PHYSICAL REVIEW A VOLUME 58, NUMBER 2 AUGUST 1998

Theory of shape-preserving short pulses in inhomogeneously broadened three-level media

Ashiqur Rahman and J. H. Eberly
Rochester Theory Center for Optical Science and Engineering and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 14 April 1998)

We introduce an ansatz that permits a treatment of the effects of Doppler broadening in the theory of intense steady-state pulse pairs propagating in three-level (Λ or V) media. We have derived analytic solutions for pulse amplitudes, for group and phase velocities, and for the probability amplitudes of the atoms in the transmission medium. The solutions are different for the Λ and V cases. [S1050-2947(98)50308-3]

PACS number(s): 42.50.Md

Optical pulse propagation in a three-level medium under two-photon or double one-photon resonance conditions has been studied extensively theoretically and experimentally by various authors since the 1970s [1–3]. It has been studied in connection with self-induced transparency and simultons [4–9], lasing without inversion [10–12], phaseonium [13,14], electromagnetically induced transparency [15], and other related topics [16–21]. A constant focus of interest has been on mechanisms by which two pulses can travel through an otherwise opaque three-level medium, sometimes without changing their initial temporal shapes. Although no general analytic solution has been found so far that covers all aspects of propagation, some special analytic solutions have been found by restricting the parameter regimes or by using special input pulses. For example, simultons [4] were obtained in 1981, in which two solitonlike pulses with temporal shapes of ‘‘sech’’ propagate without changing through a three-level medium that could be either V type, asymptotically in the ground state, or cascade-type with asymptotically clamped equal inversions, both cases in the absence of detuning. Explicit *N*-soliton for V media have been given by Steudel [8].

Another solitonlike solution was obtained more recently [9], which shows that two pulses with amplitudes given by sech and ‘‘tanh’’ can propagate without change through a three-level Λ-type medium that is asymptotically in the ground state. From the intensity (square of amplitude) point of view, this latter solution describes a bright-soliton dark-soliton pair.

However, most of these earlier investigations have avoided two difficult questions: (i) the possible existence or nonexistence of a ‘‘global’’ restriction on pulse pairs similar to the area theorem discovered for two-level media by McCall and Hahn [22]; and (ii) a practical method to deal with Doppler broadening (or other kinds of inhomogeneous broadening), commonly a prominent broadening process and frequently the dominant one for atomic and molecular vapors and optical crystals of many kinds. The studies of Konopnicki and Eberly [4] included numerical experiments showing pulse reshaping toward their predicted sech steady-state form was observed in Doppler-broadened media. This is reminiscent of area theorem behavior, but no formulas for area were found.

In this paper we address the second question. We have found analytic expressions for the amplitudes of solitonlike pulses propagating through Λ- and V-type three-level media when Doppler broadening is included, under the reasonable approximation (for Λ and V systems) that the two transitions have equal shifts. The expressions we have found describe bright-dark and bright-bright soliton pairs, respectively. Λ and V systems are illustrated in Fig. 1.

The electric-field vector for the two optical pulses can be written as

$$\mathbf{E}=\hat{\mathbf{x}}\mathcal{E}_a(z,t)e^{i(k_a z-\omega_a t)}+\hat{\mathbf{x}}\mathcal{E}_b(z,t)e^{i(k_b z-\omega_b t)}+\text{c.c.}, \quad (1)$$

where $k_a c=\omega_a$ and $k_b c=\omega_b$, and $\mathcal{E}_a$ and $\mathcal{E}_b$ are the amplitudes of the electric fields of the two pulses. We assume that the two fields interact separately with their respective transitions, but they are nevertheless coupled by the nonlinear constraint that in all cases the two transitions share a common level of the atoms (level 2 in the figure). The Rabi frequencies that correspond to these fields are given by $\Omega_a(\equiv 2d_{12}\mathcal{E}_a/\hbar)$ and $\Omega_b(\equiv 2d_{23}\mathcal{E}_b/\hbar)$, where d_{ij} is the dipole moment between levels i and j. The evolution equations for the atomic levels with complex amplitudes C_1, C_2, and C_3 can be obtained for either Λ or V media from Schrödinger's equation as

