Nonlocality, counterfactuals, and quantum mechanics

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Stapp [Am. J. Phys. **65**, 300 (1997)] has recently argued from a version of the Hardy-type experiments that quantum mechanics must be nonlocal, independent of any additional assumptions such as realism or hidden variables. I argue either that his conclusions do not follow from his assumptions or that his assumptions are not true of quantum mechanics and can be interpreted as assigning an unwarranted level of reality to the value of certain quantum attributes. [S1050-2947(98)06408-7]

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In a recent paper [1] Stapp has argued that quantum mechanics is nonlocal. He bases this conclusion on a weak definition of locality for a physical theory and an examination of the predictions of quantum mechanics for a specific two-particle Hardy-type [2,3] experiment. It has already been known since the 1950s from the theorem of Bell [4] that quantum mechanics violated the correlations expected in any theory that is both local and is a ''hidden variable'' theory, in which the statistics arises out of the lack of knowledge of the values of local hidden variables that determine the outcome of the experiment itself. However, Stapp's argument claims to strengthen that conclusion by arguing that no local theory can give the same predictions as quantum mechanics. In other words, he reaches the conclusion that quantum mechanics is nonlocal.

In his paper, Stapp claims that his proof that quantum mechanics is nonlocal applies only if one assumes that locality holds in all frames of a special relativistic system. However, I will take this as a given, that locality is to be taken to apply to any two systems when causally separated from each other. The temporal order of experiments that are spacelike separated is assumed to be irrelevant.

To begin, let me first present the argument in my own way. Stapp phrased his argument in the form of a logical calculus, which I will examine later. He considers a very general situation in which measurements are made at two locations, L (for left) and R (for right). At each location two possible measurements are made that he designates by $L1$ and $L2$ for the left and $R1$ and $R2$ for the right. In this first section I will specify his generic situation by having the measurements made on the spins of two particles, one particle labeled L on the left and the other, R, on the right. These two particles are placed into a specific correlated state. For purposes of illustration, I will assume that these are two spin-1/2 particles, and that in the spin-z basis for each particle the state is given by

$$|\Psi\rangle = N\bigg(\cos(\theta)|+\rangle|+\rangle + \sin(\theta)|+\rangle|-\rangle + \frac{1+\sin^2(\theta)}{\cos(\theta)}|-\rangle|+\rangle - \sin(\theta)|-\rangle|-\rangle\bigg), \qquad (1)$$

where the state $|\alpha\rangle|\beta\rangle$ refers to the left particle being in the α eigenstate of its spin-z operator and the right particle is in the β eigenstate of its spin-z operator. θ is a fixed parameter, and N is a normalization given by

$$N = \cos(\theta)/\sqrt{2[1+\sin^2(\theta)]}.$$

Writing this state in the form

$$|\Psi\rangle = N\bigg[|+\rangle[\cos(\theta)|+\rangle + \sin(\theta)|-\rangle] + |-\rangle\bigg(\frac{1+\sin^2(\theta)}{\cos(\theta)}|+\rangle - \sin(\theta)|-\rangle\bigg)\bigg], \qquad (2)$$

we see that the $|+\rangle$ eigenstate of the left particle is perfectly correlated with the eigenstate $[\cos(\theta)|+\rangle + \sin(\theta)|-\rangle]/\sqrt{2}$ for the right particle, which is the + eigenstate of the operator $\sigma_\theta = \cos(2\theta)\sigma_z + \sin(2\theta)\sigma_x$ for the right particle. In other words, if σ_z is measured on the left particle and is found to have value +, and σ_θ is measured on the right, then it will always be found to have value +.

