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Dense coding in photonic quantum communication with enhanced information capacity

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We propose a scheme for enhancing the information capacity to more than 2 bits in dense coding quantum communication that involves transmitting a polarization entangled twin photon and a subsequent joint measurement with a Bell-state analyzer. Although direct modulation of the timing position and optical frequency of the photon play no role in dense coding quantum communication, we show that the frequency-dependent phase correlation of polarization entangled twin photons can be used for this enhancement if photon twins are generated through optical parametric down-conversion. With our proposed Bell-state analyzer containing a nonlinear optical gate, the capacity can be enhanced from the reported 2 bits to 3 bits by the *local* operation of a frequency-dependent phase shift on the transmitted photon. [S1050-2947(99)04802-7]

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I. INTRODUCTION

Recent theoretical and experimental studies on quantum communication through the use of polarization entangled photon twins [1–5] show great promise for establishing new kinds of information processing with no classical counterparts. One example is the concept of dense coding in quantum communication [1]. Bob, the information receiver, prepares a pair of polarization entangled twin photons and sends one of them to Alice, the information sender, while retaining the other. Then, when Alice wants to send Bob a message, she can encode it by manipulating the photon sent by Bob and transmitting it to Bob. No one other than Bob can obtain any information from the transmitted photon. This is because the message is not encoded in the local state of the transmitted photon but is encoded in the nonlocal quantum state, Bell states, of the twin photons. The nonlocal feature of the quantum state guarantees that (i) Alice can encode 2 bits of information by preparing one of the four Bell states $|\Psi^+\rangle$, $|\Psi^-\rangle$, $|\Phi^+\rangle$, and $|\Phi^-\rangle$ through local operation on her photon and (ii) a joint measurement of the twin photons with a perfect Bell-state analyzer is the only way to decode the 2 bits of information [1].

The information capacity of a transmitted photon, in dense coding quantum communication, is determined by the number of usable nonlocal quantum states of twin photons [1,5]. Here a question arises as to whether Alice can enhance the capacity of her photon or not, by utilizing an additional degree of freedom, i.e., the time or frequency domain, by means of pulse position modulation or optical frequency modulation. At first, capacity enhancement seems to be possible by using another degree of freedom. The question here is, however, whether or not it is possible by manipulating only one of the twin photons. This is the question in line with the dense coding of a photon. Then the answer apparently appears to be ''no,'' because the modulation of one photon generally makes the twin photons distinguishable, and this destroys the two-photon interference that is necessary for dense coding quantum communication [5]. If Alice can modulate the nonlocal quantum state of twin photons in the time or frequency domain by manipulating only one photon, there is a possibility of further dense coding.

This paper proposes a local operation and joint measurement scheme for enhancing the information capacity from 2 bits to 3 bits in dense coding quantum communication. Here a frequency-dependent phase correlation between twin photons is employed as a degree of freedom in addition to the entanglement in the polarization variables. Alice can manipulate the nonlocal quantum state by applying a frequency-dependent phase shift to her photon provided the entangled twin photons have a spectral correlation. Section II explains the basic concept of quantum communication with entangled twin photons, where two Bell states $|\Psi^+\rangle$ and $|\Psi^-\rangle$ are considered. In Sec. III, we show that frequency-dependent phase operation can enhance the capacity from 1 bit to 2 bits for a pair of two Bell states $|\Psi^+\rangle$ and $|\Psi^-\rangle$. In Sec. IV, we propose a perfect Bell-state analyzer containing a nonlinear optical gate, and finally, we show an implementation for enhancing the capacity from 2 bits to 3 bits in dense coding quantum communication.

