PHYSICAL REVIEW A VOLUME 58, NUMBER 5 NOVEMBER 1998

Induced optical spatial solitons

D. Bortman-Arbiv, A. D. Wilson-Gordon, and H. Friedmann
Department of Chemistry, Bar-Ilan University, Ramat Gan 52900, Israel

(Received 3 August 1998)

We show that under suitable conditions, a weak nondegenerate probe may become spatially confined in a Doppler-broadened medium composed of two-level atoms due to the effect of a strong resonant copropagating pump beam. The *induced spatial solitons* result from the interplay of diffraction and induced focusing of the probe. Realistic conditions that include effects of probe absorption and pump reshaping on propagation are examined. In order to avoid absorption of the near-resonant probe, which would make the strong induced focusing unobservable, the unique properties of the dead zone are exploited. [S1050-2947(98)50811-6]

PACS number(s): 42.65.Sf, 42.65.Tg, 03.40.Kf

Spatial solitons in single beams have been the object of intensive theoretical and experimental research for the last three decades [1–4]. These solitons evolve from nonlinear changes in the refractive index of the medium, induced by the light-intensity distribution. When the self-focusing effect of the refractive index exactly compensates the effect of diffraction, the beam becomes self-trapped and is called a spatial soliton. The nonlinear effects that are responsible for spatial soliton formation are usually Kerr-like [1], where the changes in the refractive index are proportional to the light intensity. Higher-order nonlinearities are also important in determining the changes in the refractive index, as in the case where an initially Gaussian beam propagates in a medium consisting of two-level atoms [3,4]. Incident beams that deviate slightly from a self-trapped solution may exhibit oscillating spatial patterns [3,4].

In this Rapid Communication we show that the often discussed nonlinear effects of spatial confinement or oscillating spatial patterns in single strong beams can be produced also in weak probes. We describe the conditions for *nondissipative spatial confinement* of a nondegenerate weak probe beam propagating in a Doppler-broadened two-level atomic medium, due to the influence of a strong copropagating pump beam. It is well known [5,6] that a Doppler-broadened medium exhibits a frequency range of almost zero probe absorption, called the dead zone. We exploit the properties of the dead zone in order to minimize absorption of the probe during its propagation through the atomic medium. We show that an initially Gaussian probe beam can propagate through the atomic medium without being absorbed, with a profile that shows focusing and diffraction effects.

The term *spatial soliton* applies to the case where diffraction is compensated by the nonlinearity of the refractive index created by the soliton itself. Thus, due to self-induced focusing, the soliton creates its own waveguiding effect, provided this effect is not destroyed by absorption. Here, we propose to use the term *induced spatial soliton* for the case where the nonlinearity of the refractive index for a weak probe is *not* due to the probe field itself but is induced by a strong pump field. Then the waveguiding effect is induced by the pump-induced focusing of the probe beam. In other words, an intense pump field confers the soliton property to a weak probe by means of the cross-focusing effect. Since sufficiently large values of the nonlinear refractive index can only be obtained when the probe is near resonance, in which case the probe absorption is also high, the effect would be unobservable. Therefore, by creating a dead zone, we eliminate probe absorption without cancelling the pump-induced focusing of the probe.

Spatial reshaping of one beam due to the effect of a second beam has been extensively studied. According to theoretical models, spatially varying refraction plays a key role in the formation of conical emission [7,8]. First, when combined with diffraction, it determines the spatial profile of the strong pump. Second, it has a strong influence on the propagation of the two weak beams, the probe and the beam generated by four-wave mixing. In another context, Stentz *et al.* [9] confirmed experimentally Agrawal's theoretical prediction [10] that the presence of a strong pump beam can induce focusing and deflection of a weak beam even in a self-defocusing medium. This was achieved by displacing the axis of the copropagating probe beam from that of the pump beam, so that the axial symmetry of the field experienced by the probe is distorted and the field intensity at the center of the transverse profile of the probe is lower than at part of its periphery. Recent experiments in atomic Rb indicate that reshaping of the probe beam in an electromagnetically induced transparency (EIT) experiment results from the combined effect of the nonlinear refractive index *and* absorption induced by the strong field [11,12]. In the EIT experiment, transverse variation of absorption contributes to probe reshaping: regions of good and poor EIT experienced by the probe beam lead to focusinglike and defocusinglike reshaping effects. Whereas spatial variation of both the refractive index *and* absorption is important in predicting probe beam reshaping in the EIT experiment, in our work the probe profile exhibits focusing and defocusing effects without being absorbed. Spatial confinement of the probe beam results from mutual compensation of diffraction and the pump-induced spatially varying refractive index. As in the case discussed by Agrawal [10], focusing of the probe is obtained in a defocusing medium. Here, however, the axis of the copropagating probe is not displaced, allowing transverse confinement of the probe beam.

