Interferometric measurements of the position of a macroscopic body: Towards observation of quantum limits

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(Received 2 September 1997; revised manuscript received 24 September 1998)

An optomechanical sensor suitable for the study of quantum effects has been developed and characterized. The sensor reads out the vibrations of a microfabricated miniature silicon mechanical oscillator which forms one end mirror of a high finesse Fabry-Pérot cavity. The mechanical quality factor is up to $Q=300\,000$ at 300 K and rises up to $Q=4\times10^6$ at 4 K. The thermal noise of the oscillator has been measured in the time and frequency domains at room temperature and at 4.5 K. The prospects for observing the standard quantum limit are discussed. [S1050-2947(99)07002-X]

PACS number(s): 03.65.Bz, 42.50.−p

I. INTRODUCTION

At the quantum level, a measurement disturbs the system being measured. The backaction due to the quantum nature of the probe is usually very small and has not yet been observed for a macroscopic system. The existence of the quantum backaction leads to the appearance of a quantum limit in the accuracy of measurements, the standard quantum limit (SQL) [1]. For measurements of the position of a harmonic oscillator of mass m and resonance frequency ω_m, this is $\Delta x_{\mathrm{SQL}}=\sqrt{\hbar/2m\omega_m}$. For a laser interferometric measurement, the backaction can be understood in a straightforward manner [2,3]. The probing laser wave carries quantum fluctuations. For a coherent state of light, the quantum uncertainties of phase and intensity are equal and proportional to the square root of the laser power P. The intensity quantum fluctuations cause radiation pressure noise which leads to displacement noise whose uncertainty scales with $\sqrt{P}$. At high enough laser power this noise will become appreciable. On the other hand, the read-out uncertainty due to quantum noise scales with $1/\sqrt{P}$, since the signal is $\sim P$, and dominates at low laser powers. An optimum laser power exists that minimizes the sum of the read-out noise and that due to backaction. The measurement accuracy corresponding to this optimum readout precision is the standard quantum limit. The SQL is of fundamental interest, because it limits the sensitivity with which an external force acting on the macroscopic body may be detected using coherent light [4].

The observation of quantum effects with optomechanical sensors requires the development of oscillators with low $m\omega_m$, their integration into a sensitive read-out system, and minimization of the oscillator's thermal noise, which can easily mask the quantum effects. While optomechanical systems have received much theoretical attention [5], only few experimental studies have been performed after the first observation of radiation pressure effects in an optical cavity [6]. Concerning small-scale sensors, Pang and Richard [7] have reported a room-temperature optical sensor designed for the read-out of the excitation of a Weber bar antenna. Nonlinear mechanical effects were studied on microresonators [8,9] and backaction effects were investigated on a 0.3 kg cryogenic electromechanical sensor [10].

In this paper we present an optomechanical system, based on a Fabry-Pérot cavity, whose features approach the regime where observation of quantum backaction and of the standard quantum limit should be possible: it exhibits a high cavity finesse in order to enhance the backaction for a given external laser power, and a high mechanical quality factor, to reduce Brownian noise.

II. THEORETICAL CONSIDERATIONS

The optomechanical sensor we discuss is sketched in Fig. 1. The Fabry-Pérot cavity consists of a rigid incoupling mirror and a mirror oscillator. A narrow linewidth laser wave is coupled into the cavity and the reflected wave is extracted using a combination of a polarizing beamsplitter and a quarter-wave plate. The oscillator motion causes a cavity length change and thus a detuning between the read-out laser frequency and the resonator frequency. It is detected using a standard frequency modulation (FM) technique [11]: The laser light is phase modulated to produce sidebands at an offset frequency exceeding the cavity bandwidth. The photodetec-

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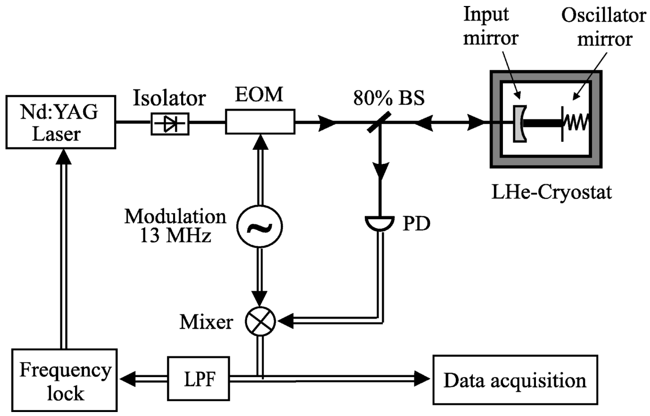