II. QUANTUM COMMUNICATION WITH ENTANGLED TWIN PHOTONS VIA TWO-PHOTON INTERFERENCE

First, we briefly explain an simple optical configuration for a quantum communication scheme with entangled twin photons. It can be modeled by two-photon interference of the Hong-Ou-Mandel type [5,6] as shown in Fig. 1. The polarization entangled photon twins are generated by means of type-II noncollinear spontaneous parametric down-conversion (SPDC) [7,8]. We regard the photon propagating through path A as one to be sent to Alice (step 1 in Fig. 1). Here we define this photon as the signal. Bob keeps the remaining ''idler photon.'' The initial quantum state of the photon twins is $|\Psi^+\rangle$ which is expressed as $(|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B)/\sqrt{2}$. Here H and V are the horizontal and vertical linear polarization states, respectively. The subscripts A and B denote the photons sent to Alice and kept by Bob, respectively. Alice's encoder is composed of a linear polarization beam splitter (LPBS), a phase shifter for the vertical

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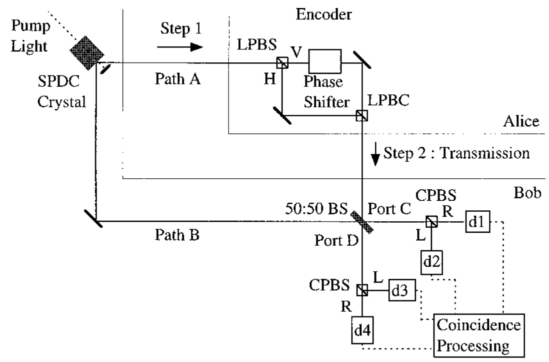

FIG. 1. Basic configuration of a quantum communication system with entangled twin photons. Step 1, sending the signal photon to Alice in advance; step 2, transmitting the signal photon to Bob after encoding information.

polarization component and a linear polarization beam combiner (LPBC).

With this optical configuration, Alice can utilize two Bell states $|\Psi^+\rangle$ and $|\Psi^-\rangle$, where the latter is $(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)/\sqrt{2}$. By operating the phase shift π for the vertical polarization component of her signal photon, Alice can change the nonlocal quantum state from $|\Psi^+\rangle$ to $|\Psi^-\rangle$ and encode 1 bit of information in the states. Then Alice transmits her signal photon to Bob (step 2 in Fig. 1). The transmitted signal photon and Bob's idler photon are superposed by a 50:50 beam splitter (BS), which is positioned symmetrically in relation to the optical path lengths. The configuration is similar to that of the Hong-Ou-Mandel two-photon interferometer [6]. A circular polarization beam splitter (CPBS) is inserted in each output direction of the BS, as shown in Fig. 1. Here R and L are the right and left circular polarization states, respectively. Finally, the photons are counted by single-photon detectors numbered $d1-d4$. Bob can decode the information encoded in the entangled quantum states by registering coincidence counting of them [5,9,10]. Table I summarizes the registration results in the detectors for the two Bell states $|\Psi^+\rangle$ and $|\Psi^-\rangle$. Two photons are routed to the same output ports with the same circular polarization state for $|\Psi^+\rangle$ while they are directed to different output ports with different circular polarization states for $|\Psi^-\rangle$. In this way, 1 bit of information can be encoded in the nonlocal quantum states through local operation on the signal photon and it can be decoded by the joint measurement.

If Alice adopts an additional Bell state $|\Phi^+\rangle$, the information capacity can be increased from 1 bit to 1.53 bits in the two-photon interferometer. She can prepare Bell states $|\Phi^\pm\rangle=(|H\rangle_A|H\rangle_B \pm |V\rangle_A|V\rangle_B)/\sqrt{2}$ by rotating the linear polarization through 90° [1,5]. For the Bell state $|\Phi^+\rangle$, two photons are routed to the same BS output ports with different circular polarization states. This registration result can be distinguished from those for $|\Psi^+\rangle$ and $|\Psi^-\rangle$. However, $|\Phi^-\rangle$ leads to the same registration result as that for $|\Psi^+\rangle$. Thus the number of usable quantum states is limited to 3 and the information capacity is limited to 1.53 bits as long as Bob employs a two-photon interferometer as an imperfect Bell-state analyzer, which is the only joint measurement demonstrated experimentally to date [5,9,10].

TABLE I. Bob's registration results for Alice's manipulations (polarization antiparallel, without enhancement).