Writing the state as

$$|\Psi\rangle = N\{(|+\rangle + |-\rangle)[\cos(\theta)|+\rangle + \sin(\theta)|-\rangle] + 2\tan(\theta)|-\rangle[\sin(\theta)|+\rangle - \cos(\theta)|-\rangle]\}, \qquad (3)$$

we see that the $[\cos(\theta)|+\rangle + \sin(\theta)|-\rangle]/\sqrt{2}$ eigenstate of the right particle's σ_θ operator is perfectly correlated with the eigenstate $(|+\rangle + |-\rangle)/\sqrt{2}$ of σ_x of the left particle. In other words, if σ_θ is measured on the right and found to have value +, then if σ_x is measured on the left, it is always found to have value +.

Finally, we can write the state as

$$|\Psi\rangle = N\bigg[(|+\rangle + |-\rangle)\frac{1}{\cos(\theta)}|+\rangle + (|+\rangle - |-\rangle)\bigg(\frac{-\sin^2(\theta)}{\cos(\theta)}|+\rangle + \sin(\theta)|-\rangle\bigg)\bigg] \qquad (4)$$

and we see that the state $(|+\rangle + |-\rangle)/\sqrt{2}$ of the left particle is perfectly correlated with the state $|+\rangle$, the plus eigenstate of the σ_z operator, of the right particle.

We thus can construct a chain of perfect correlations. If we have measured σ_z of the left particle and found it to be

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$+$, then we can predict with certainty that if we had measured σ_θ of the right, the answer would have been $+$. If we measured σ_θ of the right and found a value of $+$, then we would know with certainty that if we had measured σ_x of the left we would have gotten a value of $+$. Finally, if we had measured σ_x of the left particle and gotten $+$, we would know with certainty that we would have gotten the $+$ eigenstate if we had measured σ_z for the right particle.

In essence, the Stapp argument is that we can use the logical chain

$$\sigma_{Lz}=+\Rightarrow\sigma_{R\theta}=+\Rightarrow\sigma_{Lx}=+\Rightarrow\sigma_{Rz}=+ \tag{5}$$

to infer that $\sigma_{Lz}=+\Rightarrow\sigma_{Rz}=+$ with certainty. However, a direct calculation shows that, if $\sigma_{Lz}=+$, one has only a probability of $\cos^2(\theta)$ that if one also measures σ_{Rz} one will get a value of $+$. If one chooses θ very near $\pi/2$, one can make this probability as small as one desires. In other words, the inferred value of σ_{Rz}, given that σ_{Lz} is measured to have value $+$ and given the chain of inferences via the perfect correlation, is with high probability exactly the opposite of what quantum mechanics would predict would be obtained in a measurement. Stapp essentially argues that the truth of the chain of inferred values can be justified by an appeal only to locality. Thus, if correct, it is relativistic locality alone, with no further assumptions, which allows one to carry out the line of inferences. Since quantum mechanics violates the conclusion of that inferential chain, his conclusion is that quantum mechanics must be nonlocal.

Now, if one believed that attributes of a system had values that the measurements simply revealed, and if measurements made in one causally disconnected region could not influence those values of attributes associated with some other causally disconnected region, then the truth of that inferential chain would be immediately obvious. This is essentially a strong form of hidden variables theory (where those values can be derived from the values that some set of hidden variables have in any particular realization of the experiment), and this analysis shows that quantum mechanics is in disagreement with such a hidden variables theory. However, a weaker form of hidden variables theory is that such values are context sensitive, i.e., that the value of an attribute may depend on the types of experiments that are actually carried out on the system. This is where the assumption of locality comes in as a restriction on the types of context sensitivity of the values that attributes can have. In particular, locality is usually used to argue that value that a variable attains must be independent of the choice of experiment carried out in a causally disconnected region (although correlations clearly mean that the value need not be independent of the values obtained for measurements in disconnected regions). Thus the context for the value assigned to an attribute can depend on the experiments carried out in the causally connected region surrounding the experiment, but not on the experiment carried out in the causally disconnected region.