FIG. 1. Schematic of the optomechanical sensor system. The incoupling mirror is rigidly mounted; the other cavity mirror is the torsional oscillator. EOM, electro-optic modulator; BS, beam splitter; PD, photodiode; LPF, low pass filter.

tor signal obtained from the reflected light is mixed with a local oscillator and low-pass filtered to yield an error signal proportional to the detuning of laser frequency from the cavity frequency. The frequency components around ω_m provide information about the mechanical oscillation. The low-frequency part of the error signal at frequencies substantially smaller than ω_m is used for stabilizing the laser frequency to the average cavity frequency, thus compensating for any relative drift. The servo locking bandwidth is chosen sufficiently smaller than ω_m. For a detailed theoretical treatment of the optomechanical sensor, we refer to Ref. [12].

First we review the SQL, which is usually considered in the absence of dissipation, i.e., for observation times $\tau \ll \tau_m = 2Q/\omega_m$, the mechanical relaxation time of the oscillator. Further below we also consider the case $\tau \gg \tau_m$ (Ref. [13]). For a FM measurement of the position of the movable mirror, the read-out uncertainty due to shot noise is

$$\Delta x_{\mathrm{SN}}^2 = \frac{S_{\mathrm{SN}}}{2\tau} = \frac{\lambda hc}{128\eta P \mathcal{F}^2 \tau} \frac{[J_0(\epsilon)\alpha]^2 + 2J_1(\epsilon)^2}{(r_1-\alpha)^2 J_0(\epsilon)^2 J_1(\epsilon)^2}, \quad (1)$$

where S_{SN} is the low-frequency power spectral density of displacement [14]. We use one-sided spectral densities. η is the quantum efficiency of the photodetector, $\mathcal{F}$ the finesse of the cavity, and r_1 and r_2 are amplitude reflection coefficients of the incoupling and the rear mirror, respectively. J_n are Bessel functions with ϵ being the phase-modulation index. The parameter α is the on-resonance carrier field reflection coefficient and is given by $\alpha = [r_1 - r_2(1-a_1^2)]/(1-r_1 r_2)$, where a_1^2 is the power loss of the input coupler. This expression is valid for a measurement time τ longer than the cavity decay time, weak modulation, and perfect mode match of the laser wave into the resonator. For simplicity, we consider in the following the case of a lossless input coupler ($a_1^2 = 0$), perfect impedance match ($r_1 = r_2$), and high finesse ($r_1 \simeq r_2 \simeq 1$), so that

$$\Delta x_{\mathrm{SN}}^2 = \frac{\lambda hc}{64\eta J_0(\epsilon)^2 P \mathcal{F}^2 \tau}, \quad (2)$$

with $\mathcal{F} = 2\pi/(2 - r_1^2 - r_2^2)$. Note that this is independent of ϵ as $\epsilon \to 0$ because of the assumption of precise impedance match.

The backaction due to radiation pressure fluctuations is described by the momentum uncertainty transfered by the circulating light to the oscillator:

$$\Delta p_{\mathrm{BA}} = 2\hbar k \sqrt{\frac{\lambda P J_0(\epsilon)^2 \tau}{hc}} \frac{\mathcal{F}}{\pi}. \quad (3)$$

Here, $2\hbar k$ is the momentum transfer per photon and the other terms are the uncertainty of the photon number hitting the oscillator mirror. $\mathcal{F}/\pi$ is the power enhancement for the impedance-matched cavity. Note the linear dependence on $\mathcal{F}$, which results from the fact that the signal to quantum noise ratio of the intensity of the light circulating within the cavity must be the same as that outside the cavity. In the time interval τ, the uncertainty in the radiation pressure force is given by $\Delta F_{\mathrm{BA}} = \Delta p_{\mathrm{BA}}/\tau$. For the undamped harmonic oscillator the resulting uncertainty in the position is

$$\Delta x_{\mathrm{BA}}^2 = \left(\frac{\Delta F_{\mathrm{BA}} \tau}{2m\omega_m}\right)^2 = \frac{hJ_0(\epsilon)^2 P \mathcal{F}^2 \tau}{\pi^2 c \lambda m^2 \omega_m^2}. \quad (4)$$

This equation is valid at resonance and when $1/\omega_m \ll \tau \ll \tau_m$. Expressions essentially equal to Eqs. (2) and (4) may be obtained also for the case of side-of-fringe detection, when a small detuning is chosen.