Alice's phase shift setting $|\Psi(\phi_1,\phi_2,\phi_3,\phi_4)\rangle$	Bob's registration results
$|\Psi^+\rangle$ $[=|\Psi(0,0,0,0)\rangle]$	Two photons are routed to the same detectors; either $d1$, $d2$, $d3$, or $d4$.
$|\Psi^-\rangle$ $[=|\Psi(0,0,\pi,\pi)\rangle]$	$(d1,d3)$, $(d2,d4)$

III. FREQUENCY-DEPENDENT PHASE SHIFT

To enhance the information capacity of the transmitted signal photon, Alice should be able to change the nonlocal quantum state of the photon twins by manipulating only her signal photon. Frequency-dependent phase shifting meets these requirements provided the twin photons generated through SPDC [7,8] exhibit both spectral and phase correlation. The quantum state can be expressed as follows by adopting the spectral correlation of twin photons;

$$|\Psi\rangle \propto \int_{-W}^{W} \{(\exp[i\phi_H(\omega_d+u)]|H,\omega_d+u\rangle)_A|V,\omega_d-u\rangle_B + (\exp[i\phi_V(\omega_d-u)]|V,\omega_d-u\rangle)_A|H,\omega_d+u\rangle_B\}du. \tag{1}$$

For theoretical simplicity, we assume a nearly monochromatic pump frequency $2\omega_d$ and a rectangular spectral broadening of 2 W around the center frequency ω_d, which is the degenerate frequency. These assumptions are valid and do not lose generality because our proposed scheme is independent from an actual spectral profile of the signal photon with the proviso that the spectral profile is symmetric around the degenerate frequency ω_d, which is usually satisfied. As the optical phases of the photons are uncertain while their sum has a definite value, the phase shift of one photon can alter the absolute phase of the whole system. Polarization-dependent relative phase $\phi(\omega)$ is defined by $\phi(\omega) = \phi_H(\omega_d+u) - \phi_V(\omega_d-u)$. The phase shifts $\phi(\omega)=0$ and $\phi(\omega)=\pi$ correspond to $|\Psi^+\rangle$ and $|\Psi^-\rangle$, respectively. Alice can further utilize the frequency dependence of ϕ_H and ϕ_V.

For a pair of the two Bell states $|\Psi^+\rangle$ and $|\Psi^-\rangle$, she applies a different phase shift to her photon depending on whether the frequency is higher or lower than the degenerate frequency ω_d. Figure 2(a) shows our modified configuration for the quantum communication system. An optical wavelength decombiner (WLDCOM) splits the wave packet of her photon into higher (ω_H) and lower (ω_L) frequency parts in the frequency domain, as illustrated in Fig. 2(b). Each partial wave packet undergoes a polarization-dependent phase shift of 0 or π. They are then recombined via a wavelength combiner (WLCOM) as shown in Fig. 2(b) and the photon is transmitted to Bob. The two-photon quantum-state input into the BS can be represented as

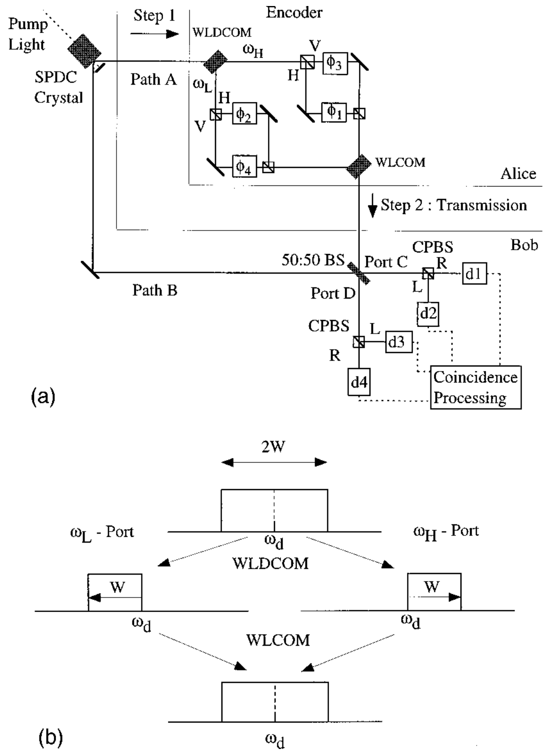

FIG. 2. (a) Modified configuration of the quantum communication system with entangled twin photons. (b) Division and recombination of the photon wave packet in the frequency domain via WLDCOM and WLCOM, respectively.