However, within quantum mechanics, attributes do not have values unless those attributes are actually measured. Thus, if σ_z of particle L has value $+$, it is inappropriate within quantum mechanics to argue that σ_{Lx} must have some value. It was not measured and thus one cannot talk about the value that it has. Talking about the the values of nonmeasured attributes is termed ''counterfactual'' reasoning. Clearly the inference chain in Eq. (5) relies on counterfactual reasoning, since within the context of quantum mechanics, only one of a noncommuting set of operators can be measured at any one time. One attitude toward any such chain of arguments would be to disallow all counterfactuals. However, as Stapp argues, such a procedure would be to disallow the reasoning that physicists often engage in, even in quantum situations. His example is classical, where such counterfactual reasoning is unexceptional, but quantum examples could also be found. However, in the quantum case one must be extremely careful in carrying out such arguments, and must ensure that one is not assuming a form of quantum realism—that quantum attributes have values even if they have not been measured—together with the counterfactual discussion.

I will assume that such counterfactual statements may legitimately be made in certain circumstances. Given that one has established a correlation of system A (which since I am making a generic argument I will use instead of, say, L of the above specific example) with system B (instead of the specific R from above), then one can make measurements on system B, and on the basis of the known correlations, make inferences about system A, even if system A has not been directly measured. After all, if such reasoning were disallowed, the whole of the von Neuman argument about measurement would be invalid. In von Neuman's discussion, it is precisely the use of correlations of measuring apparatuses with systems that allows us to deduce properties of the system from measurements made on the measuring apparatus, even though no direct ''measurement'' has been made on the system.

However, great care is required so that in such counterfactual statements one does not import a notion of reality. In particular, the truth of the statement made about system A, which relies on the measurement made on system B and on the correlations that have been established between A and B in the state of the joint system, is entirely dependent on the truth of the actual measurement that has been made on system B. To divorce them is to effectively claim that the statement made about A can have a value in and of itself, and independent of measurements that have been made on A. This notion is equivalent to asserting the reality of the statement about A independent of measurements, a position contradicted by quantum mechanics.

Thus in the above system, measuring σ_θ on particle R and finding value $+$ can lead one to assigning a value of $+$ to σ_x of particle L, even if that attribute were not directly measured, due to the correlation between the two particles. However, that value for σ_{Lx} is entirely dependent on the fact that σ_θ was actually measured and found to have a certain value on particle R. In particular, causality cannot be used to argue that the inferred (as opposed to measured) value of σ_{Lx} must be independent of what was measured at particle R. Although the two measurements may be causally disconnected, they are not logically disconnected. The value can be assigned to σ_{Lx} is logically tied to the actual measurement of $\sigma_{R\theta}$ and its value.

Without the extension of the concept of locality to such inferred values, I will argue below that the chain of reasoning used by Stapp to establish Eq. (5) is broken. In all cases at most one attribute (either σ_{Lz} or σ_{Lx} for the left particle or σ_{Rz} or $\sigma_{R\theta}$ for the right particle) is measured at each of the particles. That measurement may be used to infer some counterfactual value at the other particle, but in each case that chain of inference cannot be extended sufficiently to obtain the conclusion of Eq. (5). Since his argument breaks down, there is no contradiction between the statement that quantum mechanics is local—i.e., that measured (as opposed to inferred) values must be independent of the attribute measured in a causally disconnected region—and that the quantum outcomes obey the rules that they do.

Let me finally examine Stapp's argument in detail. Stapp postulates three requirements that he claims a local theory should satisfy. He gives labels to particles, variables, and outcomes, and uses the notation of a logic calculus. He refers to two measurement situations, *L* and *R*, which are spacelike separated (i.e., causally disconnected). These would correspond to my particles each with spins on the left and right mentioned above. Thus in order to make contact with his notation, I will assume that the measurement of σ_z at *L* is designated by his specific measurement situation on the left, $L1$, while measuring σ_x at *L* is designated by his $L2$. Measuring σ_z on particle *R* is $R1$ and σ_θ on the particle *R* is $R2$. His outcomes *a* and *b* are mapped onto my obtaining the values $-$ and $+$ on $L1$ or, more specifically, on obtaining the values $-$ and $+$ on a measurement of σ_z on the particle at *L*. Similarly, *c* and *d* refer to obtaining the value of $+$ and $-$ on $L2$ (σ_{Lx}). *e* and *f* refer to obtaining values $-$ and $+$ on $R1$, (σ_{Rz}) and *g* and *h* to values $+$ and $-$ on $R2$ $(\sigma_{R\theta})$. By assumption, *L* and *R* are measured in causally disconnected regions. Thus in all cases one can assume that *L (R)* is measured after any measurement is made of R (L), and the free choice of which measurement is actually made on *L (R)* should have no impact on the outcomes of measurements made on R (L).