$$i\frac{\partial}{\partial\tau}C_1=\Delta_1 C_1-\frac{1}{2}R_a^* C_2, \quad (2a)$$

$$i\frac{\partial}{\partial\tau}C_2=\Delta_2 C_2-\frac{1}{2}R_a C_1-\frac{1}{2}R_b C_3, \quad (2b)$$

$$i\frac{\partial}{\partial\tau}C_3=\Delta_3 C_3-\frac{1}{2}R_b^* C_2, \quad (2c)$$

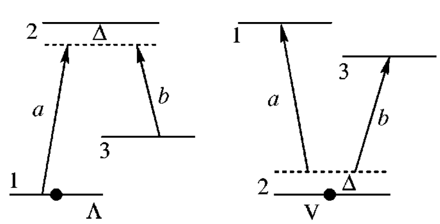

FIG. 1. Schematic energy-level diagrams for three-level atomic media: Λ (left) and V (right). The pulses *a* and *b* (driving transitions 1–2 and 2–3, respectively) are in two-photon resonance with overall detuning Δ.

 © 1998 The American Physical Society

TABLE I. Parameters for different types of medium.

	Λ	V
R_a	Ω_a	Ω_a^*
R_b	Ω_b	Ω_b^*
Δ_1	0	Δ
Δ_2	Δ	0
Δ_3	0	Δ
P_a	$C_1^*C_2$	$C_2^*C_1$
P_b	$C_3^*C_2$	$C_2^*C_3$

where the parameters Δ_1, Δ_2, and Δ_3 are related to the detuning Δ shown in Fig. 1 and the parameters R_a and R_b are related to the Rabi frequencies Ω_a and Ω_b, as given in Table I for Λ or V media.

Similarly, the evolution equations for the fields can be obtained from the slowly varying Maxwell equations [23] as

$$\frac{\partial}{\partial\zeta}\Omega_a = i\mu\int P_a g(\Delta)d\Delta, \tag{3a}$$

$$\frac{\partial}{\partial\zeta}\Omega_b = i\mu\int P_b g(\Delta)d\Delta, \tag{3b}$$

where P_a and P_b are the polarizations given in Table I, and $g(\Delta)$ is the distribution of detunings resulting from inhomogeneous (Doppler-type) broadening. The propagation coefficient μ is assumed to be equal for all transitions and is given by $\mu = 4\pi d^2\mathcal{N}\omega/(\hbar c)$, where $\mathcal{N}$ is the density of atoms in the medium. Equations (2) and (3) are written in local-time coordinates ζ and τ in the frame propagating with velocity c in the medium $\tau \equiv t - z/c$ and $\zeta \equiv z$.

Equations (2) and (3) are nonlinear and there is no generally known method to solve them. We will introduce a complex detuning-dependent factorization as an ansatz, and we will see that this permits previously known on-resonance solutions to be extended from zero detuning to any arbitrary finite detuning. The key steps in determining the solutions for the Λ-type medium are summarized below. The derivation of solutions for the V-type medium is similar.

We now introduce a factorization ansatz for the upper-level amplitude, motivated by a similar step taken by McCall and Hahn [22] for the absorptive Bloch vector component in a two-level system (see also [19], p. 82). Our factorization is

$$C_2(\zeta,\tau;\Delta) \equiv f(\Delta)c_2(\zeta,\tau), \tag{4}$$

where the lower-case letters (c_1, c_2, c_3) denote the level amplitudes for $\Delta = 0$, and $f(\Delta)$ satisfies $f(0) = 1$, but otherwise must be determined. The factorization ansatz then leads to the following useful relation

$$i\frac{\partial}{\partial\tau}C_1 = -\tfrac{1}{2}\Omega_a^* f(\Delta)c_2 = if(\Delta)\frac{\partial}{\partial\tau}c_1, \tag{5}$$