$$|\Psi(\phi_1,\phi_2,\phi_3,\phi_4)\rangle_{\text{in}} = \frac{1}{\sqrt{4W}}\int_0^W \{(\exp[i\phi_1]|H,\omega_d+u\rangle)_A|V,\omega_d-u\rangle_B + (\exp[i\phi_2]|H,\omega_d-u\rangle)_A|V,\omega_d+u\rangle_B + (\exp[i\phi_3]|V,\omega_d+u\rangle)_A|H,\omega_d-u\rangle_B + (\exp[i\phi_4]|V,\omega_d-u\rangle)_A|H,\omega_d+u\rangle_B\}du. \tag{2}$$

The ket vector $|\Psi\rangle_{\text{in}}$ is normalized so that ${}_{\text{in}}\langle\Psi|\Psi\rangle_{\text{in}}=1$ and the subscript in means the input state to the 50:50 BS. Two-photon interference arises from the pair of the first and last terms in Eq. (2). Another pair of the second and third terms also brings about two-photon interference.

If Alice arranges all the phase shifts so that they are 0, the input state $|\Psi(0,0,0,0)\rangle$ becomes $|\Psi^+\rangle$. Similarly, $|\Psi(0,0,\pi,\pi)\rangle$ corresponds to $|\Psi^-\rangle$. When Alice sets the phase shifts as $(\phi_1,\phi_2,\phi_3,\phi_4)=(0,\pi,\pi,0)$ by adopting frequency-dependent phase operation, the quantum state of the two photons output from the circular polarization beam splitters is represented by

$$|\Psi(0,\pi,\pi,0)\rangle_{\text{out}} = -i\frac{1}{\sqrt{4W}}\int_0^W \{|R,+u\rangle_C|L,-u\rangle_C - |R,-u\rangle_C|L,+u\rangle_C + |R,-u\rangle_D|L,+u\rangle_D - |R,+u\rangle_D|L,-u\rangle_D\}du, \tag{3}$$

TABLE II. Bob's registration results for Alice's manipulations (polarization antiparallel, with enhancement).

Alice's phase shift setting $\|\Psi(\phi_1,\phi_2,\phi_3,\phi_4)\rangle$	Bob's registration results
$\|\Psi(0,\pi,\pi,0)\rangle$	$(d1,d2)$, $(d3,d4)$
$\|\Psi(0,\pi,0,\pi)\rangle$	$(d1,d4)$, $(d2,d3)$

where the degenerate frequency ω_d is omitted for simplicity. R and L are the right and left circular polarization states, respectively. Thus two photons are routed to the same BS output ports: i.e., C or D with different circular polarization states. For $(\phi_1,\phi_2,\phi_3,\phi_4)=(0,\pi,0,\pi)$, the output quantum state becomes

$$|\Psi(0,\pi,0,\pi)\rangle_{\text{out}} = -i\frac{1}{\sqrt{4W}}\int_0^W \{|R,+u\rangle_C|R,-u\rangle_D - |R,-u\rangle_C|R,+u\rangle_D - |L,+u\rangle_C|L,-u\rangle_D + |L,-u\rangle_C|L,+u\rangle_D\}du. \tag{4}$$

Here, the two photons are routed to different output ports with the same circular polarization states. Therefore, the four quantum states $|\Psi(0,0,0,0)\rangle(=|\Psi^+\rangle)$, $|\Psi(0,0,\pi,\pi)\rangle(=|\Psi^-\rangle)$, $|\Psi(0,\pi,\pi,0)\rangle$ and $|\Psi(0,\pi,0,\pi)\rangle$ prepared by Alice are completely distinguishable to Bob. The relations between the frequency-dependent phase shifts and the registration results are summarized in Table II. Thus Alice can successfully enhance the information capacity of her photon from 1 bit to 2 bits by applying frequency-dependent phase shift to her photon. In other words, Alice can prepare the four different nonlocal quantum states through local operation on her signal photon and Bob can distinguish them perfectly with the two-photon interferometer. Here we should note that the asymmetric polarization beam splitting H-V and R-L between Alice and Bob is necessary for discriminating these four states.

As the capacity is limited to 1.53 bits as long as Bob employs a two-photon interferometer as an imperfect Bell state analyzer, this capacity enhancement to 2 bits by means of frequency-dependent phase operation may offer a practical advantage for certain kinds of quantum communication with entangled twin photons.