The total error is the quadrature sum of the individual errors, $\Delta x_{\mathrm{tot}}^2 = \Delta x_{\mathrm{SN}}^2 + \Delta x_{\mathrm{BA}}^2$, since the shot noise originates from the sidebands and is uncorrelated with the shot noise of the carrier that enters the cavity and causes the radiation pressure fluctuations. Minimizing the total error with respect to τ we are led to the SQL,

$$\Delta x_{\mathrm{SQL}}^2 = \frac{\hbar}{2\sqrt{\eta} m \omega_m}, \quad (5)$$

at the optimum measurement time

$$\tau_{\mathrm{opt}} = \frac{\pi \lambda c m \omega_m}{8\sqrt{\eta} J_0^2(\epsilon) P \mathcal{F}^2}. \quad (6)$$

Note that the usual SQL value is only reached for an ideal photodetector with $\eta = 1$. A nonunity quantum efficiency implies loss of information and thus an increase in the minimum uncertainty. Expression (6) implies that there is a well-defined total optical energy $E_{\mathrm{SQL}} = P\tau_{\mathrm{opt}}$ that must be expended in order to reach the SQL. This energy is partially absorbed in the photodetector and partially dissipated in the cavity. In an application where the temporal change of an external force acting on the oscillator is to be monitored, the power P would be chosen such that τ_{opt} is equal to the desired sampling time. Note that the product $\Delta x_{\mathrm{SN}}(\tau) \Delta p_{\mathrm{BA}}(\tau) = \hbar/2\sqrt{\eta}$ satisfies the uncertainty relation.

The condition for the dissipation being negligible is equivalent to requiring that the Brownian noise $\Delta x_{\mathrm{therm}}(\tau)$ is small during the measurement time [15]. Its value for $\tau \ll \tau_m$ is [16]

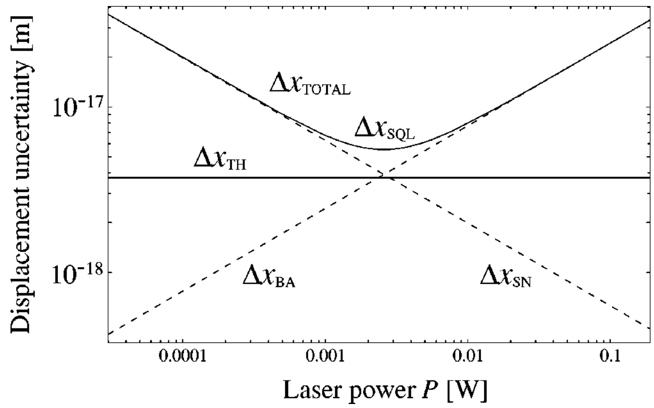

FIG. 2. Calculated short-time displacement uncertainties resulting from read-out shot noise, quantum backaction noise, and thermal noise as a function of the laser power. The backaction regime, where Δx_{BA} dominates, can be reached with reasonable laser power. Parameters for calculations are $T=2$ K, finesse $\mathcal{F}=80\,000$, measurement time $\tau=12\,\mu$s, mechanical quality factor $Q=1.5\times10^7$, detection efficiency $\eta=1$.