His locality conditions, in his modal logic calculus are

$$LOC1{:}Ru\wedge Lv\wedge i\Rightarrow Ru'\Box\!\!\rightarrow Lv\wedge i, \tag{6}$$

or in words, a change in the choice of which experiment the experimenter at *R* will carry out does not affect the outcome of the experiment at *L* where Lv is measured. *u* refers to one of 1 or 2, and u' refers to the other of the two. v refers to 1 or 2, and *i* refers to one of the possible outcomes of the measurement Lv. $\wedge$ means ''and'' and the symbol $Ru'\Box\!\!\rightarrow$ means ''if Ru is replaced by Ru' in the previous expression.'' This definition of locality is unexceptional if we limit ourselves to $Lv\wedge i$ actually measured. I discuss this further below when I go through Stapp's argument step by step:

$$LOC2\text{: If }\ (Lv\Rightarrow[(Ru\wedge j)\Rightarrow(Ru'\Box\!\!\rightarrow Ru'\wedge j')]) \tag{7}$$

$$\text{Then }\ (Lv'\Rightarrow[(Ru\wedge j)\Rightarrow(Ru'\Box\!\!\rightarrow Ru'\wedge j')]). \tag{8}$$

This says that If, under the condition that Lv is measured (at a late time) at *L*, one can deduce that some condition prevails relating measurements only on the right hand side, *Then* that relation on the right must also be true if the experimentalist on the left side had decided to make some other measurement (at that later time). In other words, the truth of a statement pertaining exclusively to possible events in region *R* cannot depend on which free choice is to be made by the experimenter in region *L*.

I have inserted the ''at a late time'' to emphasize that these two measurements are causally unrelated and thus one can assume that *L* is measured at a time later than *R*. Note that in $LOC1$, it is *R* that is taken to be the later, while in $LOC2$ it is *L*. Under the notion of relativistic causality, this is a completely legitimate procedure. Either of two causally separated events can be taken to be the earlier in relativity. In his paper, Stapp questions the legitimacy of this procedure. I do not, and see no possible reason for doing so. In my opinion, if quantum mechanics is to be a local theory it should be local under the full relativistic definition.

$LOC2$ can be rewritten as

$$\begin{aligned} LOC2{:}(&Lv\wedge Ru\wedge j\\ &\Rightarrow Ru'\Box\!\!\rightarrow Lv\wedge Ru'\wedge z)\\ &\Rightarrow Lv'\Box\!\!\rightarrow Ru\wedge j(Lv'\wedge Ru\wedge j\\ &\Rightarrow Ru'\Box\!\!\rightarrow Lv'\wedge Ru'\wedge j). \end{aligned} \tag{9}$$

To me this more clearly states the content of the above principle and says that, if from the fact that Lv is measured and Ru has the value *j* you can infer that if Ru' had been measured instead it would have had value j', then that inference must by independent of whether you measured Lv or Lv'.

If it were true that one could deduce solely from the fact that a measurement had been made at *L* that some relation on the right-hand side must hold, then I would agree that this requirement would be reasonable. However, if the truth of the relation on the right-hand side depended not only on which measurement had been made on the left, but *also* on the actual value obtained on the left, then no such locality relation would obtain. If it is the value obtained on the left, even if that value is obtained at a later time, which allows one to deduce the relation on the right, then that relation on the right cannot be independent of what it is that is measured on the left, but rather is tied to that measured value. To assume otherwise, to assume that the relations between possible measurements on the right are independent of the values of the outcomes on the left which were used to derive those relations is, in my opinion, simply another form of realism.

