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Excitability in a nonlinear optical cavity

Weiping Lu, Dejin Yu, and Robert G. Harrison
Department of Physics, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom

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We establish a nonlinear optical cavity to be excitable. Excitability in this system is shown to originate from the combined dynamical effects of nonlinear intracavity field saturation and temperature-dependent absorption in the medium on two different time scales. The model may be experimentally realized using optical bistable devices with possible useful applications. [S1050-2947(98)50508-2]

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Excitability has long been studied as an important class of dynamical phenomena in biological and chemical systems. In local regions, an excitable system exhibits a long excursion (pulse) in phase space for a superthreshold perturbation, the magnitude and width of such a pulse being independent of this perturbation. For spatially extended systems, excitability underlies wave propagation and formations, such as cardiac muscle and nerve wave fronts and spirals under the FitzHugh-Nagumo model [1,2] and chemical excitation waves in the Belousov-Zhabotinsky reaction [3]. However, it is only recently that excitable features have been revealed in nonlinear systems that have physical mechanisms different from those of biological and chemical interactions, such as in liquid crystals [4] and most recently an injected laser [5]. In this Rapid Communication, we establish a nonlinear optical cavity to be excitable. We show that its excitable behavior occurs in a small parameter window close to a bistable operating region and is attributable to the combined dynamical effects of nonlinear intracavity field saturation and temperature-dependent field absorption in the medium on two different time scales. We argue that such excitability may be experimentally realized in popular optical bistable devices. Possible applications of optical excitability are discussed.

We consider a unidirectional ring cavity containing a homogeneously broadened two-level nonlinear medium; a classical model description for earlier investigations of optical bistability and instabilities in the late 1970s [6]. This system is described by the Maxwell-Bloch equations, a set of three coupled complex equations, which can be simplified as a single real equation under the resonant operation condition and good cavity limit, $k_1 \ll \gamma_\perp, \ \gamma_\parallel$,

$$\frac{\partial E}{\partial t} + c\,\frac{\partial E}{\partial z} = -\frac{c\alpha E}{1+E^2/E_s^2}, \tag{1}$$

where k_1 is the cavity linewidth, and $\gamma_\perp$ and $\gamma_\parallel$ are the transverse and parallel relaxation rates of the medium. E is the intracavity slowly varying amplitude, t the time, z the coordinate in the wave propagating direction, and c the velocity of light. $E_s = \hbar\sqrt{\gamma_\perp \gamma_\parallel}/\mu$ is the saturation field amplitude and $\alpha = g\mu N/2\hbar c\gamma_\perp$ is the unsaturated absorption coefficient, where g is the field-polarization coupling coefficient, μ the modulus of the dipole moment, $\hbar$ the Planck constant, and N is the number of atoms. The cavity imposes two boundary conditions on the input and output signals, E_I and E_T, respectively, both of which are real; $E_T = \sqrt{T}E(L,t)$ and $E(0,t) = \sqrt{T}E_I + (1-T)E(L,t-(l-L)/c)$. Here T is the intensity transmission coefficient of both input and output couplers, and L and l are the lengths of the nonlinear medium and the cavity, respectively. Under the mean-field approximation, $\alpha L \ll 1$, $T \ll 1$, and the cooperation parameter $C = \alpha L/2T$ is finite; the dynamics of the optical field $E = E(L,t) \equiv E_T/\sqrt{T}$ at the output end is governed by the following ordinary differential equation [6]:

$$k_1^{-1}\,\frac{dE}{dt} = E_0 - E - \frac{2CE}{1+E^2/E_s^2}, \tag{2}$$

where $E_0 = E_I/\sqrt{T}$ is the normalized incident field amplitude at $z=0$ and $k_1 = cT/l$. It is well known that Eq. (2) under steady-state conditions gives an S-shaped bistable relation between E_0 and E. We note that, although Eq. (2) is rigorously derived in a ring cavity configuration, it is also commonly accepted as being a good approximation for a nonlinear Fabry-Perot cavity where the interference effects of the intracavity counterpropagating waves are negligible.

