperfine levels coupled primarily via ground-state collisions, spontaneous emission is certainly negligible. In addition, the fact that the atoms are in a trap changes the situation somewhat from the free-space geometry considered in our earlier work, since the atomic sample can easily be tightly confined in the transverse dimensions and hence does not suffer from free-space diffraction. Our main result is to demonstrate that a trapped condensate can then be used as a phase-conjugate mirror for a weak atomic beam, thereby effectively ''time-reversing'' it.

Section II describes our physical model, and derives the coupled-wave equations for the three components of the condensate in the Hartree regime. This is applied in Sec. III to the discussion of matter-wave phase conjugation in two-dimensional atomic traps. We concentrate explicitly on the undepleted pump regime, and show how the phase-conjugate signal depends explicitly on the relative scattering lengths of the hyperfine levels involved. Finally, the possible experimental verification of our predictions, as well as a summary and outlook, are given in Sec. IV.

II. PHYSICAL MODEL

We consider a condensate of ^{23}Na atoms in their $F=1$ hyperfine ground state, with three internal atomic states $|F=1,m=-1\rangle$, $|F=1,m=0\rangle$, and $|F=1,m=1\rangle$ of degenerate energies in the absence of magnetic fields. It is described by the three-component vector Schrödinger field

$$\mathbf{\Psi}(\mathbf{r},t)=\{\Psi_{-1}(\mathbf{r},t),\Psi_0(\mathbf{r},t),\Psi_1(\mathbf{r},t)\}, \tag{1}$$

which satisfies the bosonic commutation relations

$$[\Psi_i(\mathbf{r},t),\Psi_j^\dagger(\mathbf{r}',t)]=\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'). \tag{2}$$

Accounting for the possibility of two-body collisions, its dynamics is described by the second-quantized Hamiltonian

$$\mathcal{H}=\int d\mathbf{r}\,\mathbf{\Psi}^\dagger(\mathbf{r},t)H_0\mathbf{\Psi}(\mathbf{r},t) + \int \{d\mathbf{r}\}\mathbf{\Psi}^\dagger(\mathbf{r}_1,t)\mathbf{\Psi}^\dagger(\mathbf{r}_2,t)V(\mathbf{r}_1-\mathbf{r}_2)\mathbf{\Psi}(\mathbf{r}_2,t)\mathbf{\Psi}(\mathbf{r}_1,t), \tag{3}$$

where the single-particle Hamiltonian is

$$H_0=\mathbf{p}^2/2M+V_{\text{trap}} \tag{4}$$

and the trap potential is of the general form

$$V_{\text{trap}}=\sum_{m=-1}^{+1}U(\mathbf{r})|F=1,m\rangle\langle F=1,m|. \tag{5}$$

Here $\mathbf{p}$ is the center-of-mass momentum of the atoms of mass M and $U(\mathbf{r})$, the effective dipole trap potential for atoms in the $|1,m\rangle$ hyperfine state, is independent of m for a nonmagnetic trap.

The general form of the two-body interaction $V(\mathbf{r}_1-\mathbf{r}_2)$ has been discussed in detail in Refs. [9,11]. We reproduce its main features for the sake of clarity. Consider situations where the hyperfine spin $F_i=1$ of the individual atoms is preserved. We label the hyperfine states of the combined system of hyperfine spin $\mathbf{F}=\mathbf{F}_1+\mathbf{F}_2$ by $|f,m\rangle$ with $f=0,1,2$ and $m=-f,\ldots,f$. In the shapeless approximation, it can then be shown that the two-body interaction is of the general form [9]

$$V(\mathbf{r}_1-\mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2)\sum_{f=0}^{2}\hbar g_f\mathcal{P}_f, \tag{6}$$

where

$$g_f=4\pi\hbar a_f/M, \tag{7}$$

$\mathcal{P}_f\equiv\Sigma_m|f,m\rangle\langle f,m|$ is the projection operator which projects the pair of atoms into a total hyperfine f state, and a_f is the s-wave scattering length for the channel of total hyperfine spin f. For bosonic atoms only even f states contribute, so that

$$\begin{aligned}V(\mathbf{r}_1-\mathbf{r}_2)&=\hbar\delta(\mathbf{r}_1-\mathbf{r}_2)(g_2\mathcal{P}_2+g_0\mathcal{P}_0)\\&=\frac{\hbar}{2}\delta(\mathbf{r}_1-\mathbf{r}_2)(c_0+c_2\mathbf{F}_1\cdot\mathbf{F}_2).\end{aligned} \tag{8}$$