Let us now look in detail at Stapp's argument. Again I will use his notation:

$$\begin{aligned}(1)LOC1{:}&(R2\wedge L2\wedge c)\\ &\Rightarrow[R1\Box\!\!\rightarrow(R2\wedge L2\wedge c)].\end{aligned} \tag{10}$$

At face value this is just the unexceptional statement that if $L2$ is measured to have value *c*, then the truth of having obtained that value is independent of what is (or will be) measured at *R*. In other words, if you had thought that $R2$ was measured at *R* and you knew that the outcome of the measurement of $L2$ had been *c*, then you would never find that outcome to be surprising if you were told that it had actually been $R1$ that had been measured at *R*. Which measurement was done at *R* has no effect on the outcome of a measurement made at *L*.

However, this meaning of $LOC1$, although certainly true in quantum mechanics, is insufficient to derive his conclusion, since it demands that $L2$ had actually been measured and had the given outcome. It ties the meaning of this locality assumption to the actuality of the measurement and its outcome. This does not allow counterfactual replacement of, for example, L, since the truth of this statement is, under this interpretation tied to the truth of the measurement of the attribute $L2$ actually having been carried out on L, and on having obtained that specific outcome.

Another possible meaning is that, if we carried out another experiment in which we kept all of the conditions on the experiment the same (except for the outcomes of the measurements and which measurement we made at R), then we would obtain the same value for $L2$. This interpretation is clearly wrong about quantum mechanics, since the outcome of any measurement is a random process underdetermined by the conditions of the experiment, unless that outcome is a certainty.

Finally, and this is what I suspect Stapp means, this statement could be taken to mean that if one is somehow able to infer that $L2$ has value c, then it remains true that $L2$ has value c under replacement of $R1$ with $R2$, even if the outcome of the measurement made of $R1$ was crucial in drawing the inference that $L2$ has value c. This interpretation of $LOC1$ is, I would argue, a form of realism, in that it claims that the value to be ascribed to $L2$ is independent of the evidence used to determine that value. Clearly this is true of a hidden variables theory, in which $L2$ has a value independent of any evidence used to determine that value. The evidence simply reveals the value. However, I would claim that, under this interpretation, $LOC1$ is not true of quantum mechanics. If the value obtained in a measurement of $R2$ was crucial in determining that $L2$ had value c, then $R2$ cannot be replaced by $R1$, even if it is causally disconnected or comes later in time. If the measurement of $R2$ was not necessary (logically) in determining the value of $L2$, then of course the acausal relation also makes the value of $L2$ physically independent of $R2$ and it can be replaced by $R1$ without changing the conclusion that $L2$ has value c.

Since this is key to my argument below, let me restate this. In quantum theory, one can determine the value of an operator in many ways. One can directly measure that value (where the term ‘‘measure’’ is taken as a primitive of the theory of quantum interpretation) or one can infer that value because one has dynamically correlated that system (in this case the L particle) with some other system (a measuring apparatus) and then measured something on that second system. (This is the essence of the von Neuman analysis, where he showed that under certain conditions, the second type of measurement with an apparatus was equivalent in its predictions to the first type.) In the example examined by Stapp, there are two possible meanings to the term $L2\wedge c(\sigma_{Lx}=+)$. One can either directly measure the value of $L2$ (σ_{Lx}) and find it to be c (+), or one can, because of the correlations between the two particles, regard R as a measuring apparatus for L. Obtaining the value g (+) for $R2$ ($\sigma_{R\theta}$) can then be used to infer the value of c (+) for $L2$ (σ_{LX}). In the former case, the value of $L2$ obtained by the measurement is clearly independent of which measurements are carried out in the causally disconnected region, and $LOC1$ applies. If, however, the value of $L2$ is only inferred from the value obtained from the ‘‘measuring apparatus’’ R, then that value is clearly not independent of the measuring apparatus R. In other words, if in $LOC1$ the value obtained for $L2$ (σ_{LX}) is inferred from some measurement on R, then $LOC1$ is false, no matter what the causal relation is between L and R:

$$(2)QM:(L2\wedge R2\wedge g)\Rightarrow(L2\wedge R2\wedge c). \tag{11}$$

This is to encompass the claim that if we know that both $L2$ (σ_{LX}) and $R2$ ($\sigma_{R\theta}$) were measured, and we know that $R2$ had value g (+), then $L2$ must have had value c (+). It is, however, important to note that this statement can have two possible interpretations. In one the value of c for $L2$ was inferred from the initial state and the measured value of $R2$, while in the other case $L2$ was measured and actually found to have value c. In that case the $\Rightarrow$ must be interpreted as ‘‘it is consistent with’’ rather than ‘‘it is inferred that’’:

$$(3)QM:(L2\wedge R1\wedge c)\Rightarrow(L2\wedge R1\wedge f). \tag{12}$$

This is a statement similar to the previous one for a different implication that can be drawn from the Hardy state:

$$(4)\text{LOGIC}:(L2\wedge R2\wedge g)\Rightarrow(R1\Box\!\!\rightarrow(L2\wedge R1\wedge f). \tag{13}$$

The claim is that this arises solely by logic out of the substitution of Eqs. (2) and (3) into Eq. (1). However, it is now clear that the meanings ascribed to $LOC1$ and to the statements in the logical calculus are crucial. To fill in the steps of this reasoning, I get

$$\begin{aligned}&(L2\wedge R2\wedge g)\\ &\quad\Rightarrow L2\wedge c\wedge R2\wedge g)\Rightarrow(L2\wedge c\wedge R2)\\ &\quad\Rightarrow(R1\Box\!\!\rightarrow(R2\wedge L2\wedge c)\rightarrow L2\wedge c\wedge R1\\ &\quad\Rightarrow L2\wedge c\wedge R1\wedge f\Rightarrow L2\wedge R1\wedge f.\end{aligned} \tag{14}$$

There are now a number of meanings that can be attached to this sequence of statements.

In the first, $L2$ is assumed to have value c only as an inference drawn from the fact that $R2$ had value g. $L2$ is not independently measured and found to have value c. Thus the third step in this expansion does not now follow since $L2$ has value c only because $R2$ has value g. Within quantum mechanics, one cannot assume the truth of a statement ($L2$ has value c) independent of the evidence for that statement ($R2$ has value g). One cannot therefore use $LOC1$ to replace $R2$ by $R1$, since one no longer has any independent evidence that $L2$ has value c. Were there some independent evidence for the value of $L2$ (a direct measurement, or a correlation between $L2$ and some other system such that knowing the outcome for that other system would imply the value for $L2$) then one could use $LOC1$ to carry out the (counterfactual) replacement of $R2$ with $R1$.

The second interpretation of this statement is that $L2$ has actually been measured at L. If $L2$ was actually measured, then the first four inferences follow, but the last inference is no longer true. The fact that $L2$ was measured and had value c and that $R1$ was measured and had value f does not imply that $L2$ was measured and that $R1$ was measured and had value f independent of the value measured for $L2$. The truth of $R1$ having value f is an inference drawn from the actually measured value of $L2$, and is not itself an independently measured value (it could not be since $R1$ is counterfactual). Just as in my previous comments about $LOC1$, the value associated with $R1$ cannot be separated from the evidence used to ascribe that value to it, as the last inference attempts to do (i.e., it claims that counterfactual $R1$ has value f even if it is not asserted that $L2$ has value c, but only that $L2$ was measured).