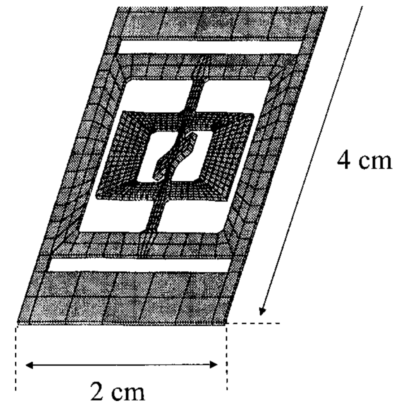

FIG. 3. The employed silicon torsional oscillator. The antisymmetric oscillation mode calculated using a finite element simulation is shown (with exaggerated amplitude). The torsion angle of the outer frame is too small to be visible.

$$\Delta x^2_{\mathrm{therm}}=\frac{2k_BT\tau}{m\omega_m^2\tau_m},\tag{7}$$

so that the condition $\Delta x_{\mathrm{therm}}(\tau_{\mathrm{opt}})<\Delta x_{\mathrm{SQL}}$ translates to

$$\tau_{\mathrm{opt}}<\frac{\hbar Q}{2\sqrt{\eta}k_BT}.\tag{8}$$

This is much more stringent than $\tau_{\mathrm{opt}}<\tau_m$, since in practice $k_BT/\hbar\omega_m\gg1$. Since we also need $\tau_{\mathrm{opt}}>1/\omega_m$, the mechanical quality factor must satisfy

$$Q>\sqrt{\eta}\,\frac{2k_BT}{\hbar\omega_m}.\tag{9}$$

Thus, low mechanical loss is a crucial requirement to suppress thermal noise during the observation time. Equations (6) and (8) imply a criterion for the required laser power,

$$\frac{\pi\lambda cm\omega_m^2}{8J_0(\epsilon)^2\mathcal{F}^2\sqrt{\eta}}>P_{\mathrm{SQL}}>P_{\mathrm{min}}=\frac{\pi\lambda cm\omega_m^2}{8J_0(\epsilon)^2\mathcal{F}^2}\,\frac{2k_BT}{\hbar\omega_mQ}.\tag{10}$$

Typically, a significant fraction of the input laser power P will be dissipated in the cavity [this fraction is $J_0(\epsilon)^2$ under the assumptions made above]. As it is desirable to keep this thermal load as small as possible, especially when operating the sensor at cryogenic temperatures, $\mathcal{F}^2$ represents an important figure of merit: low optical loss is an important requirement. In addition, the oscillator should have both low mass and low spring constant $m\omega_m^2$. Equation (10) also shows that the use of optical read-out is more favorable than microwaves.

Experimentally, the SQL would be revealed by determining $\Delta x^2_{\mathrm{tot}}$ from oscillation time evolution traces $x(t)$ of duration τ_{opt} and studying its dependence on laser power. The scale of $\Delta x^2_{\mathrm{tot}}$ can be obtained with the help of a calibration signal (see below). Figure 2 shows the theoretical dependence of Δx_{tot} and its contributions on power, for a set of parameters to be discussed below.

We now turn to the quantum limits occurring for observation times exceeding the mechanical relaxation time τ_m [3]. This regime is of interest for applications seeking to detect a force that is constant on the time scale of τ_m. The power spectral density of the displacement noise consists of three contributions,

$$S_{\mathrm{tot}}(\omega)=S_{\mathrm{SN}}(\omega)+S_{\mathrm{BA}}(\omega)+S_{\mathrm{therm}}(\omega),\tag{11}$$

where ω is the detection frequency, not necessarily equal to the mechanical resonance frequency. We note that we can simply add the three contributions, without taking into account any cross correlations, since the intensity noise at the sideband frequency causing S_{SN} is uncorrelated to the low-frequency noise driving the oscillator and causing S_{BA}.

The shot-noise contribution has been given above in Eq. (1). The displacement power spectral density due to quantum backaction is given by

$$S_{\mathrm{BA}}(\omega)=2\tau\chi^2(\omega)\Delta F^2_{\mathrm{BA}}=8\frac{\hbar k}{c}PJ_0(\epsilon)^2\left(\frac{\mathcal{F}}{\pi}\right)^2\chi^2(\omega),\tag{12}$$

where $\chi^{-2}(\omega)=m^2(\omega^2-\omega_m^2)^2+m^2\omega^2\omega_m^2/Q^2$ is the mechanical susceptibility. Finally, the one-sided spectral density of the displacement of the mass due to thermal excitation is given by the well-known expression

$$S_{\mathrm{therm}}(\omega)=\frac{4k_BTm\omega_m}{Q}\chi^2(\omega).\tag{13}$$

The backaction dominated regime, i.e., where backaction noise exceeds thermal noise, occurs for laser powers satisfying the same Eq. (10) derived above in the short-term case. A point to note is that the absence of an explicit condition on Q such as Eq. (9) broadens the range of sensor configurations with which quantum effects may be observed, since P_{min} can be reasonably low even for medium Q, provided this is compensated by the other parameters.