We consider the interaction between a cw electromagnetic field of the form $\tilde{\mathbf{E}}(\mathbf{r},t)=\Sigma_{J=L,P}\tilde{\mathbf{E}}_J(\mathbf{r},t)=\Sigma_{J=L,P}\mathbf{E}_J(\mathbf{r})e^{-i(\omega_J t-\mathbf{k}_J\cdot\mathbf{r})}+\text{c.c.}$, and a medium consisting of two-level atoms. $\mathbf{E}_L(\mathbf{r})$ and $\mathbf{E}_P(\mathbf{r})$ represent the slowly varying envelope amplitude of the strong pump field and the weak probe field, respectively, and ω_L and ω_P are the pump and probe frequencies with wave vectors $\mathbf{k}_L$ and $\mathbf{k}_P$. The two laser beams propagate in the z direction through the atomic medium. The intensity of the two beams as they propagate is determined by solving the coupled Maxwell-Bloch equations [13]

$$-\nabla^2\tilde{\mathbf{E}}_J(\mathbf{r},t)+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\tilde{\mathbf{E}}_J(\mathbf{r},t)=-\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\tilde{\mathbf{P}}_J(\mathbf{r},t), \quad (1)$$

where $\tilde{\mathbf{P}}_J(\mathbf{r},t)$ with $J=L,P$ is the induced polarization in the medium, which may be calculated from the density operator $\hat{\rho}$, $\tilde{\mathbf{P}}=N\langle\tilde{\mu}\rangle_D=N\langle\mathrm{Tr}(\hat{\rho}\hat{\mu})\rangle_D$, where N is the atomic number density and $\hat{\mu}$ is the dipole moment operator. The medium consists of two-level atoms with lower level $|a\rangle$, upper level $|b\rangle$, and resonance frequency ω_0. For this two-level system, the Doppler-averaged density matrix elements are calculated from the steady-state Bloch equations (see, for example, [14]).

Assuming the paraxial approximation, the coupled amplitude equations take the form

$$\frac{\partial}{\partial z}U_L=\frac{i}{4L_D}\frac{\partial^2}{\partial\xi^2}U_L+\frac{i}{L_{NL}}\alpha_L(|U_L|^2)U_L, \quad (2)$$

$$\frac{\partial}{\partial z}U_P=\frac{i}{4L_D}\frac{\partial^2}{\partial\xi^2}U_P+\frac{i}{L_{NL}}\alpha_P(|U_L|^2)U_P, \quad (3)$$

where we have introduced the normalized variables $\xi=x/\sqrt{2}w_{0L}$ and $U_{L,P}(\xi,z)=A_{L,P}(\xi,z)/A_{0L,}$ and where $L_D=kw_{0L}^2$ is the diffraction length, the wave vector is $k=|\mathbf{k}_P|\simeq|\mathbf{k}_L|=2\pi/\lambda_L$, and w_{0L} is the initial spot size of the pump transverse profile. Note that we have restricted our analysis to one transverse dimension only [9]; that is, we have considered diffraction effects only in the x direction. The parameter $L_{NL}=\hbar/\pi kN\mu^2T_2$ is a characteristic length indicating the strength of the nonlinear term. Here the definition of the parameter L_{NL} is different from that defined for a Kerr-type medium [10], since it does not depend on the pump laser intensity. In the definition of $U_{L,P}(\xi,z)$, A_{0L}^2 is the dimensionless peak intensity of the incident pump, and $2A_{L,P}=\mu T_2E_{L,P}/\hbar$ are the dimensionless pump and probe Rabi frequencies, respectively; T_2 is the transverse decay time and μ is the transition dipole moment. The Doppler-averaged complex coefficients $\alpha_{L,P}=\langle\rho_{ba}(\omega_{L,P})\rangle_D/U_{L,P}$ have been calculated for a copropagating configuration, from the steady-state Bloch equations for the case of a strong pump and a weak probe; see Ref. [14].