which implies that C_1 and $f(\Delta)c_1$ differ by an additive Δ-dependent constant at most. Since we assume the medium to be in level 1 asymptotically, independent of detuning, this gives a definite value for the constant. The same argument also provides the precise relation between C_3 and its zero-detuning counterpart c_3, so we get

$$C_1 = f(\Delta)c_1 + 1 - f(\Delta), \tag{6}$$

$$C_3 = f(\Delta)c_3. \tag{7}$$

With these two relations we return to the equation for C_2. It is easy to see that this then provides an expression for $f(\Delta)$ and an important connection between c_2 and Ω_a:

$$f(\Delta) = \frac{1}{1 + i\Delta\tau_\Lambda}, \tag{8}$$

$$c_2 = \frac{i\tau_\Lambda}{2}\Omega_a, \tag{9}$$

where τ_Λ is an auxiliary constant whose connection to the pulse duration will be obtained below.

Now it is trivial to obtain solutions for all three c's by inserting into the c equations the new connection between c_2 and Ω_a and the two known pulse amplitude expressions derived earlier for on-resonance media [4,9], namely,

$$\Omega_a(\zeta,\tau) = A\ \mathrm{sech}(K_\Lambda\zeta - \tau/\tau_p),$$

$$\Omega_b(\zeta,\tau) = B\ \tanh(K_\Lambda\zeta - \tau/\tau_p).$$

When we add the relevant $f(\Delta)$ factors, the full solutions are

$$C_1(\zeta,\tau;\Delta) = \frac{\tanh(K_\Lambda\zeta - \tau/\tau_p) + i\Delta\tau_\Lambda}{1 + i\Delta\tau_\Lambda}, \tag{10a}$$

$$C_2(\zeta,\tau;\Delta) = \left(\frac{2i}{A\tau_p}\right)\frac{\mathrm{sech}(K_\Lambda\zeta - \tau/\tau_p)}{1 + i\Delta\tau_\Lambda}, \tag{10b}$$

$$C_3(\zeta,\tau;\Delta) = -\left(\frac{B}{A}\right)\frac{\mathrm{sech}(K_\Lambda\zeta - \tau/\tau_p)}{1 + i\Delta\tau_\Lambda}, \tag{10c}$$

where τ_p is clearly the pulse duration, which satisfies a specific relation with τ_Λ:

$$\frac{A\tau_\Lambda}{2}\frac{A\tau_p}{2} = 1. \tag{11}$$

We also require the following constraint on the amplitudes:

$$A^2 - B^2 \equiv 4/\tau_p^2, \tag{12}$$

which was found previously for the zero-detuning solutions [9].

These solutions are so far the only solutions of the Schrödinger equations. To be physical they must also satisfy the Maxwell equations. Inspection shows that the field amplitude expressions are indeed compatible with the Maxwell equations and with the existence of a spread of detunings, via the Doppler distribution function $g(\Delta)$, with the right choice of K_Λ and a single modification: a ζ-dependent ''carrier wave'' phase factor $\phi(\zeta)$ must be attached to pulse a, so that its solution function becomes

$$\Omega_a(\zeta,\tau) = A\ \mathrm{sech}(K_\Lambda\zeta - \tau/\tau_p)e^{i\phi(\zeta)}. \tag{13}$$

The Schrödinger equations show that the same phase factor must also be attached to C_2 and C_3, and these are the only changes. After substituting Eq. (13) into the complex Maxwell equations (3) we find the following equation for K_Λ and the phase:

$$K_\Lambda = \frac{\mu}{2\tau_\Lambda} \int \frac{g(\Delta)}{\Delta^2 + (1/\tau_\Lambda)^2} d\Delta, \tag{14a}$$

$$\frac{d}{d\zeta}\phi(\zeta) = \frac{\mu}{2} \int \frac{\Delta g(\Delta)}{\Delta^2 + (1/\tau_\Lambda)^2} d\Delta. \tag{14b}$$

We can define a new wave-vector shift K'_Λ by

$$\phi(\zeta) \equiv K'_\Lambda \zeta, \tag{15}$$

which is physically equivalent to a modification of the refractive index and is similar in appearance to the index change known for two-level self-induced transparency (see [23], p. 96), but here τ_Λ is not the pulse duration.