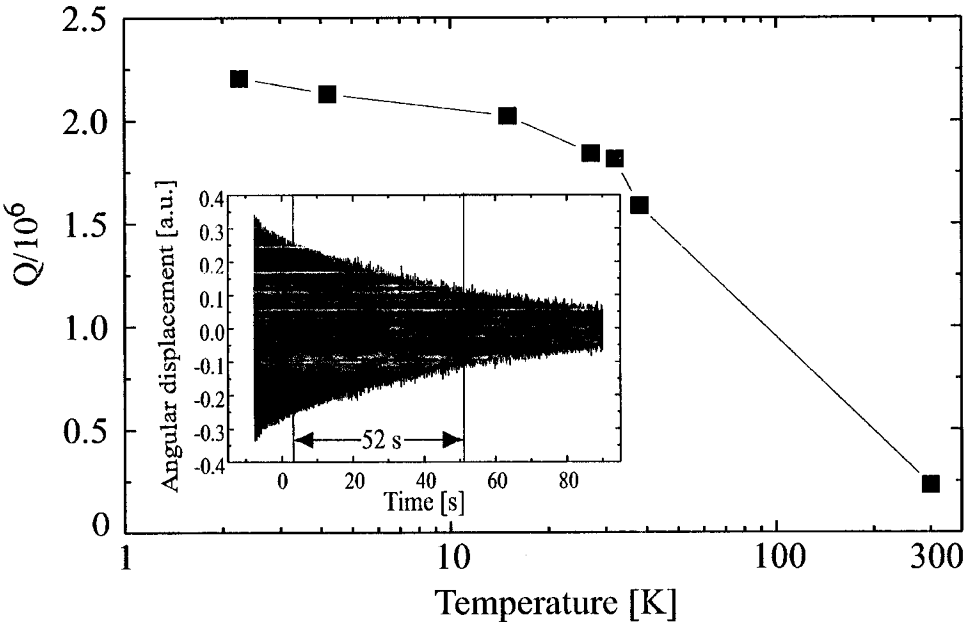

FIG. 4. Temperature dependence of the quality factor of a coated torsional silicon oscillator. The inset shows a decay measurement of an uncoated oscillator at 2 K after switching off the resonant excitation. The decay time of 52 sec corresponds to Q= 4.3 million.

To verify the possibility to observe the SQL in the long-time regime, we minimize $S_{\mathrm{SN}}+S_{\mathrm{BA}}$ with respect to laser power to find S_{SQL}. Its value relative to thermal noise is

$$\frac{S_{\mathrm{SQL}}(\omega)}{S_{\mathrm{therm}}(\omega)}=\frac{1}{2\sqrt{\eta}}\frac{\hbar\omega_m}{k_B T}\frac{\chi(\omega_m)}{\chi(\omega)}, \tag{14}$$

at a power

$$P_{\mathrm{SQL}}(\omega)=\frac{1}{2}P_{\min}\frac{S_{\mathrm{SQL}}(\omega)}{S_{\mathrm{therm}}(\omega)}. \tag{15}$$

For nonresonant detection, $\chi(\omega)/\chi(\omega_m)$ becomes small ($\leqslant 1/Q$ for ω sufficiently different from ω_m), and may thus offer the possibility of achieving the SQL: for $\omega \ll \omega_m$ the thermal noise is pushed below the SQL, $S_{\mathrm{therm}}<S_{\mathrm{SQL}}$, under essentially the same conditions (9) and (10) derived for short-time measurements. For ω sufficiently larger than ω_m, on the other hand, the SQL may be reached without requiring a minimum value for Q.