Here an experimental feasibility of the frequency-dependent phase shift should be mentioned. As it is impossible to split the frequency spectrum sharply at $u=0$ actually, we suppose that the spectral transmittance of each output port of a practical WLDCOM and WLCOM can be represented by a Lorentzian with a peak frequency ω_H or ω_L to simulate long tails around the peak. The peak frequency deviation from the center frequency ω_d is given by $w=|\omega_H|=|\omega_L|$ and half width at half maximum of the peak is Γ. Because of the spectral overlap between the two output ports, error occurs in the coincidence counting results with a finite probability. For the ratio $\Gamma/w=\frac{1}{5}$, even though Alice sets the phase shift arrangement as $|\Psi(0,\pi,\pi,0)\rangle$, Bob obtains the wrong result $(d1,d4)$ or $(d2,d3)$, which correspond to $|\Psi(0,\pi,0,\pi)\rangle$, with the probability of approximately 0.2%. Such a kind of WLDCOM, for example, $2W\sim200$

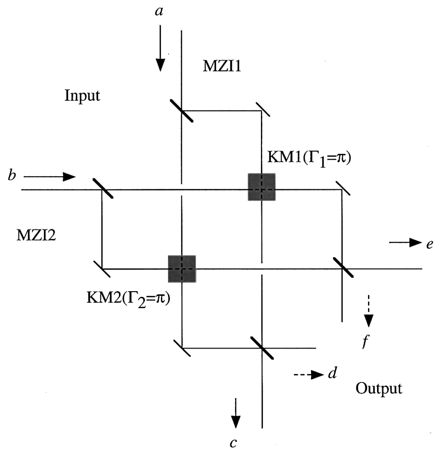

FIG. 3. Configuration of our proposed nonlinear optical gate (NOG). KM, optical Kerr medium; MZI, Machzehnder interferometer.

GHz and $\Gamma \sim 10$ GHz, have been realized at least for the wavelength band ($\lambda = 1.3 - 1.5$ μm) as an important device for wavelength division multiplexing optical fiber communication system [14].

IV. CAPACITY ENHANCEMENT IN DENSE CODING QUANTUM COMMUNICATION

The four Bell states $|\Psi^+\rangle, |\Psi^-\rangle, |\Phi^+\rangle$, and $|\Phi^-\rangle$ are mutually orthogonal quantum states for polarization entangled twin photons and these states can be prepared by a local operation on the signal photon. The information capacity of the signal photon, therefore, is 2 bits in dense coding quantum communication if a perfect Bell-state analyzer is available. Moreover, the number of mutually orthogonal quantum states can be increased to eight through the local operation of the frequency-dependent phase shift for each Bell state. Therefore, we can expect to enhance the capacity to 3 bits provided the eight nonlocal quantum states are distinguishable from each other. In this section, we propose a scheme for enhancing the capacity from 2 bits to 3 bits. First, we describe the nonlinear optical gate (NOG) necessary for our perfect Bell-state analyzer and then outline an implementation for this enhancement.

A. Nonlinear optical gate

The configuration of our proposed nonlinear optical gate is shown in Fig. 3. Here two identical Mach-Zehnder interferometers MZI1 and MZI2 intersect and optical Kerr media KM1 and KM2 are inserted at the two points where their optical paths cross. In each optical Kerr medium, cross phase modulation (XPM) between two photons shifts their optical phases [11,12]. The signal and idler photons are input into the NOG from ports *a* and *b*, respectively. the phase shifts imposed by XPM are Γ_1 and Γ_2 for KM1 and KM2, respectively and are assumed to be polarization independent. The MZI1 output ports are ports *c* and *d*, and the MZI2 output ports are *e* and *f*. By using the above notation, the output states $|\Psi\rangle_{\rm out}$ of the two photons can be represented by