In an absorptive medium the small signal absorption coefficient α is in general a function of the medium temperature and radiation frequency. For simplicity, we assume that a linear relation between absorption coefficient α and temperature change Θ is valid for a finite window of temperature range at the input optical frequency, that is,

$$\alpha = \alpha_0 + \alpha_1 \Theta(E^2), \tag{3}$$

where α_0 is the absorption coefficient at the equilibrium temperature Θ_0 and Θ the temperature change induced by the intracavity optical field. α_1 is the absorption coupling coefficient, which can be either positive or negative, depending on the types of materials used and the frequency of the input optical field. We restrict ourselves to the condition of $\Delta\alpha \equiv \alpha_1\Theta \ll \alpha_0$.

The temperature change in the nonlinear medium is described by the heat flow equation

$$c_v \rho\,\frac{\partial \Theta}{\partial t} = a\nabla^2\Theta + Q, \tag{4}$$

where c_v is the specific heat at constant volume, and ρ and a the equilibrium density and thermal conductivity of the medium, respectively. $\nabla^2 = \nabla_\perp^2 + \partial^2/\partial z^2$ is the three-dimensional (3D) Laplacian. $Q = \frac{1}{2}\alpha\epsilon_0 c n E^2$ is the heat source arising from the absorbed optical intensity in the cav-

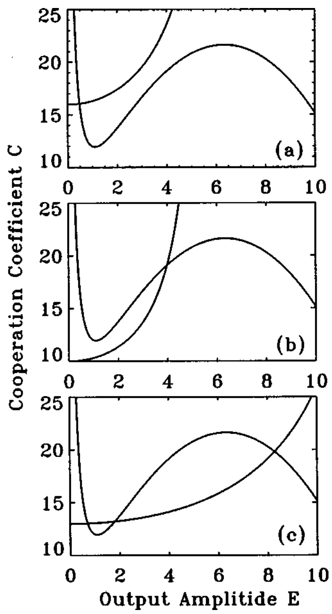

FIG. 1. Equations (6) have three types of solutions, corresponding to (a) an excitable system, (b) a system undergoing a Hopf bifurcation, and (c) a bistable system, respectively.

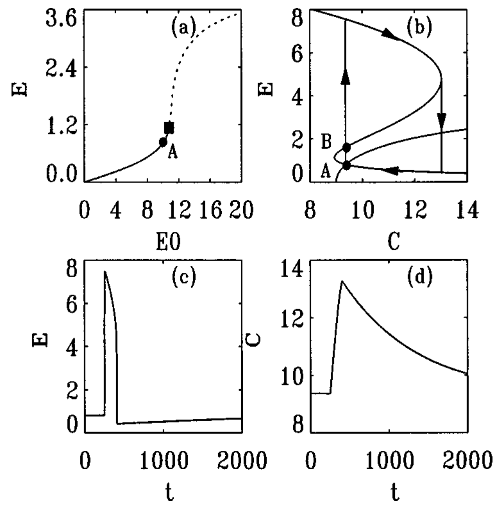

FIG. 2. (a) Steady-state relation of the input E_0 to the output E for $C_0=9$ and $\eta=0.06$. The solid and dotted lines correspond to stable and unstable steady-state solutions, which are separated by a Hopf bifurcation (marked by a solid square) at $(E_0,E)=(10.80,1.13)$ for $\epsilon=0.001$. (b) Phase-space illustration of excitability for $E_0=10$, $C_0=9$, and $\eta=0.06$. (c) and (d) Time sequences of the fast and slow variables, E and C, after excitation.

ity, where ϵ_0 is the vacuum permittivity and n is the linear refractive index. Under the mean-field approximation, the optical field E is z-independent and so is the temperature change Θ induced by the absorption through Eq. (4). When the boundary effect on the temperature variations in the longitudinal direction is negligible, typically in a bulk medium, its temperature change can be considered to be uniform in this direction. The transverse diffusion, on the other hand, can be approximated as a source of heat dissipation when the dynamics of the transverse heat distribution of the system is not taken into account. Equation (4) is then simplified as

$$c_v\rho\,\frac{\partial\Theta}{\partial t}=-\frac{a}{\pi\omega_p^2}\,\Theta+\tfrac{1}{2}\epsilon_0 nc\alpha E^2(t), \tag{5}$$

where ω_p is the radius of the input optical beam spot in the medium.