The coefficients $\alpha_{L,P}$ account for self-induced absorption and refraction of the pump and for cross-induced (pump-induced) absorption and refraction of the probe, and determine the pump and probe propagation conditions. In particular, the choice of parameters, such as pump and probe detunings Δ_L and Δ_P, the rate of collisional dephasing $1/T_2$, and the pump Rabi frequency A_L, determines the transverse variation of $\alpha_{L,P}$. The purpose of this work is to show that under certain circumstances the probe beam can become confined along the propagation path through the medium, due to the effect of the pump beam. In order to obtain the desired effect, two major requirements must be met. First, the probe absorption, which is proportional to Im α_P, should be small and, second, the refractive index, which is determined by Re α_P, should lead to a converging wave front by induced focusing of the probe. We show that, when these conditions are fulfilled, the interplay between pump-induced focusing effects and the diffraction-induced spreading of the probe beam leads to the confinement of the probe along its propagation path in the medium.

Let us analyze the requirement of small probe absorption. Often, absorption is simply neglected in propagation calculations [3,10] in order to emphasize the interplay between refraction and diffraction. We found that when absorption is included, the effects are often damped or even wiped out. Thus, in order to make the theoretical calculations realistic, it is *essential* to include probe absorption. In our work probe absorption is avoided by exploiting the properties of the dead zone. This zone, which is characterized by almost zero absorption, is obtained when the Doppler width D is larger than the pump generalized Rabi frequency $(\Delta_L^2+4V_L^2)^{1/2}$ where $V_L=A_L/T_2$, so that the stimulated emission of atoms with Doppler-shifted detunings smaller than the Rabi frequency is cancelled by the absorption of Doppler-shifted atoms with detunings larger than the Rabi frequency. These cancellation effects do not exist for the nonlinear probe refractive index, so that it survives Doppler broadening. Thus, the dead zone has the unique advantage of almost zero absorption of the probe accompanied by relatively large values of its nonlinear refractive index [5,6].

We now turn to the second requirement, that of an induced focusing of the probe. This induced focusing is determined by the nonlinear refractive index, which is usually written in the form $n=n_0+n_2I$, where n_2 is a constant for the case of a Kerr medium [15] and I is the intensity. The Kerr coefficient n_2 determines the spatial behavior of the beam. Thus a beam whose transverse intensity profile decreases monotonically from the center to the periphery is focused if $n_2>0$ and defocused if $n_2<0$. This expression for n is only valid for weak fields but can readily be generalized for more intense fields by replacing n_2 by dn/dI [16,17]. In our calculations we consider a resonant pump beam and a copropagating detuned probe beam. We have checked from the exact nonlinear dispersion relations that for this setup, four-wave mixing is negligible due to the lack of phase matching. When n was plotted as a function of the square of the pump Rabi frequency $(2V_LT_2)^2$ [16,17], we found that an extremum is obtained when $2V_L$ is equal to the absolute value of the pump-probe detuning $\delta=\omega_P-\omega_L$. This extremum is a maximum for $\delta>0$ and a minimum for $\delta<0$. Thus the generalized Kerr coefficient dn/dI of the probe is itself strongly nonlinear near $2V_L=|\delta|$: for $\delta<0$ and $2V_L<|\delta|$, $dn/dI<0$ and the probe is defocused, whereas for $\delta<0$ and $2V_L>|\delta|$, $dn/dI>0$ and the probe is focused. The extrema coincide with the low-intensity limit of the dead zone. These observations lead us to suggest that in an experimental setup in which the probe is red-detuned, and the transverse intensity profile is chosen such that $2V_L>|\delta|$ throughout most of the profile, the pump will lead to induced focusing and hence confinement of the probe beam. Note that the *induced* focusing is obtained in the frequency region $\delta<0$, which corresponds to *self*-defocusing. (Actual self-defocusing would occur if the probe beam were sufficiently strong. However, in the weak probe limit assumed here, self-defocusing that is nonlinear in the probe intensity is negligible.)