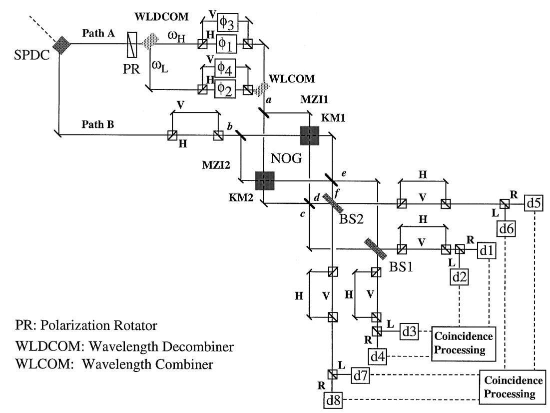

FIG. 4. Modified configuration of the dense coding quantum communication with an enhanced capacity.

$$|\Psi\rangle_{\rm out} = \frac{1}{4}\{(\exp[i\Gamma_1] - \exp[i\Gamma_2])(|d\rangle|e\rangle + |c\rangle|f\rangle) + (\exp[i\Gamma_1] + \exp[i\Gamma_2] + 2)|c\rangle|e\rangle + (\exp[i\Gamma_1] + \exp[i\Gamma_2] - 2)|d\rangle|f\rangle\}, \quad (5)$$

where $|c\rangle$ means $|1\rangle_c$. For $\Gamma_1 = \Gamma_2 = \pi$, two photons are routed to ports *d* and *f*, respectively, whenever they are input simultaneously. By contrast, if they are launched at different times and do not interact with each other in the optical Kerr media, i.e., $\Gamma_1 = \Gamma_2 = 0$, they are output from ports *c* and *e*, respectively.

B. Perfect Bell-state analysis and capacity enhancements to 3 bits

By employing our proposed NOG, the two-photon interferometer described in the previous section can be extended so that Bob can decode three bits of information. As shown in Fig. 4, the two-photon interferometer is further modified as follows. (i) A polarization rotator is set in path *A* so that Alice can prepare not only $(|\Psi^\pm\rangle = (|H\rangle_A|V\rangle_B \pm |V\rangle_A|H\rangle_B)/\sqrt{2}$ but also $(|\Phi^\pm\rangle = (|H\rangle_A|H\rangle_B \pm |V\rangle_A|V\rangle_B)/\sqrt{2}$. (ii) Optical delay lines are inserted in all the optical paths of the *V*-polarization components. The imposed delay time τ must be sufficiently long so that the *H*- and *V*-polarization components do not overlap each other in the time domain. (iii) The signal and idler photons are launched into NOG ports *a* and *b*, respectively. (iv) The two photons output from NOG ports *d* and *f* are superposed by a 50:50 beam splitter (BS2) and those from ports *c* and *e* are superposed by another 50:50 beam splitter (BS1). (v) In each BS output port, the *H*-polarization component undergoes a time delay τ through an optical delay line after the linear polarization beam splitting. (vi) Then the *H*- and *V*-polarization components are superposed again in the time domain and each photon is directed to a circular polarization beam splitter (CPBS). Here single-photon detectors are renumbered $d1 - d8$ as shown in Fig. 4.

Two-photons are routed to BS1 for $|\Psi^\pm\rangle$. This is because the linear polarization states of the signal and idler photons are antiparallel and these photons pass through optical Kerr media at different times. Even if two photons arrive at BS1

TABLE III. Bob's registration results for Alice's manipulations (polarization parallel, without enhancement).

Alice's phase shift setting $\|\Phi(\phi_1,\phi_2,\phi_3,\phi_4)\rangle$	Bob's registration results
$\|\Phi^+\rangle$ $[=\|\Phi(0,0,0,0)\rangle]$	$(d5,d6)$, $(d7,d8)$
$\|\Phi^-\rangle$ $[=\|\Phi(0,0,\pi,\pi)\rangle]$	Two photons are routed to the same detectors; either $d5$, $d6$, $d7$, or $d8$.

TABLE IV. Bob's registration results for Alice's manipulations (polarization parallel, with enhancement).

Alice's phase shift setting $\|\Phi(\phi_1,\phi_2,\phi_3,\phi_4)\rangle$	Bob's registration results
$\|\Phi(0,\pi,\pi,0)\rangle$	$(d5,d8)$, $(d6,d7)$
$\|\Phi(0,\pi,0,\pi)\rangle$	$(d5,d7)$, $(d6,d8)$

at different times, two-photon interference occurs between the $|H\rangle_A|V\rangle_B$ and $|V\rangle_A|H\rangle_B$ terms [13]. Therefore, the registration results summarized in Tables I and II are valid for $\{|\Psi(0,0,0,0)\rangle(=|\Psi^+\rangle),|\Psi(0,0,\pi,\pi)\rangle(=|\Psi^-\rangle)\}$ and $\{|\Psi(0,\pi,\pi,0)\rangle,|\Psi(0,\pi,0,\pi)\rangle\}$, respectively.