Equation (5) describes a dynamical relation of the temperature change to the intracavity field amplitude. Using the relation in Eq. (3), it can be easily transformed into the evolution equation for $\alpha(E)$. The optical system comprising both types of nonlinearities is then described by the following coupled equations:

$$k_1^{-1}\,\frac{dE}{dt}=-E-\frac{2CE}{1+E^2}+E_0,$$
$$k_2^{-1}\,\frac{dC}{dt}=-C+\eta CE^2+C_0, \tag{6}$$

where E and E_0 have been renormalized to E_s. The constant $k_2=a/(c_v\rho\pi\omega_p^2)$ is the relaxation rate of the absorptive coefficient and $C_0=\alpha_0 L/2T$ the cooperation coefficient at Θ_0. $\eta=\epsilon_0 nc\alpha_1\pi\omega_p^2E_s^2/(2a)$ is the field-absorption coupling coefficient, which is restricted to $\eta\ll 1$ to satisfy $\Delta\alpha\leq\alpha_0$. The steady-state solutions of Eqs. (6) are determined by the intersection points of the two nullclines $E_0=E+2CE/(1+E^2)$ and $C_0=C(1-\eta E^2)$. Three basic cases, corresponding to three types of solutions, can be distinguished as illustrated in Fig. 1. The nullclines intersect at (a) a single point lying on low or high branches of the bistable curve, (b) a single point lying on the middle branch, and (c) three points, each lying on a different branch of the bistable curve. The three cases describe an excitable system, a system undergoing a Hopf bifurcation, and a bistable system, respectively. Equations (6) are in many respects similar to the FitzHugh-Nagumo model, but in our case the relation between the two variables in the second equation (with slow time scale) is quadratic for $\eta\ll 1$. The absorptive bistable model is now a special case of a constant absorption coefficient, $\alpha=\alpha_0$.

We now focus on the parameter region of excitability for which only a single intersection occurs in the low branch of the bistable curve as depicted in Fig. 1(a). Excitable behavior occurs under the condition of $\epsilon\equiv k_2/k_1\ll 1$, i.e., the time scale of optical field dynamics is much faster than that of the temperature change. The system in this parameter region gives a unique input-output relation, and its stability depends on the parameters C_0, η as well as the ratio ϵ. The steady-state solutions, as shown in Fig. 2(a), are stable on increasing the input field amplitude until a Hopf bifurcation occurs at $E_0=10.80$ and $E=1.13$ for $C_0=9$, $\eta=0.06$, and $\epsilon=0.001$, above which the output signal shows a periodic motion for a constant input. Excitable behavior occurs in a small region just below the Hopf bifurcation. The mechanism for such behavior can be best explained in the (C,E) phase space of Fig. 2(b). While the steady state A in the (E_0,E) curve, which corresponds to the intersecting point in Fig. 2(b), is linearly stable in the sense that A immediately relaxes back to its original state for a small perturbation, it is however unstable once a larger excitation, referred to as the superthreshold perturbation, pushes this state to a position across the middle branch of the bistable curve, denoted as B in Fig.

2(b). For the latter case, the system first switches to the high branch of the bistable curve, then follows this branch to the right, jumps back to the low branch, and eventually relaxes back to A, forming a long excursion in the phase space. The value of the superthreshold perturbation is therefore defined to be the distance between the steady state (in the low branch) and the middle branch points in the vertical direction. A typical time pulse signal of both the fast (E) and slow (C) variables under a superthreshold perturbation is shown in Figs. 2(c) and 2(d). As a distinct feature of excitability, such a long excursion in phase space or pulse signal in time is independent of the exact external perturbation once it is a superthreshold perturbation. Notice that, as shown in Fig. 2(c), after excitation the system spends a long time along the low bistable branch, the refractory period, when it is not susceptible to small perturbation until returning to the steady state A. As a common feature of excitability, the existence of two different time scales in the system, i.e., $\epsilon \ll 1$, is *a priori*. When the two time scales are of the same order of magnitude, the long excursion phenomenon disappears and is replaced by a short pulse, the width and amplitude of which are dependent on the perturbation.

