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Embedded Bell-state analysis

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We discuss a method for distinguishing the four orthogonal Bell states of two entangled particles. Because the scheme relies only on linear optical elements, it should be realizable with current technology. The new feature is that the Bell states must be embedded in a larger Hilbert space. That is, the correlated particles must be entangled in more than one degree of freedom. [S1050-2947(98)50310-1]

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Entangled states of particles form the cornerstone of the newly emerging field of quantum information. They have been proposed for use in certain quantum cryptography schemes, in which the nonlocal correlations of the particles are employed to allow a sender and a receiver to share a secret ''key'' [1]. In a different manifestation of quantum communication, one may use such correlated pairs to enable ''quantum dense coding'' [2,3], whereby more than one bit of information may be encoded in a single two-state system. However, one must be able to distinguish among the various possible entangled states. This is also true in the related phenomenon of ''quantum teleportation'' [4–6]—an unknown quantum state may be ''teleported'' between two parties as long as each of them possesses one of a pair of entangled particles, and can make appropriate joint measurements of two particles. The entire new field of quantum computation has at its heart the ability to prepare and manipulate various superposition states involving multiple particles, which form the bits of the computer [7]. Entangled states may result whenever elementary gate operations are applied to input states in a quantum superposition (e.g., when the control bit of a controlled-not gate is in a superposition of 0 and 1).

Perhaps the simplest examples of entangled states of two particles, e.g., photons, are the polarization-entangled ''Bell states'' [8]:

$$|\psi^{\pm}\rangle=(|H_1,V_2\rangle\pm|V_1,H_2\rangle)/\sqrt{2},$$
$$|\phi^{\pm}\rangle=(|H_1,H_2\rangle\pm|V_1,V_2\rangle)/\sqrt{2}, \tag{1}$$

where H and V denote horizontal and vertical polarization, respectively. These states are central in many tests of Bell's inequalities, and have been produced experimentally via the nonlinear process of spontaneous parametric down-conversion [9].

The states (1) are clearly orthogonal (they form the complete maximally entangled basis of the two-particle polarization Hilbert space), and it is simply assumed, in many theoretical proposals, that one can distinguish among them with a single joint measurement of the two particles [10]. Unfortunately, until now there has been no way to experimentally distinguish all four of the Bell states, although there are fairly straightforward interferometric schemes to identify two of the four states, with the others giving the same detection signal [3,11,12]. In fact, although to our knowledge it has not been rigorously proved, it is commonly believed that a nonlinear process, such as is available in cavity-QED experiments [13], is necessary for a complete analysis.

We will show that a complete analysis of Bell states *is possible with only linear optical elements*, by working in a larger Hilbert space, i.e., by employing additional degrees of freedom—the particles must also be entangled in these new variables. In essence, these schemes use the additional entanglement to permit a second interferometric measurement, which can distinguish between the remaining two Bell states. The first proposal employs time-energy entanglement and is easier experimentally, especially since such an entanglement comes automatically in continuous-wave parametric down-conversion [14]. The other scheme, relying on momentum

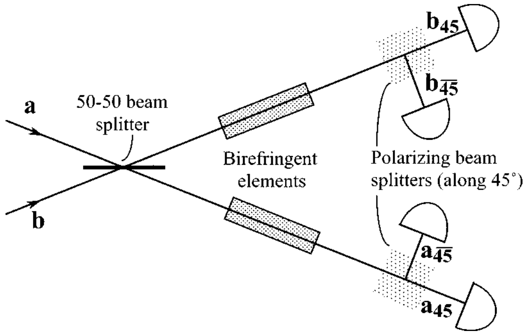

FIG. 1. Setup to allow identification of all four polarization Bell states, by employing the intrinsic time-energy correlations of the down-converted photon pairs. Interference at the 50-50 beam splitter renders ψ^- distinguishable from the others; the birefringent element (with axes along the horizontal and vertical directions) separates ψ^+; interference at the polarizing beam splitters distinguishes ϕ^- from ϕ^+.