In this expression,

$$\begin{aligned}c_0&=2(g_0+2g_2)/3,\\c_2&=2(g_2-g_0)/3.\end{aligned} \tag{9}$$

Substituting this form of $V(\mathbf{r}_1-\mathbf{r}_2)$ into the second-quantized Hamiltonian (3) leads to

$$\begin{aligned}\mathcal{H}=\sum_m&\int d\mathbf{r}\Psi_m^\dagger(\mathbf{r},t)\left[\frac{\mathbf{p}^2}{2M}+U(\mathbf{r})\right]\Psi_m(\mathbf{r},t)\\&+\frac{\hbar}{2}\int d\mathbf{r}\{(c_0+c_2)[\Psi_1^\dagger\Psi_1^\dagger\Psi_1\Psi_1+\Psi_{-1}^\dagger\Psi_{-1}^\dagger\Psi_{-1}\Psi_{-1}\\&+2\Psi_0^\dagger\Psi_0(\Psi_1^\dagger\Psi_1+\Psi_{-1}^\dagger\Psi_{-1})]+c_0\Psi_0^\dagger\Psi_0^\dagger\Psi_0\Psi_0\\&+2(c_0-c_2)\Psi_1^\dagger\Psi_1\Psi_{-1}^\dagger\Psi_{-1}\\&+2c_2(\Psi_1^\dagger\Psi_{-1}^\dagger\Psi_0\Psi_0+\text{H.c.})\}.\end{aligned} \tag{10}$$

This form of the Hamiltonian is quite familiar in quantum optics, where it describes four-wave mixing between a pump beam and two side modes, which are identified with the field operators Ψ_0 and $\Psi_{\pm1}$ in the present situation. Specifically, we observe that the three terms in the two-body Hamiltonian which are quartic in one of the field operators only, i.e., of the form $\Psi_i^\dagger\Psi_i^\dagger\Psi_i\Psi_i$, can be readily interpreted as self-defocusing terms, corresponding to the fact that the two-body potential is, for a positive scattering length and a scalar field, analogous to a defocusing cubic nonlinearity in optics. The terms involving two ''modes,'' i.e., of the type $\Psi_i^\dagger\Psi_i\Psi_j^\dagger\Psi_j$, conserve the individual mode populations of the modes and simply lead to phase shifts. Finally, the terms involving the central mode Ψ_0 and *both* side modes are the contributions of interest to us, since they correspond to a redistribution of atoms between the ''pump'' mode Ψ_0 and the side modes $\Psi_{\pm1}$, e.g., by annihilating two atoms in the central mode and creating one atom each in the side modes. This is the kind of interaction that leads to phase conjugation in quantum optics, except that in that case the modes in question are modes of the Maxwell field instead of the Schrödinger field. Note also that a similar mechanism is at the origin of amplification in the collective atom recoil laser (CARL) [12,13], except that in that latter case, the Schrödinger field mode coupling is induced by optical transitions in the atoms.

In the Hartree approximation, which is well justified for condensates at $T=0$, the many-body problem reduces to an effective single-particle problem for the Hartree wave function $\phi_m(\mathbf{r},t)$. It is easily shown that its dynamics is governed by the system of coupled nonlinear Schrödinger equations [14]

$$i\dot{\phi}_{-1}(\mathbf{r},t)=\frac{1}{\hbar}\left[\frac{\mathbf{p}^2}{2M}+U(\mathbf{r})\right]\phi_{-1}+N\{c_2\phi_0\phi_0\phi_1^\star+[(c_0+c_2)\times(|\phi_{-1}|^2+|\phi_0|^2)+(c_0-c_2)|\phi_1|^2]\phi_{-1}\},$$

$$i\dot{\phi}_0(\mathbf{r},t)=\frac{1}{\hbar}\left[\frac{\mathbf{p}^2}{2M}+U(\mathbf{r})\right]\phi_0+N\{c_0|\phi_0|^2\phi_0+(c_0+c_2)\times(|\phi_{-1}|^2+|\phi_1|^2)\phi_0+2c_2\phi_1\phi_{-1}\phi_0^\star\}, \quad (11)$$

$$i\dot{\phi}_1(\mathbf{r},t)=\frac{1}{\hbar}\left[\frac{\mathbf{p}^2}{2M}+U(\mathbf{r})\right]\phi_1+N\{c_2\phi_0\phi_0\phi_{-1}^\star+[(c_0+c_2)(|\phi_1|^2+|\phi_0|^2)+(c_0-c_2)|\phi_{-1}|^2]\phi_1\}.$$