If the linear polarization of the signal photon is rotated through 90° by Alice, a two-photon quantum-state input into NOG can be expressed as

$$\begin{aligned}|\Phi(\phi_1,\phi_2,\phi_3,\phi_4)\rangle_{\rm in} &= \frac{1}{\sqrt{4W}}\int_0^W\{(\exp[i\phi_3]|V,\omega_d+u\rangle)_A|V,\omega_d-u\rangle_B \\ &+(\exp[i\phi_4]|V,\omega_d-u\rangle)_A|V,\omega_d+u\rangle_B \\ &+(\exp[i\phi_1]|H,\omega_d+u\rangle)_A|H,\omega_d-u\rangle_B \\ &+(\exp[i\phi_2]|H,\omega_d-u\rangle)_A|H,\omega_d+u\rangle_B\}du. \qquad (6)\end{aligned}$$

Here two-photon interference occurs between the first and second terms in Eq. (6). Two-photon interference also arises from the pair of the third and last terms. The linear polarization states of the signal and idler photons are parallel so two photons are directed to BS2 after experiencing XPM through the NOG. If Alice arranges all the phase shifts so that they are 0, the state $|\Phi(0,0,0,0)\rangle$ becomes $|\Phi^+\rangle$ and they are guided to the same BS2 output ports with different circular polarization states. Similarly, $|\Phi(0,0,\pi,\pi)\rangle$ corresponds to $|\Phi^-\rangle$ and two photons are directed to the same single-photon detector. Table III summarizes the registration results for single-photon detectors $d5$–$d8$ with the different phase shift arrangements $\{|\Phi(0,0,0,0)\rangle(=|\Phi^+\rangle), |\Phi(0,0,\pi,\pi)\rangle(=|\Phi^-\rangle)\}$. Tables I and III show that a perfect Bell-state analysis can be accomplished and that the information capacity of the signal photon becomes 2 bits without enhancement.

When Alice sets the phase shifts at $|\Phi(0,\pi,\pi,0)\rangle$ by applying a frequency-dependent phase shift to her signal photon, the two photons are routed to the different BS2 output ports with the same circular polarization states. In a similar way, for $|\Phi(0,\pi,0,\pi)\rangle$, two photons are directed to different BS2 output ports with different circular polarization states. Table IV shows these registration results in detectors $d5$–$d8$. Therefore, the four quantum states $|\Phi(0,0,0,0)\rangle(=|\Phi^+\rangle)$, $|\Phi(0,0,\pi,\pi)\rangle(=|\Phi^-\rangle)$, $|\Phi(0,\pi,\pi,0)\rangle$, and $|\Phi(0,\pi,0,\pi)\rangle$ prepared by Alice are completely distinguishable to Bob. If the nonlinear optical gate does not function, the registration results for these four states are identical to those for $|\Psi(0,\pi,\pi,0)\rangle$, $|\Psi(0,0,0,0)\rangle(=|\Psi^+\rangle)$, $|\Psi(0,\pi,0,\pi)\rangle$, and $|\Psi(0,0,\pi,\pi)\rangle(=|\Psi^-\rangle)$, respectively.

In this way, as summarized in Tables I–IV, the number of mutually orthogonal nonlocal quantum states can be increased to eight by the local operation of a frequency-dependent phase shift on the signal photon and these states can be distinguished by a joint measurement employing the nonlinear optical gate. Therefore Alice can enhance the capacity of one photon from 2 bits to 3 bits in dense coding quantum communication even though she seems to modulate a local redundancy of the transmitted photon in the frequency domain. The frequency-dependent optical phase, however, is not the redundancy attributed to the local quantum state of each entangled twin photon as mentioned in Sec. III. The frequency-dependent phase shift applied to one photon does not change the local quantum state of the photon. Therefore this enhancement does not contradict quantum mechanics.