We now consider the pump beam propagation, as described by Eq. (2). It is usually assumed that, due to saturation, the intensity profile of the pump remains constant during propagation through the medium. This assumption is not valid in our case and we take into account pump reshaping during propagation, which is mainly due to nonlinear absorption. For the waist size chosen in our calculations, the effect of diffraction on pump reshaping is small and there is no effect of refraction on the reshaping of the pump since the nonlinear part of the refractive index of a resonant beam is zero. This absorption is small near the center of the transverse pump profile where the high beam intensity required to obtain the dead zone is highly saturating. However, absorption at the low-intensity wings of the pump can still lead to reshaping of the pump, which in turn modifies the focusing effects experienced by the probe on propagation. This effect can be reduced by introducing dephasing collisions that decrease T_2 and hence the value of Im α_L. However, T_2 cannot be decreased too far, because of the limited value of D and the requirement that the dead zone can only be obtained for $D \gg 1/T_2$. The probe propagates nearly lossless in the medium as long as the pump Rabi frequency remains larger than the pump-probe detuning.

Equations (2) and (3) are solved numerically using the Fourier split step method [18]. We assume that the transverse profiles of the two propagating beams are initially Gaussian,

$$U_L(\xi,0)=\exp[-\xi^2],$$
$$U_P(\xi,0)=(A_{0P}/A_{0L})\exp[-\xi^2(w_{0L}^2/w_{0P}^2)], \tag{4}$$

where w_{0L} and w_{0P} are the initial pump and probe waist sizes. We assume the probe waist to be smaller than that of the pump so that the probe profile remains within the dead zone for an experimentally reasonable propagation length. Then the pump induces focusing of the probe without absorption, and the interplay between this induced focusing and diffraction leads to spatial confinement of the probe.

In the numerical results presented here, the pump is resonant with the atomic transition, the pump-probe detuning $\delta T_2=-20$ and $T_2 \sim 0.4 T_1$. The initial dimensionless pump and probe Rabi frequencies are $2A_{01}=40$ and $A_{02}=10^{-2}A_{01}$ and the Doppler width $DT_2=50$. In Fig. 1 we plot the initial transverse profile of the probe together with its absorption [$\mathrm{Im}(\alpha_P)$] and the deviation from unity of its refractive index [$\mathrm{Re}(\alpha_P)$]. This is equivalent to plotting the probe absorption and refractive index as a function of pump intensity, since the plots take into account the initial Gaussian transverse profile of the pump. Near the axis (small ξ) where the pump is intense, $\mathrm{Re}(\alpha_P)$ increases with increasing pump intensity, thereby inducing cross focusing of the probe. Moreover, in that region the probe absorption is zero. Here the probe waist size is initially half the pump waist size, that is, $w_{0L}=2w_{0P}$, and thus almost all the probe beam is contained in the region where focusing is induced by the pump and absorption is zero. In order to illustrate how these factors affect the propagation of the probe, we plot in Fig. 2 the probe transverse intensity profile as a function of the propagation distance, for the same parameters as in Fig. 1 and for

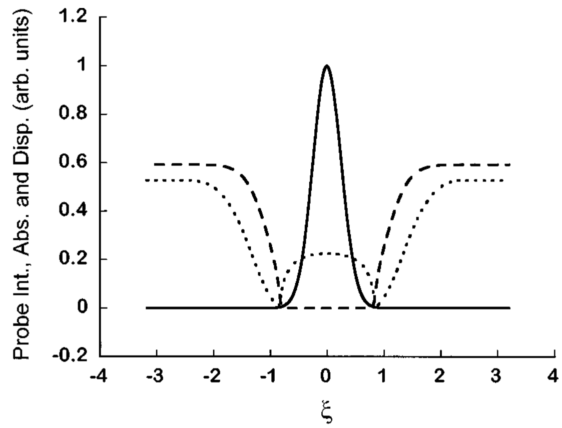

FIG. 1. The initial probe profile (solid line), the transverse variation of the probe dispersion (dotted line), and the probe absorption (dashed line) are illustrated on the same transverse scale, with their intensities in arbitrary units.