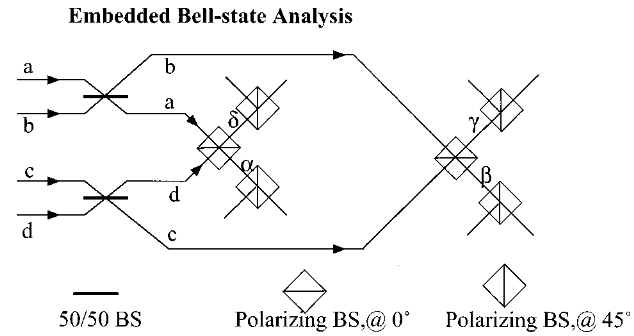

FIG. 2. Schematic of the setup to identify all four polarization Bell states when the photons are also momentum-entangled.

entanglement, is technically more involved, but could be employed even in cases where the absolute timing of the photons is critical. Both schemes put challenging requirements on the single-photon detectors used. We discuss how to meet these with current technology.

The setup for a total polarization Bell-state analysis, relying on additional time-energy entanglement, is shown in Fig. 1. The first step is to interfere the two photons on a 50-50 beam splitter, which converts incident spatial modes $a \rightarrow (a + ib)/\sqrt{2}$; $b \rightarrow (ia + b)/\sqrt{2}$. It is then easy to show that of the four states (1), only ψ^- will result in one photon in each of the beam-splitter output ports; for the other three states both photons will end up in the same output port. The physical reason for this is that ψ^- has a singletlike character—the spin part is antisymmetric, and so too must be the spatial part, in order to preserve the total bosonic nature. Therefore, the photons act effectively fermionic at the beamsplitter (which affects only the spatial modes), and always end up in different output ports. For the other three states the spatial part of the wave function is symmetric (i.e., bosonic), so the photons always exit the same port of the beam splitter. Note that one can already distinguish ψ^- from the rest on this basis alone. In what follows, therefore, we need to consider only one of the output ports, the a mode (identical analysis is made in the b mode).

In a recent experiment to demonstrate the principles of quantum dense coding [3], the next step was to analyze the photons in the H-V basis using a polarizing beam splitter. This readily distinguishes ψ^+ from $\phi^\pm$, but cannot distinguish ϕ^+ from ϕ^-. If one were to instead analyze in the 45-$\overline{45}$ basis (where $\overline{45}$ represents $-45°$), then ϕ^+ and ϕ^- *would* be distinguishable from each other, but no longer from ψ^+. The essence of our technique lies in using extra correlations shared by the photons to enable a discrimination of ψ^+ from ϕ^+ from ϕ^-.

To achieve this, the photons are passed through a strongly birefringent material of length L, whose axes are oriented in the H-V basis. The effect is to separate the H and V parts of the wave function *temporally* by an amount $\Delta nL/c$, where Δn is the difference in the refractive indices. When this is done, ϕ^+ (for which any subsequent detections will happen simultaneously) becomes distinguishable from $\psi^\pm$ (detections will, in principle, be temporally distinguishable). Although in principle one only needs to separate the components by more than the correlation time of the down-conversion photons (less than $\sim$100 femtoseconds), in practice the relative delay must be measureable by current detectors, so $\sim$1 ns is required [15]. What remains is to identify ϕ^+ from ϕ^-.

We use polarization analysis in the 45-$\overline{45}$ basis, which transforms $a_H \rightarrow (a_{45} - a_{\overline{45}})/\sqrt{2}$, $a_V \rightarrow (a_{45} + a_{\overline{45}})/\sqrt{2}$; and similarly for b_H and b_V. It is instructive here to write out the resulting states $\phi^\pm$:

$$\begin{aligned}\phi^+ \Rightarrow &\{a_{45}(t_h)a_{45}(t_h) - 2a_{45}(t_h)a_{\overline{45}}(t_h) + a_{\overline{45}}(t_h)a_{\overline{45}}(t_h)\}\\ &+ \{a_{45}(t_v)a_{45}(t_v) + 2a_{45}(t_v)a_{\overline{45}}(t_v)\\ &+ a_{\overline{45}}(t_v)a_{\overline{45}}(t_v)\} + \{b(t_h)\} + \{b(t_v)\}\end{aligned} \tag{2a}$$