The V solutions differ in interesting small ways from the Λ solutions, and we give the full solutions for the V-type media:

$$C_1(\zeta,\tau;\Delta) = i\left(\frac{A\tau_p}{2}\right) \frac{\mathrm{sech}(K_V\zeta - \tau/\tau_p)}{1 + i\Delta\tau_p} e^{iK'_V\zeta}, \tag{16a}$$

$$C_2(\zeta,\tau;\Delta) = \frac{\tanh(K_V\zeta - \tau/\tau_p) + i\Delta\tau_p}{1 + i\Delta\tau_p}, \tag{16b}$$

$$C_3(\zeta,\tau;\Delta) = i\left(\frac{B\tau_p}{2}\right) \frac{\mathrm{sech}(K_V\zeta - \tau/\tau_p)}{1 + i\Delta\tau_p} e^{iK'_V\zeta}, \tag{16c}$$

$$\Omega_a(\zeta,\tau) = A\ \mathrm{sech}(K_V\zeta - \tau/\tau_p) e^{iK'_V\zeta}, \tag{16d}$$

$$\Omega_b(\zeta,\tau) = B\ \mathrm{sech}(K_V\zeta - \tau/\tau_p) e^{iK'_V\zeta}, \tag{16e}$$

where

$$K_V = \frac{\mu}{2\tau_p} \int \frac{g(\Delta)}{\Delta^2 + (1/\tau_p)^2} d\Delta, \tag{17a}$$

$$K'_V = \frac{\mu}{2} \int \frac{\Delta g(\Delta)}{\Delta^2 + (1/\tau_p)^2} d\Delta. \tag{17b}$$

As in the Λ case, the solution is constrained by a relationship between the amplitude and width of the pulses, in this case given by

$$A^2 + B^2 = \frac{4}{\tau_p^2}. \tag{18}$$

When the pulses are detuned from line center and the line-shape function is not symmetric with respect to $\Delta = 0$, the integrals in Eqs. (14b) and (17b) give a nonzero value for K', which in turn gives rise to the complex phase. To demonstrate this effect, we have taken a medium with a Gaussian line-shape function:

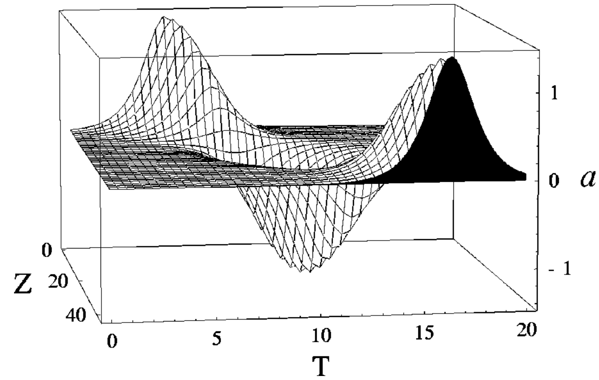

FIG. 2. Plot of the real part of the pulse "a" [$\equiv \mathrm{Re}(\Omega_a) T_2^*$] as a function of time $T(\equiv \tau/T_2^*)$ and space $Z(\equiv \mu\zeta T_2^*)$ for a V-type medium with Doppler broadening. The parameters used in this plot are $\Delta_0 = 1/T_2^*$, $\tau_p = T_2^*$, and $A = \sqrt{2}/T_2^*$, which gives $K\zeta = 0.260Z$ and $K'\zeta = 0.144Z$.

$$g(\Delta) = \frac{T_2^*}{\sqrt{2\pi}} e^{-[(\Delta-\Delta_0)T_2^*]^2/2}, \tag{19}$$

where Δ_0 is the detuning from line center and T_2^* is the inhomogeneous lifetime. Figure 2 shows propagation of pulse "a" in a V-type medium with such Doppler broadening. The spatial oscillations due to nonzero K' are evident from the figure.