Just as in the familiar quantum optics case, we consider in the following a situation where the central mode, described by the Hartree wave function ϕ_0, is strongly populated initially, while the side modes $\phi_{\pm 1}$ are weakly populated. In other words, we consider the phase conjugation of a weak atomic beam from a reasonably large condensate. In that case, it is appropriate to introduce the matter-wave optics equivalent of the undepleted pump approximation, whereby

$$\dot{\phi}_0\simeq 0. \quad (12)$$

In that case, the problem reduces to a set of coupled-mode equations for the two side modes $\phi_{\pm 1}$, the central mode acting as a catalyst for the coupling between them.

III. PHASE CONJUGATION IN DIPOLE TRAPS

In what follows we consider atomic samples confined in a two-dimensional harmonic trap. The trap potential $U(\mathbf{r})$, which is as we recall independent of the atomic internal state m [9] for a dipole trap, is taken to be of the harmonic form

$$U(\mathbf{r})=M\omega_0^2(x^2+y^2)/2 \quad (13)$$

for simplicity. That is, we assume that the dipole trap confines the atoms in the transverse plane (x,y), but not in the longitudinal direction z. This geometry allows one to consider side modes propagating along that axis, rather than bouncing back and forth in an elongated trap. In case of tight confinement in the transverse direction, we can assume to a good approximation that the transverse structure of the condensate is not significantly altered by many-body interactions and is determined as the ground-state solution of the transverse potential.

Expressing the Hartree wave function associated with the hyperfine level m as

$$\phi_m(\mathbf{r},t)=\varphi_\perp(x,y)\varphi_m(z,t)e^{-i\omega_0 t}, \quad (14)$$

we then have

$$\hbar\omega_0\varphi_\perp(x,y)=\left[-\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+\frac{M\omega_0^2}{2}(x^2+y^2)\right]\varphi_\perp(x,y). \quad (15)$$

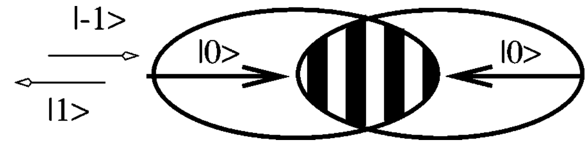

FIG. 1. Geometry of the matter-wave phase conjugation with three-component vector fields.

Substituting this expression into Eqs. (11) and projecting out the transverse part of the wave function yields the coupled one-dimensional Gross-Pitaevskii (coupled-mode) equations

$$i\dot{\varphi}_{-1}(z,t)=-\frac{\hbar}{2M}\frac{\partial^2}{\partial z^2}\varphi_{-1}+N\eta\{c_2\varphi_0\varphi_0\varphi_1^\star+[(c_0+c_2)\times(|\varphi_{-1}|^2+|\varphi_0|^2)+(c_0-c_2)|\varphi_1|^2]\varphi_{-1}\},$$

$$i\dot{\varphi}_0(z,t)=-\frac{\hbar}{2M}\frac{\partial^2}{\partial z^2}\varphi_0+N\eta\{c_0|\varphi_0|^2\varphi_0+(c_0+c_2)\times(|\varphi_{-1}|^2+|\varphi_1|^2)\varphi_0+2c_2\varphi_1\varphi_{-1}\varphi_0^\star\}, \quad (16)$$

$$i\dot{\varphi}_1(z,t)=-\frac{\hbar}{2M}\frac{\partial^2}{\partial z^2}\varphi_1+N\eta\{c_2\varphi_0\varphi_0\varphi_{-1}^\star+[(c_0+c_2)\times(|\varphi_1|^2+|\varphi_0|^2)+(c_0-c_2)|\varphi_{-1}|^2]\varphi_1\},$$

where

$$\eta=\frac{\int dxdy|\varphi_\perp(x,y)|^4}{\int dxdy|\varphi_\perp(x,y)|^2}. \quad (17)$$