III. EXPERIMENTAL RESULTS

In our experiments, the movable mirror is a silicon torsional oscillator (Fig. 3). Such oscillators have already found use in condensed matter studies [17]. The size of the whole structure is 4 cm × 2 cm × 380 μm. Its design and room-temperature properties were reported before [18]. It consists of two coupled oscillators, an inner frame and a 2 mm × 4 mm vane connected by torsion bars (400 μm width). The whole structure has many oscillation modes, among which are a symmetric, low-frequency torsional mode and an anti-symmetric, high-frequency torsional mode. Only the anti-symmetric mode has a high-Q resonance due to low stress concentrations in the beams connecting the two coupled oscillators to the frame. The resonance frequency of this oscillation is $\omega_m/2\pi \simeq 26$ kHz. The oscillator was fabricated as follows: a thick oxide layer was thermally grown on both sides of a highly polished commercial p-type 5 Ω/cm silicon wafer with $\langle 100\rangle$ orientation. The oscillator was defined by double-sided lithography and the structure was etched in a 60 °C KOH solution. After fabrication of the structure and removal of the oxide, the vane was coated on one side with a high-reflectivity dielectric coating for 1064 nm. When the oscillator is part of the optical cavity, the laser beam is reflected at a distance $L=1$ mm from the torsional axis, so that the torsional motion leads to a cavity length change. For a torsional oscillator, the effective mass m in the expressions given above is I/L^2, where I is the moment of inertia. Here $I=9.6\times 10^{-12}$ kg m^2, leading to $m \simeq 10$ mg.

A. Mechanical quality factor

Mechanical damping depends both on gas pressure and temperature. Best values are obtained in vacuum and at low temperatures, but usually still depend on clamping loss. To characterize the properties of the oscillator, a simple optical setup was used. The oscillator is clamped to a mount attached to a piezoelectric transducer in a cryostat vacuum chamber. A HeNe laser is shone on the torsional vane and the reflected light is detected with a segmented photodiode, which gives a signal proportional to the angular displacement of the torsional oscillator. The oscillator is mechanically excited at resonance. After the excitation is turned off, the amplitude decay is measured using a lock-in amplifier as a demodulator with a slightly detuned reference frequency.

Figure 4 shows the Q factor of a coated oscillator in the temperature range 2 K–40 K. The increase of the quality factor at low temperatures is due to a reduction in phonon scattering as the phonon number decreases. The highest cryogenic Q factor we observed was 4.3 million (inset of Fig. 4). Between room and liquid helium temperature the resonance frequency increases by about 100 Hz, due to changes in the elastic constants.

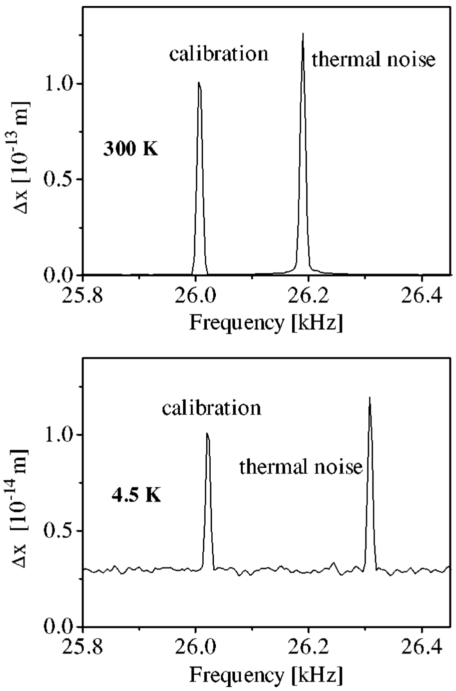

FIG. 5. The spectrum of the cavity detuning error signal at room temperature and 4.5 K. The peak at 26.2 kHz for the room-temperature measurement and 26.3 kHz for the measurement at 4.5 K has its origin in the Brownian motion of the mechanical oscillator. The calibration peak at 26 kHz is obtained by frequency-modulating the laser. The observed shift of the oscillator's resonance is due to the increased spring constant at lower temperatures. The measurement bandwidth was 4 Hz. Note the different vertical scales of the two coordinate systems. The thermal noise shows a tenfold decrease at 4.5 K compared to room temperature.