Again, it is the fact that in quantum mechanics there is a difference between the value of an attribute as inferred from the value of some other attribute, and the value as directly measured that breaks this logical chain. Stapp must use the independence of the value of an attribute from the way in which that value was inferred to carry through his argument. His logical calculus does not distinguish between the various ways a value for an attribute can be determined, and thus assumes that that value is independent of that determination, an unwarranted assumption in quantum theory, and at least for me, an assumption of realism for the value of an attribute.

It is on this expression (or rather on a slight rewriting of this expression) that Stapp now uses $LOC2$:

$$(5)\text{LOGIC}:(L2)\Rightarrow(R2\wedge g\Rightarrow R1\Box\!\!\rightarrow R1\wedge f). \quad (15)$$

This is a rewriting of Eq. (4). Again, however, this expression does not capture the essence of Eqs. (1), (2), and (3) where it would only be the fact of $L2$ actually having been measured to have the value c which allows us to make these inferences, and not simply that $L2$ has been inferred to have value c from measurements made on $R2$, or that $R1$ had value f inferred from the measured value of c for $L2$. It would be true if the proposition $(L2)$ were replaced with $(L2\wedge c)$, where this would be interpreted as ''$L2$ actually having been measured and found to have value c'' but this would again not allow him to use his $LOC2$ to draw his ultimate conclusion:

$$(6)LOC2:(L1)\Rightarrow(R2\wedge g\Rightarrow R1\Box\!\!\rightarrow R1\wedge f). \quad (16)$$

This is the statement that if one has derived some expression about R that is true, then that expression should be independent of what it is that has been measured on the left-hand side (e.g., that measurement on the left-hand side could have been made long after whatever measurement had been made on the right). However, this neglects the fact that the inference was made entirely based not only on the fact that $L2$ was measured on the left, but also that the value obtained for $L2$ was in fact the value c. If $R2$ was measured, then $R1$ was not in fact measured. The whole basis for the inference that $R1$ had value f is that $L2$ was measured to have value c. Again, if the value of $R1$ were something that was real, something that had an existence independent of the means used to determine what it was, then this substitution would make sense. But in quantum mechanics, the value is not independent of the means. The inference of the value of $R1$ is entirely based on the truth of the statement that $L2$ was measured to value c. Thus it is not simply locality that enters into $LOC2$ but also some notion of the independent reality of the value of $R1$, even if it has not itself been directly measured.

The truth of statement 4 ultimately rests on the additional assumption that the value of an attribute [e.g., $R1$ (σ_{Rz}) being f (+)], is independent of the evidence— $L2$ being g or $\sigma_{Lx}=+$—which was used to infer that value for $R1$. $R1$, being counterfactual, cannot have actually been measured in some independent way. Its value is only inferred from the value obtained for $L2$, and as such is not independent of that value, even if $L2$ was only measured much later. That assumption of independence of value from evidence is, I would claim, the heart of realism, and is contrary to quantum mechanics.

His analysis does raise interesting issues in our understanding of quantum mechanics. As mentioned above the von Neuman analysis of the measurement process does assert that one can use the correlations between systems (a measuring apparatus and some system one is measuring) to make statements about the system that is being measured. (If one sees the pointer on a properly constructed and operating volt meter point to 10 V, then one can use that to infer that the system one is measuring the voltage of has 10 V as its value.) To deny the use of such inferences drawn from such correlations between a measuring apparatus and the measured system would be to destroy our theory of almost all physical measurements. However, the Stapp argument shows that one must treat such inferential measurements carefully. Once one has used the measuring apparatus to infer the value of some measured quantity, one must keep in mind that inferred value depends crucially on that process used to carry out the measurement, and is not independent of that measuring apparatus. Although usually one can get away with an elimination of the measuring apparatus and regarding the inference of a value as being equivalent to the direct measurement of that value (measurement here being used in its primitive sense in the interpretation of quantum mechanics), there are situations, as in these Hardy-type experiments, in which such sloppiness about the meaning of measurement can lead one into error.

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