$L_{NL}=1.1\times10^{-3}$ cm and $L_D=15$ cm. Initially, the nonlinear refraction term is strong enough, due to the relatively large number of atoms in the medium, to induce focusing of the probe profile. This is shown in Fig. 2(a) ($z=0.5$ cm), where the slight initial focusing pulls the probe beam towards the center. On further propagation, the probe exhibits reshaping of its intensity profile due to the interplay between the nonlinear refractive index and diffraction, which gives rise to a successive focusing and defocusing. In Fig. 2(b) slight diffraction at the beam center is followed by focusing in Fig. 2(c). Diffraction then broadens the central part of the profile, as shown in Fig. 2(d). At $z=1$ cm [Fig. 2(e)] the previously defocused intensity is refocused due to refraction. We note that the effect of the refractive index becomes more pronounced as the pump and probe propagate in the medium. This is caused by the reshaping of the pump on propagation mainly due to nonlinear absorption, which in turn leads to continuous change in the behavior of the probe absorption

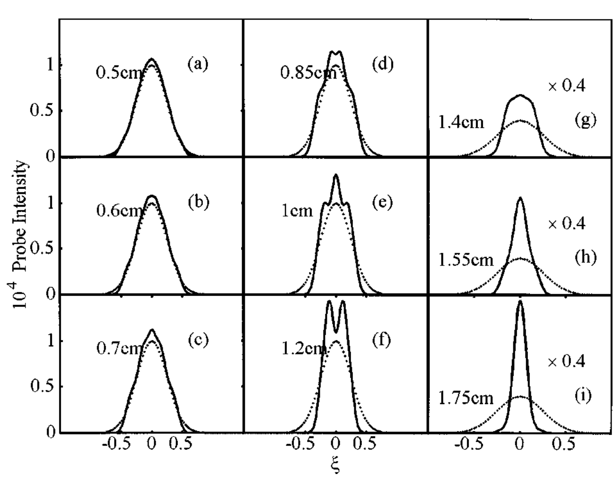

FIG. 2. The transverse intensity profile of the probe copropagating with a pump, as a function of $\xi=x/\sqrt{2}w_{0L}$ for various values of propagation length z, for a relatively wide probe beam and a strong nonlinearity. The solid line indicates the propagated probe and the initial probe profile (dotted line) is shown for comparison. The pump and probe parameters are $A_{0L}=20$, $\delta T_2=-20$, $T_2=0.43\times T_1$, $DT_2=50$, $L_D=15$ cm, and $L_{NL}=1.1\times10^{-3}$ cm.

and refractive index. The region of the dead zone is gradually reduced and the focusing induced by the refractive index is enhanced. In particular, the wings experience progressively stronger focusing. We see this clearly in Fig. 2(f) ($z=1.2$ cm), where the wings of the probe are focused and the near axis region is diffracted producing a dip at the center. Further propagation to $z=1.55$ cm [Fig. 2(h)] strongly focuses the probe. Focusing persists until $z\sim 2$ cm, where probe absorption becomes significant. At this stage the mean pump intensity is reduced to approximately 30% of its initial value due to its reshaping on propagation; the dead zone disappears causing increased probe absorption. For the case of a smaller probe waist and a smaller number of atoms in the medium, diffraction would initially dominate and would be followed by cycles of focusing and defocusing until the dead zone disappears.

Inspection of Eqs. (2) and (3) shows that the combined evolution of the pump and probe beams in the two-level medium can be characterized by a single parameter L_D/L_{NL} and a dimensionless propagation distance, z/L_D. A practical consequence is that one can probe confinement in a longer medium, provided the parameters L_D and L_{NL} are modified appropriately.

An experimental realization of nondissipative induced confinement of a probe beam, based on the parameters of Fig. 2, can be designed using, for example, a medium of sodium atoms. The $3\ ^2S_{1/2}-3\ ^2P_{3/2}$ transition of sodium with transition wavelength $\lambda_0=589$ nm forms an effective two-level system. For $N\sim 10^{13}$ atoms/cm^3 and pump waist $w_{0L}\sim 120\ \mu$m, the diffraction length is $L_D\sim 15$ cm and the nonlinear coefficient is $L_{NL}\sim 10^{-3}$ cm. The transverse decay time T_2 is reduced due to collisions, with $T_2\sim 0.4T_1$, and the Doppler width is $DT_2\sim 50$. A pump laser with initial intensity $I\sim 430$ W/cm^2 would be required to induce the confinement of a copropagating probe beam in the medium of length $L\sim 2$ cm. Relying on these rough estimates, we expect that in a suitable experiment, it should be possible to observe probe beam confinement.