The time scale for which these solutions are valid can be recognized by noting that Eqs. (2) and (3) are written in the absence of T_1 and T_2 terms (no homogeneous broadening). So the solutions must be regarded as short-pulse approximations, which are valid when the pulse width τ_p of the sech pulse is shorter than T_1 and T_2. There is no restriction on τ_p with respect to the inhomogeneous broadening time T_2^*, i.e., τ_p can be longer or shorter than T_2^*.

Actually, the inhomogeneous broadening time T_2^* only affects the values of K and K', which determine the group velocities [$v_g = 1/(K\tau_p)$ in the moving frame] and phase velocities ($v_{ph} = c + K'/\omega$). Figure 3 shows the dependence of K and K' on the ratio of τ_p/T_2^*. We see from the figure that

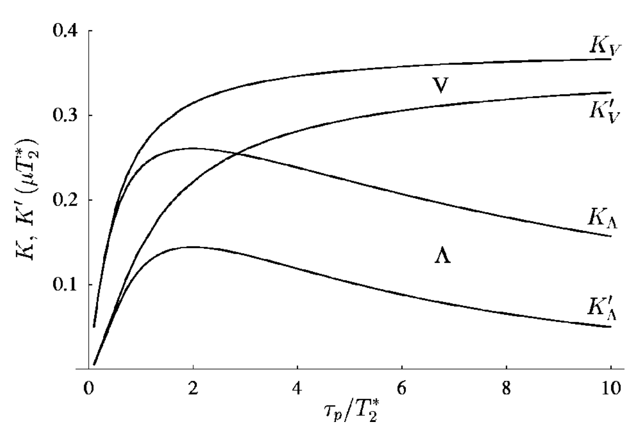

FIG. 3. Plot of K and K' (in units of μT_2^*) as a function of τ_p/T_2^* for Λ- and V-type media. Parameters used are $\Delta_0 = 1/T_2^*$ in $g(\Delta)$ and $B = 1/T_2^*$ in the Λ-type media.

when the pulse width is ultrashort (shorter than the inhomogeneous broadening time $\tau_p \ll T_2^*$), the values of K and K' increase with the increase of τ_p for both Λ-type and V-type media. However, when the pulse width is merely ''short,'' i.e., short enough to ignore homogeneous relaxation but still much longer than the inhomogeneous broadening time ($\tau_p \gg T_2^*$), the values of K and K' decrease in the case of a Λ-type medium and increase in the case of a V-type medium, as τ_p increases.

The reason for this behavior is that, for a Λ-type medium, K and K' depend on τ_p through $\tau_\Lambda \equiv \tau_p/(1+\tau_p^2 B^2/4)$. For a V-type medium the equivalent of τ_Λ is simply τ_p. So for $\tau_p \ll T_2^*$, we have $\tau_\Lambda \sim \tau_p$ and K and K' behave similarly for both types of medium. But for $\tau_p \gg T_2^*$, we have $\tau_\Lambda \sim 1/\tau_p$ and K and K' behave oppositely for Λ-type and V-type media.

Note that it is also possible (and it could sometimes be more physical) to regard the pulse amplitudes A and B as the primary independent parameters instead of τ_p. Then the relation

$$\frac{2}{\tau_p} = \sqrt{A^2 \pm B^2}$$

can be used to eliminate τ_p everywhere in either the V(+) or the Λ(−) case. This leads to amplitude-dependent expressions for group and phase velocities in an obvious way.

As a final result, we point out that the constraints in Eqs. (12) and (18) require that the areas of the input sech pulse be greater than 2π for Λ-type media and less than 2π for V-type media. Also note that the solution pulses for the Λ-type medium correspond to a ''bright'' (sech) and ''dark'' (tanh) soliton, whereas both of the pulses (sech) in the V-type medium correspond to a bright soliton. Note that these solutions reduce to the well-known 2π sech pulse for self-induced transparency in a two-level medium [22] if level 3 and pulse ''b'' are eliminated (by putting $B=0$ and $C_3=0$).