$$\begin{aligned}\phi^- \Rightarrow &\{a_{45}(t_h)a_{45}(t_h) - 2a_{45}(t_h)a_{\overline{45}}(t_h) + a_{\overline{45}}(t_h)a_{\overline{45}}(t_h)\}\\ &- \{a_{45}(t_v)a_{45}(t_v) + 2a_{45}(t_v)a_{\overline{45}}(t_v)\\ &+ a_{\overline{45}}(t_v)a_{\overline{45}}(t_v)\} + \{b(t_h)\} + \{b(t_v)\},\end{aligned} \tag{2b}$$

where the labels t_h and t_v represent the delays experienced in the birefringent element by the horizontal and vertical components, respectively. For photons uncorrelated in time-energy, the states (2) are still indistinguishable. However, as has been demonstrated in several experimental violations of a Bell's inequality for time-energy variables [16], photons created via spontaneous parametric down-conversion are automatically entangled in energy and time [14], so that terms arising from photons created at different times [e.g., $a_{45}(t_h)a_{45}(t_h)$ and $a_{45}(t_v)a_{45}(t_v)$] can be *coherent* with each other. Consequently, due to interference, the states [2(a) and 2(b)] simplify tremendously and, in fact, become experimentally distinguishable:

$$\begin{aligned}\phi^+ \Rightarrow &\{a_{45}(t)a_{45}(t) + a_{\overline{45}}(t)a_{\overline{45}}(t)\\ &+ b_{45}(t)b_{45}(t) + b_{\overline{45}}(t)b_{\overline{45}}(t)\}\end{aligned} \tag{3a}$$

$$\phi^- \Rightarrow \{a_{45}(t)a_{\overline{45}}(t) + b_{45}(t)b_{\overline{45}}(t)\}. \tag{3b}$$

Thus, the four Bell states can be distinguished due to different detection events: only for ψ^- does one detect one photon at each of the two sides of the first beam splitter; for the other three states both photons are registered on one side of this beam splitter. ψ^+ is further characterized by detecting two photons with a time separation of $t_h - t_v$. For ϕ^+ we expect simultaneous detection of two photons at a single detector, whereas for ϕ^-, one simultaneously registers one photon at each of two detectors. Note that the time difference $t_h - t_v$ must be less than the pump coherence length for the desired interference to occur. Similarly, had we considered a short *pulsed* pump to produce photon pairs with a short correlation time (thus permitting us to separate ψ^+ from $\phi^\pm$), there would not be the necessary coherence [17]. The photons must not have a well-defined creation time, i.e., they must be ''time''-entangled.

For our second scheme (Fig. 2) it is necessary that the photons begin simultaneously entangled in polarization and in (momentum) direction [19]:

$$\begin{aligned}\Psi^\pm &= \{(a_H b_V \pm a_V b_H) + (c_H d_V \pm c_V d_H)\}/2\\ \Phi^\pm &= \{(a_H b_H \pm a_V b_V) + (c_H d_H \pm c_V d_V)\}/2.\end{aligned} \tag{4}$$

For example, these states can be generated via two crystals of the sort used to generate polarization-entangled states, coherently pumped by a single laser. Such ''hyper''-entangled states (i.e., jointly entangled in more than one degree of freedom) may also be available directly from a single suitably chosen down-conversion crystal [18].

The first step in the analysis is identical with the previous case—the photons are mixed on 50-50 beam splitters. The role of the birefringent element (which separated H and V components into two different *times*) is played by polarizing beam splitters, which separate the H and V components into two different *directions*: $a_H \to \alpha_H$, $a_V \to \delta_V$, etc. Moreover, these allow us to mix our initial spatial modes, e.g., both a_H and d_V are transformed into the spatial mode α. After the 50-50 beam splitters and the polarizing beam splitters, our Bell states become

$$\begin{aligned}\Psi^+ &= i(\alpha_H\delta_V + \alpha_V\delta_H + \beta_H\gamma_V + \beta_V\gamma_H)/2,\\ \Psi^- &= (\alpha_H\gamma_V + \alpha_V\gamma_H - \beta_H\delta_V + \beta_V\delta_H)/2,\\ \Phi^\pm &= i\{(\alpha_H\alpha_H + \beta_H\beta_H + \gamma_H\gamma_H + \delta_H\delta_H) \pm (\alpha_V\alpha_V + \beta_V\beta_V\\ &\quad + \gamma_V\gamma_V + \delta_V\delta_V)\}/(2\sqrt{2}).\end{aligned} \tag{5}$$