B. Sensitivity of the interferometer

The optomechanical sensor is housed in a LHe cryostat. The oscillator mirror is glued with stycast epoxy at its four corners to an invar holder. This holder is then attached to a larger invar block, on which a rigid incoupling mirror is mounted. The cavity length is 1 cm [free spectral range (FSR) equal to 15 GHz]. The concave input coupler with 10 cm radius of curvature has a transmissivity $1-r_1^2=50$ ppm. The finesse of the cavity is $\mathcal{F}=15\,000$, limited by the loss of the oscillator mirror. The resonator is thus undercoupled. A 300 mW diode-pumped monolithic single-frequency Nd:YAG laser is used, with typical laser powers incident onto the cavity around 5 mW. A phase-modulation frequency of 13 MHz with modulation index $\epsilon=0.8$ was used. The laser intensity noise was determined by balanced homodyne detection. By comparing the difference and the sum signals of the homodyne detector, we verified that above 12 MHz the laser intensity noise level is at the shot-noise level for the 4 mW power level. The read-out sensitivity was therefore not limited by technical laser noise. Mode-match efficiency of the laser mode into the cavity was around 50%. The quantum efficiency of the InGaAs photodetector plus losses in the optical path yielded a total efficiency $\eta=70\%$. The laser could be stably frequency-locked to the cavity. The displacement sensitivity for the measurement of the oscillator motion, obtained from the error signal noise floor, is 2×10^{-16} m/$\sqrt{\mathrm{Hz}}$. The expected level from Eq. (1) is approximately $\sqrt{S_{\mathrm{SN}}}=3\times10^{-19}$ m/$\sqrt{\mathrm{Hz}}$. Electronic noise is responsible for the large difference, and will have to be eliminated in future work.

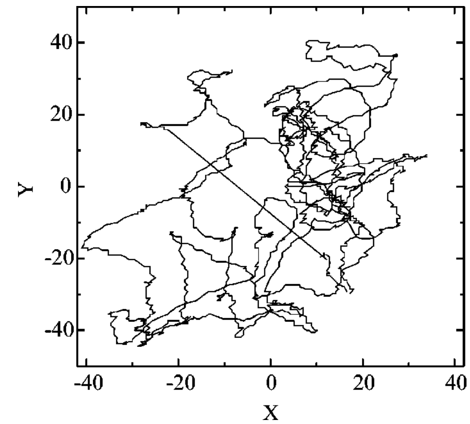

FIG. 6. Phase space trajectory of the oscillator at room temperature driven by thermal noise during a time interval of 80 s. One larger jump corresponding to a laboratory disturbance can be seen.

C. Brownian noise

The optomechanical sensor setup allows detailed studies of the Brownian motion of the moving mirror. For a frequency domain analysis, the error signal is recorded with a SRS 780 FFT spectrum analyzer. Figure 5 shows two typical results. The thermally excited motion of the torsional oscillator is clearly observed at 26.2 kHz for the room-temperature measurement and at 26.3 kHz at cryogenic temperature. To calibrate the signal, the laser frequency was modulated by an amount $\Delta\nu$ via a piezoelectric transducer glued onto the laser crystal, which has a constant transfer function below 100 kHz. A modulation frequency of 26 kHz was chosen, sufficiently separated from the mechanical resonance frequency, but sufficiently close so that the error signal gain is the same. $\Delta\nu$ was calibrated by comparison of the corresponding error signal amplitude with that obtained when scanning the laser frequency by an amount equal to a known fraction of the phase-modulation frequency. The equivalent mirror amplitude is calculated as $\Delta x_{\mathrm{cal}}=\Delta\nu\lambda/2$ FSR.

Typically the spectrum was recorded with a 0.25 s integration time. This is an order of magnitude less than τ_m at room temperature, and even less at 2 K. Therefore the spectrum was averaged 30 times for room temperature and 100 times for cryogenic measurements, to obtain an estimate of the thermal noise. The measured thermal noise levels are $\Delta x=(1.3\pm0.4)\times10^{-13}$ m at 300 K and $\Delta x=(1.3\pm0.6)\times10^{-14}$ m at 4.5 K. They agree reasonably well with the theoretical expectation $\Delta x_{\mathrm{therm}}(\tau\gg\tau_m)=\sqrt{k_BT/m\omega_m^2}$.