If this enhancement by means of a frequency-dependent phase shift is unnecessary, the Bell-state analyzer can be simplified as shown in the Appendix. To our regret, however, our proposed NOG seems experimentally infeasible until an optical Kerr medium is developed which has a sufficiently large optical nonlinearity. The two-photon coupling imposed by cavity-QED techniques is still insufficient for our purpose at this stage [15].

V. DISCUSSION

As described in previous sections, we can enhance the capacity of a photon in dense coding quantum communication provided the photon is generated through spontaneous parametric down-conversion (SPDC). This enhancement does not require any broadening in the transmission or receiver bandwidth and this seems curious in terms of conventional classical transmission theory. To clarify this point, we should discuss the information capacity per unit time.

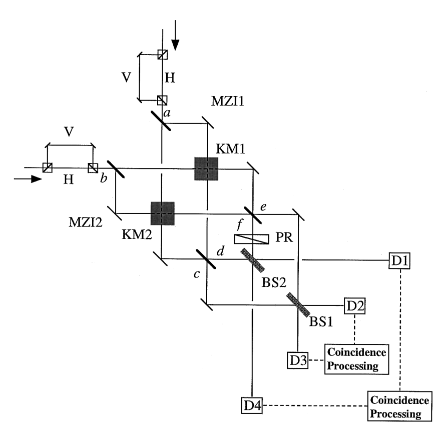

FIG. 5. Simplified configuration of the perfect Bell-state analyzer.

The repetition period T_p of the pump pulse launched into an SPDC crystal determines the number of down-converted signal photon pulses per unit time. The period, however, cannot be greatly shortened as explained in the following. The half value of the pump light frequency, i.e., degenerate frequency ω_d, must be nearly the same as the center frequency of the transmission band for the signal and idler photons so that both of them fall in the transmission band. This means that the spectral broadening $\sim 1/T_p$ of the pump pulse must be sufficiently narrower than the given transmission bandwidth 2 W for down-converted photons. Therefore, the average time interval between two successive signal photons, or the repetition period of the signal photon, is considerably longer than the time duration of each photon wave packet given by $\sim 1/2$ W.

Although the repetition period of the signal photon is limited by that of the pump pulse, the information capacity per signal photon can be enhanced by the frequency-dependent phase shift. The enhancement doubles the time duration of each wave packet but the repetition period is still longer than the doubled time duration. Therefore, the information capacity per unit time can be increased without broadening the transmission bandwidth. We can conclude that the enhancement improves an efficiency in the time domain.

VI. CONCLUSION

We proposed a scheme for enhancing the information capacity in dense coding quantum communication that involves transmission of a polarization entangled twin photon and a subsequent joint measurement using a Bell-state analyzer. It is clear that the direct modulation of local redundancies such as the pulse position and optical frequency cannot be used for capacity enhancement. We have shown, however, that the phase correlation of twin photons can be employed as a usable redundancy. The information sender can manipulate the nonlocal quantum state of twin photons through the local operation of a frequency-dependent phase shift. With our proposed Bell-state analyzer containing a nonlinear optical gate, we can enhance the capacity of a photon from 2 bits to 3 bits in dense coding quantum communication.

TABLE V. Registration results for four different Bell states.

Four Bell states	Bob's registration results
$\|\Psi^+\rangle$	Two photons are routed to the same detectors; either $D2$ or $D3$.
$\|\Psi^-\rangle$	$(D2,D3)$
$\|\Phi^+\rangle$	$(D1,D4)$
$\|\Phi^-\rangle$	Two photons are routed to the same detectors; either $D1$ or $D4$.

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APPENDIX

The Bell-state analyzer containing a nonlinear optical gate (NOG) can be simplified if this enhancement is unnecessary. The simplified configuration is shown in Fig. 5, where the number of single-photon detectors can be reduced to 4. They are numbered $D1-D4$. After experiencing a polarization-dependent time delay, each photon is launched into the NOG. The photon output from port f undergoes polarization rotation through 90° before arriving at BS2 so that $|\Phi^+\rangle$ and $|\Phi^-\rangle$ can be transformed into $|\Psi^+\rangle$ and $|\Psi^-\rangle$, respectively. The registration results in the detectors are summarized in Table V for the four Bell states.

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