While excitability in optics shares many universal features with those in biological and chemical systems, an important difference between them lies in the fact that excitability in optics is manifested in the field amplitude as an excitable optical field, whereas for the latter it is in the nonlinear media, referred to as an excitable media. In our system, the population difference and polarization follow adiabatically the intracavity optical field under the good cavity limit, that is, $D=1/(1+E^2)$ and $P=E/(1+E^2)$, respectively. Both variables show a dip in time when the optical field is excited and stay in the upper branch of the (C,E) curve.

The superthreshold phenomenon in an excitable optical system may be utilized for applications. One such example is a signal profile-reshaping device for a train of pulses that are distorted, say, through propagation in a nonlinear medium. Using such a pulse sequence as perturbations to the intracavity field amplitude E, the device reshapes the output, producing the same profile pulse train. Since such a device amplifies only the signal (superthreshold) and ignores the noise background (subthreshold), it may also be used as a signal selection device. The selection criteria can easily be controlled through the adjustment of the superthreshold value by varying the operating condition of the system. Furthermore, the optical excitable device may have potential applications through its capability of generating a coherent resonance output under stochastic noise perturbations. The phenomenon, akin to the well-known stochastic resonance effect, has been initially investigated recently in the FitzHugh-Nagumo model with a noisy driving term to the fast variable [7]. In optics, the excitable device can turn a white-noise-type signal to a nearly equal-spaced periodic time series, as shown in Fig. 3.

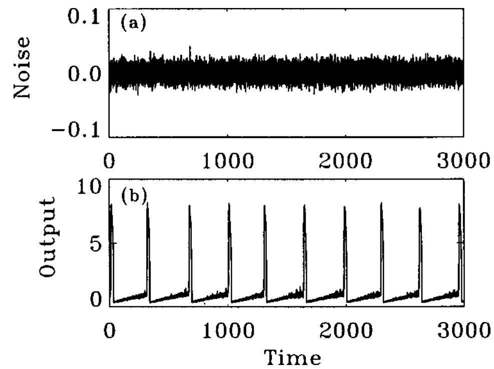

FIG. 3. Noise-induced coherence in system (6). (a) Gaussian white noise g_ω, $\langle g_\omega(t) g_\omega(s)\rangle = \delta(t-s)$, which perturbs the intracavity optical field E, $E \rightarrow E + D g_\omega$; (b) output E shows coherence enhancement under the noise perturbation. The parameters are $C_0=9$, $\eta=0.06$, $E_0=10.7$, $\epsilon=0.01$, and $D=0.2$.

An experimental observation of the excitability may be realized in optical bistable devices. The two types of nonlinearities required in Eq. (6) can be obtained, or at least approximated, by using semiconductor and other absorbing materials. Indeed, the saturation of absorption in certain types of semiconductors may be treated as that in a two-level system, whereas the thermally induced temperature changes in these materials are quasilinear in a definite temperature window for an optical input of appropriate frequency. Two different time scales in a semiconductor cavity are readily available. The fast one, $k^{-1}=l/cT$, is the cavity lifetime, on the order of $10^{-7}-10^{-8}$ s for a ring cavity length of 1 m and a transmission coefficient of 90%. The response time of the temperature change in semiconductors is dependent on the physical properties of the materials and the input optical beam size. It is generally slow due to thermal conduction in the materials, typically on the order of between 10^{-2} and 10^{-6} s. For example, in bulk ZnSe [8], which has widely been used in optical bistable devices, $D \equiv a/c_v \rho = 10^{-4}$ m^2 s^{-1}, leading to $k^{-1} \sim 3 \times 10^{-4}$ s for an optical beam size of $\omega_p = 30$ μm. Finally, we note that the excitable behavior in our model is predicted in nonbistable operating conditions and is in a small region close to the Hopf bifurcation point.

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