The physical situation we have in mind is that of a weak ''probe'' in the hyperfine state $m=-1$ propagating toward a large condensate in state $m=0$ and at rest in the dipole trap, and generating a backward-propagating conjugate matter wave in the hyperfine state $m=1$ (see Fig. 1). Hence we express the longitudinal component of the Hartree wave function as

$$\boldsymbol{\varphi}(z,t)\equiv\begin{pmatrix}\varphi_{-1}(z,t)\\ \varphi_0(z,t)\\ \varphi_1(z,t)\end{pmatrix}=\begin{pmatrix}\psi_{-1}(z,t)e^{ikz}e^{-i\omega t}\\ 2\psi_0\cos(kz)e^{-i\omega t}\\ \psi_1(z,t)e^{-ikz}e^{-i\omega t}\end{pmatrix}, \quad (18)$$

where the slowly varying envelopes ψ_m of the Hartree wave function components $m=\pm 1$ satisfy the familiar inequalities

$$\left|\frac{\partial^2}{\partial z^2}\psi_m\right|\ll k\left|\frac{\partial}{\partial z}\psi_m\right|\ll k^2|\psi_m| \quad (19)$$

and we have additionally invoked the undepleted pump approximation (12). Note that in this ansatz the ''pump'' wave function φ_0 is described by a standing wave. This spatial structure is required in order to achieve momentum conservation, a direct consequence of the fact that a standing wave can be viewed as a superposition of two counterpropagating atomic waves. A state with such a periodic spatial structure can be achieved, for instance, by interfering two condensates [15], in a grating matter-wave interferometer [16], or in CARL [13]. To the first order in the probe and signal fields, this geometry leads to a linearized system of two coupled-mode equations for the probe and condensate fields. In the stationary state they reduce to

$$i\frac{\hbar k}{2M}\frac{\partial}{\partial z}\psi_{-1}(z)=-N\eta[2(c_0+c_2)\rho_0\psi_{-1}(z)+c_2\psi_0^2\psi_1^\star(z)],$$

$$i\frac{\hbar k}{2M}\frac{\partial}{\partial z}\psi_1^\star(z)=-N\eta[2(c_0+c_2)\rho_0\psi_1^\star(z)+c_2\psi_0^{\star 2}\psi_{-1}(z)], \tag{20}$$

where $\rho_0=|\psi_0|^2$.

The form of these equations is familiar from optical phase conjugation and their solution is well known. Before giving them explicitly, though, we note that they contain two contributions. For instance, the equation for the phase-conjugate wave $\psi_1^\star$ contains a term proportional to the density ρ_0 of the condensate and the field itself. In the absence of the second term, it would simply lead to a phase shift of $\psi_1^\star$. Physically, it results from the self-interaction of the conjugate field, catalyzed by the condensate (pump) component. Its origin can be traced back to the term proportional to $\Psi_1^\dagger\Psi_{-1}^\dagger\Psi_0\Psi_0$ in the Hamiltonian (10). The second term, in contrast, couples the two side modes via the condensate and is responsible for phase conjugation. Note that it is not proportional to the condensate density ρ_0, but rather to ψ_0^2. We return to this point later on.

The general solution of Eqs. (20) reads [17]

$$\psi_{-1}(z)=\frac{e^{i\alpha z}}{\cos(|\kappa|L)}\{-ie^{-i\beta}\sin(|\kappa|z)\psi_1^\star(L)+\cos[|\kappa|(z-L)]\psi_{-1}(0)\}, \tag{21}$$

$$\psi_1(z)=\frac{e^{i\alpha z}}{\cos(|\kappa|L)}\{\cos(|\kappa|z)\psi_1(L)+ie^{-i\beta}\sin[|\kappa|(z-L)]\psi_{-1}^\star(0)\},$$