In summary we have found formulas for the amplitudes of a solitonlike pulse pair for inhomogeneously broadened Λ- and V-type three-level media. Explicit expressions for both group and phase velocities, and for the detuning-dependent Schrödinger amplitudes have also been given. We have commented on the differences between the solutions for the two types of media.

This research was partially supported by NSF Grant Nos. PHY94-15583 and PHY97-22024.

[1] T. Hänsch and P. Toschek, Z. Phys. **236**, 213 (1970).
[2] D. Grischkowsky, M. M. T. Loy, and P. F. Liao, Phys. Rev. A **12**, 2514 (1975).
[3] R. J. Cook and B. W. Shore, Phys. Rev. A **20**, 539 (1979).
[4] M. J. Konopnicki, P. D. Drummond, and J. H. Eberly, Opt. Commun. **36**, 313 (1981); M. J. Konopnicki and J. H. Eberly, Phys. Rev. A **24**, 2567 (1981).
[5] C. R. Stroud, Jr. and D. A. Cardimona, Opt. Commun. **37**, 221 (1981).
[6] A. I. Maĭmistov, Sov. J. Quantum Electron. **14**, 385 (1984); A. M. Basharov and A. I. Maĭmistov, Sov. Phys. JETP **67**, 2426 (1988).
[7] L. A. Bol'shov and V. V. Likhanskiĭ, Sov. J. Quantum Electron. **15**, 889 (1985).
[8] H. Steudel, J. Mod. Opt. **35**, 193 (1988).
[9] J. H. Eberly, Quantum Semiclassic. Opt. **7**, 373 (1995).
[10] O. A. Kocharovskaya and Ya. I. Khanin, JETP Lett. **48**, 630 (1988); O. Kocharovskaya, P. Mandel, and Y. V. Radeonychev, Phys. Rev. A **45**, 1997 (1992).
[11] S. E. Harris, Phys. Rev. Lett. **48**, 630 (1988).
[12] M. O. Scully, S.-Y. Zhu, and A. Gavridiles, Phys. Rev. Lett. **62**, 2813 (1989).
[13] M. O. Scully, Phys. Rev. Lett. **67**, 1855 (1991); Phys. Rep. **219**, 191 (1992); Quantum Opt. **6**, 201 (1994).
[14] J. H. Eberly, A. Rahman, and R. Grobe, Phys. Rev. Lett. **76**, 3687 (1996).
[15] S. E. Harris, Phys. Rev. Lett. **70**, 552 (1993); **72**, 52 (1994); Opt. Lett. **19**, 2018 (1994).
[16] M. Fleischhauer *et al.*, Phys. Rev. A **46**, 1468 (1992); M. Fleischhauer and T. Richter, *ibid.* **51**, 2430 (1995); M. Fleischhauer and A. S. Manka, *ibid.* **54**, 794 (1996).
[17] F. T. Hioe and R. Grobe, Phys. Rev. Lett. **73**, 2559 (1994); R. Grobe, F. T. Hioe, and J. H. Eberly, *ibid.* **73**, 3183 (1994).
[18] J. H. Eberly, H. R. Haq, and M. L. Pons, Phys. Rev. Lett. **72**, 56 (1994).
[19] E. Cerboneschi and E. Arimondo, Phys. Rev. A **52**, R1823 (1995).
[20] M. Jain *et al.*, Phys. Rev. Lett. **75**, 4385 (1996); **77**, 4326 (1996); A. S. Zibrov *et al.*, *ibid.* **76**, 3935 (1996).
[21] G. Vemuri, G. S. Agarwal, and K. V. Vasavada, Phys. Rev. Lett. **79**, 3889 (1997).
[22] S. L. McCall and E. L. Hahn, Phys. Rev. **183**, 457 (1969).
[23] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover Publications, Inc., New York, 1987).