In the time domain the analysis of the oscillator motion is performed by feeding the cavity detuning error signal into a two-phase lock-in amplifier, with a reference frequency equal to the resonance frequency ω_m (Ref. [9]). The integration time constant corresponds to τ. Figure 6 shows a phase plot of the two quadratures of the error signal near ω_m, showing the Brownian random walk of the oscillator at room temperature.

IV. DISCUSSION AND OUTLOOK

The optomechanical sensor presented here represents a first step towards a device that should permit the observation of the SQL. It is useful to consider the necessary improvements for this goal. On the oscillator side this requires a substantial increase in both Q and $\mathcal{F}$. A higher finesse is desirable to reduce P_{SQL} to a reasonable level of a few mW. A mirror coating on the silicon oscillator with a finesse of 80 000 appears realistic, considering the high surface quality achievable for Si wafers. The Q factor must also be improved from the present value, which barely falls short of the condition (9), in order to permit the optimum measurement time to exceed $1/\omega_m$. This should be feasible, since quality factors 10 times larger have been obtained in larger silicon torsional oscillators [17].

Assuming an oscillator as described, but with a low-temperature mechanical quality factor $Q=1.5\times10^7$ and operation at $T=2$ K, condition (9) is satisfied. The measurement time should then lie within $6\,\mu\text{s}<\tau<27\,\mu\text{s}$. Thus, there is a small window where the assumptions used to derive the SQL are approximately satisfied. For the choice $\tau_{\text{opt}}=12\,\mu\text{s}$, Fig. 2 shows the expected uncertainties. The SQL at 6×10^{-18} m would be reached at a laser power $P=2.5$ mW. At approximately this power level, we also expect to see evidence of quantum backaction in a spectral measurement centered at ω_m. The SQL power spectral density $S_{\text{SQL}}(\omega_m)$ is six orders of magnitude below the thermal noise, as Eq. (14) shows. However, in the spectral noise density off-resonance, with $|\omega-\omega_m|>1/\tau_m$, the SQL should be accessible.

Several aspects of the sensor can be identified that need significant attention in order not to mask the SQL. First, the read-out sensitivity of our setup is still limited by electronic noise and must be improved by three orders of magnitude. Second, the frequency noise of our laser is expected to be about $S_{\text{FN}}=1$ Hz^2/Hz around ω_m [19]. This corresponds to a displacement uncertainty $\lambda\sqrt{S_{\text{FN}}/2\tau_{\text{opt}}}/2\text{FSR}=7\times10^{-15}$ m, which is three orders of magnitude higher than the standard quantum limit. A reduction of laser frequency noise to the required 10^{-3} Hz^2/Hz level by means of a high finesse reference cavity has been demonstrated [20]. In our case, a combination of such an active frequency noise reduction and an increase of the FSR of the cavity can be used. Finally, another important noise source is laser intensity noise, which at 21 kHz is about 20 dB higher than the coherent state quantum noise. A reduction close to the latter level can be achieved by a feedback system [21]. On the other hand, the presence of technical intensity noise may be used in a first step to demonstrate classical backaction effects. In closing, we note that an important advantage of the mechanical oscillators used here is that thanks to their high frequency, laboratory seismic noise is not expected to be relevant.

In conclusion, we have demonstrated an optomechanical sensor at cryogenic temperatures that exhibits both low optical loss and low mechanical loss. We have shown that the thermal oscillator noise can be resolved and is in quantitative agreement with theory. The equivalent force sensitivity is, from Eq. (13), 1.3×10^{-14} N/$\sqrt{\text{Hz}}$. With substantial improvements in the cavity finesse, mechanical quality factor, electronics and laser frequency, and amplitude stability, the sensor opens the possibility to observe the standard quantum limit of position measurement of a macroscopic body using interferometry.

ACKNOWLEDGMENTS

We are grateful to J. Mlynek, in whose laboratories this work was performed, for his encouragement and support. We are indebted to S. Vyatchanin for enlightening discussions and important suggestions, and V. Braginsky and R. Onofrio for discussions. I.T. acknowledges support from the Alexander von Humboldt Foundation. This work has also been generously supported by the Rotary Club (S.S.). We thank R. Storz for advice and help in manufacturing oscillators, S. Seel and D. Reinelt for help with the cryogenic Q measurements, and P. Münstermann and M. Spiegel for help with cleaning and clean-etching the oscillators.

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