where

$$\alpha=2N\eta(c_0+c_2)\rho_0, \tag{22}$$

$$\kappa=\frac{N\eta c_2\psi_0^2}{\hbar k/2M}, \tag{23}$$

and

$$e^{i\beta}=\kappa/|\kappa|. \tag{24}$$

For the probe $\psi_{-1}(0)$ incident at $z=0$ and no incoming conjugate signal $\psi_1(L)=0$, the conjugate wave in the input plane $z=0$ becomes

$$\psi_1(0)=-ie^{-i\beta}\tan(|\kappa|L)\psi_{-1}^\star(0), \tag{25}$$

which demonstrates that the interaction of the probe and the condensate results in the generation of a counterpropagating phase-conjugated signal. Note that the intensity of the conjugate wave exceeds that of the incoming wave for $\pi/4<|\kappa|L<3\pi/4$ and phase conjugation oscillations (PCO) [17] can occur for $|\kappa|L=\pi/2$, the so-called oscillation condition.

IV. EXPERIMENTAL FEASIBILITY AND OUTLOOK

In order to determine the feasibility of matter-wave phase conjugation in state of the art experiments, we briefly discuss the values of the oscillation parameter $|\kappa|L$ that can be achieved in current ^{23}Na Bose-Einstein condensation (BEC) experiments.

From the definition (23) we have

$$|\kappa|L\sim N\eta\frac{a_2-a_0}{3k}, \tag{26}$$

where we have taken that due to normalization $\psi_0^2L\simeq\rho_0L\sim 1$ and that [9]

$$c_2=4\pi\hbar(a_2-a_0)/3M, \tag{27}$$

with a_0 and a_2 being the singlet and triplet state scattering lengths, respectively. For sodium, these scattering lengths are estimated as [9] $(a_2-a_0)/3\sim 0.04a_2\sim 10^{-10}$ m. In the MIT optical confinement experiments [4] the number of trapped atoms is of the order of 5×10^6 to 10^7 and the transverse dipole trap frequency ω_0 is of the order of 10^4 sec^{-1}, so that the transverse ground-state size of the condensate $a_\perp$ should be determined from the Thomas-Fermi [18] rather than from the single-particle approximation, see Eq. (15). This gives [18] $a_\perp=a_{ho}(15Na_0/a_{ho})^{1/5}\sim 10a_{ho}$, where $a_{ho}\equiv\sqrt{\hbar/m\omega_0}\sim 0.5$ μm is the single-particle ground-state size in a harmonic trap. Consequently, $\eta\sim a_\perp^{-2}\sim 10^{10}$ m^{-2} and the oscillation parameter is $|\kappa|L\sim 10^7/k$ where k is as we recall the wave number of the pump side mode, see Eq. (18). In case the condensate side modes are obtained by diffraction on a standing light wave [16], we have $k=2\pi/\lambda\sim 10^7$ m^{-1} and thus $|\kappa|L\sim 1$. This means that the oscillation condition $|\kappa|L=\pi/2$ can be met in current BEC experiments.

The characteristic time scale over which the population of the conjugate side mode builds up can be estimated from Eq. (21): The characteristic length over which the population transfer between modes takes place is $2\pi\kappa^{-1}$, corresponding to a characteristic time $t_c=2\pi/\kappa v$ for atoms of velocity $v=\hbar k/M\sim 2.5\times 10^{-2}$ m/sec. In typical current BEC experiments the longitudinal size of the condensates is about ~ 300 μm so that $t_c\sim 2\pi L/v\sim 10^{-1}$ sec for $|\kappa|L\sim 1$. This time is well below the lifetime of optically trapped spin-1 Na condensates.

In addition to its interest from a nonlinear atom optics point of view, matter-wave phase-conjugation could also be used as a diagnostic tool for Bose-Einstein condensates. For instance, we noted that the parameter $|\kappa|L$ is proportional to the difference in scattering lengths between the singlet and triplet states. Hence, this quantity could in principle be inferred from phase conjugation measurements. In addition, we recall that the phase conjugate signal is not determined by the condensate density ρ_0, but rather by ψ_0^2. While the distinction between the two is expected to be minimal for large condensates, and is essentially ignored in the Hartree and undepleted pump approach of the present paper, this will no longer be the case for smaller condensates. In such situations, phase conjugation provides one with a probe of the coherence properties of the condensate. Future work will analyze these aspects of the problem, as well as the role of higher-order correlation functions in the atom statistics of the phase-conjugate mode.

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