In conclusion we have proposed a realistic system where a weak red-detuned probe shows induced solitonlike behavior, due to the influence of a resonant copropagating strong beam. This lossless propagation of the weak probe is obtained by exploiting the unique properties of the dead zone, where strong probe focusing is obtained in the absence of absorption. The lossless confined probe propagation in the medium is unaffected by pump absorption as long as the pump Rabi frequency is larger than the pump-probe detuning. Only at a propagation length where the pump Rabi frequency decreases below this detuning as a result of pump absorption and reshaping will significant probe absorption set in, leading to termination of the lossless solitonlike propagation of the probe.

The proposed effect is of practical interest since large values of the nonlinear refractive index can be obtained for a nearly resonant probe. However, near resonance, the absorption coefficient is also large, so that the effect would be unobservable. We have shown that this absorption can be eliminated so that strong induced focusing can be obtained in a lossless medium. We have also shown that this focusing persists during propagation. This allows, for example, strong focusing without the presence of an actual lens. The advantage of induced focusing of a weak probe over self-focusing is that self-focusing requires a strong beam and creates very high optical fields that produce destructive dielectric breakdown (hole burning). This is avoided when a weak probe is focused so that lensless nondestructive microscopy becomes possible.

[1] R. Y. Chiao, E. Garmire, and C. H. Townes, Phys. Rev. Lett. **13**, 479 (1964); P. L. Kelley, *ibid.* **15**, 1005 (1965); D. Grischkowsky, *ibid.* **24**, 866 (1970); E. Bjorkholm and A. Ashkin, *ibid.* **32**, 129 (1974); J. H. Marburger, Prog. Quantum Electron. **4**, 35 (1975).

[2] M. Segev, B. Crosignani, A. Yariv, and B. Fischer, Phys. Rev. Lett. **68**, 923 (1992).

[3] M. L. Dowell, B. D. Paul, A. Gallagher, and J. Cooper, Phys. Rev. A **52**, 3244 (1995).

[4] M. L. Dowell, R. C. Hart, A. Gallagher, and J. Cooper, Phys. Rev. A **53**, 1775 (1996).

[5] V. Baklanov and V. P. Chabotaev, Zh. Eksp. Teor. Fiz. **60**, 552 (1971) [Sov. Phys. JETP **33**, 300 (1971)]; **61**, 922 (1971) [**34**, 490 (1972)].

[6] G. Khitrova, P. R. Berman, and M. Sargent III, J. Opt. Soc. Am. B **5**, 160 (1988).

[7] D. J. Harter and R. W. Boyd, Phys. Rev. A **29**, 739 (1984); D. J. Harter, P. Narum, M. G. Raymer, and R. W. Boyd, Phys. Rev. Lett. **46**, 1192 (1981); R. W. Boyd, M. G. Raymer, P. Narum, and D. J. Harter, Phys. Rev. A **24**, 411 (1981).

[8] J. F. Valley, G. Khitrova, H. M. Gibbs, J. W. Grantham, and Xu Jianjin, Phys. Rev. Lett. **64**, 2362 (1990).

[9] A. J. Stentz, M. Kauranen, J. J. Maki, G. P. Agrawal, and R. W. Boyd, Opt. Lett. **17**, 19 (1992).

[10] G. P. Agrawal, Phys. Rev. Lett. **64**, 2487 (1990).

[11] R. R. Moseley, S. Shepherd, D. J. Fulton, B. D. Sinclair, and M. H. Dunn, Phys. Rev. Lett. **74**, 670 (1995).

[12] R. R. Moseley, S. Shepherd, D. J. Fulton, B. D. Sinclair, and M. H. Dunn, Phys. Rev. A **53**, 408 (1995).

[13] Robert W. Boyd, *Nonlinear Optics* (Academic Press, New York, 1992).

[14] H. Friedmann and A. D. Wilson-Gordon, Phys. Rev. A **36**, 1333 (1987).

[15] A. Javan and P. L. Kelley, IEEE J. Quantum Electron. **QE-2**, 470 (1966).

[16] H. Friedmann and A. D. Wilson-Gordon, Opt. Commun. **116**, 163 (1995).

[17] H. Friedmann and A. D. Wilson-Gordon, Phys. Rev. A **52**, 4070 (1995).

[18] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, Boston, 1989).