One can readily see from the spatial modes that Ψ^- is distinguishable from Ψ^+, and both are distinguishable from $\Phi^\pm$. The remaining step, to identify Φ^+ and Φ^-, is again performed with a polarizing beam splitter at 45°. The resulting final states are

$$\begin{aligned}\Psi^+ &= i(\alpha_{45}\delta_{45} - \alpha_{\overline{45}}\delta_{\overline{45}} + \beta_{45}\gamma_{45} - \beta_{\overline{45}}\gamma_{\overline{45}})/2,\\ \Psi^- &= (\alpha_{45}\gamma_{45} - \alpha_{\overline{45}}\gamma_{\overline{45}} - \beta_{45}\delta_{45} + \beta_{\overline{45}}\delta_{\overline{45}})/2,\\ \Phi^+ &= i(\alpha_{45}\alpha_{45} + \alpha_{\overline{45}}\alpha_{\overline{45}} + \beta_{45}\beta_{45} + \beta_{\overline{45}}\beta_{\overline{45}} + \gamma_{45}\gamma_{45} + \gamma_{\overline{45}}\gamma_{\overline{45}}\\ &\quad + \delta_{45}\delta_{45} + \delta_{\overline{45}}\delta_{\overline{45}} +)/(2\sqrt{2}),\\ \Phi^- &= -i(\alpha_{45}\alpha_{\overline{45}} + \beta_{45}\beta_{\overline{45}} + \gamma_{45}\gamma_{\overline{45}} + \delta_{45}\delta_{\overline{45}})/2.\end{aligned} \tag{6}$$

From (6) we note that each Bell state gives a different signature of detectors firing.

A discussion about detectors is now appropriate. One sees immediately that the schemes we propose here place rather strong requirements on the detectors, aside from the obvious wish for detection efficiencies that are as high as possible. First, in both schemes, the detectors should have the ability to determine the *number* of incident photons: two incident photons should give a different signal than a single one, otherwise some states cannot be identified unambiguously (due to the nonunity detector efficiency). Also, in the first scheme, the detector must be able to distinguish between two photons that are incident simultaneously, and one that is incident only a short time after the other. While such behavior is the norm for photodiodes operated in the linear mode, these are typically far too noisy to allow single-photon sensitivity. And while avalanche photodiodes operated in the geiger mode can have efficiencies in excess of 75–80% (visible) [20], they are not able to resolve multiple-photon impacts; two photons hitting the detector simultaneously initiate just as great an avalanche as one photon (although the probability of starting the avalanche will be enhanced). However, there are detectors under investigation that may solve these problems.

The first detectors are a solid-state photomultiplier and a visible-light photon counter (VLPC), produced by Rockwell International, now Boeing International. Initial tests of these devices indicated single-photon detection efficiencies over 70% [20], with indications that efficiencies in excess of 90% may be achievable. Moreover, the devices have shown some ability to discriminate the photon number [21], owing to the fact that the avalanche associated with an incident photon is limited to a small filament $\sim$10 μm diameter), compared with the total device size of 1 mm; therefore, the net effect is very roughly that of an array of $\sim$10 000 independent detection elements [22]. Recently, the VLPC was demonstrated in a high-efficiency mode ($\eta \approx 88\%$), with clearly distinguishable signals for one and two simultaneously incident photons [23].

The second type of detector, which is already commercially available, is the hybrid photomultiplier [24]. These devices basically have a photocathode at the input, which produces a photoelectron; a central accelerating region (typically 8–15 kV) that boosts the energy of the photoelectrons by a factor of several thousand; and finally a p-i-n diode or an avalanche photodiode (the latter resulting in an additional gain of about 500). While such devices have already demonstrated their ability to resolve several photoelectrons, and thus to distinguish between, e.g., one and two incident photons, their usefulness at present is extremely limited by the rather low quantum efficiencies of the photocathodes (10–20%).

In conclusion, we have shown how all four polarization Bell states may be reliably identified using only linear optical elements, with the additional requirement that the correlated particles also be entangled in other degrees of freedom (time-energy or momentum-position). By *embedding* the states of interest in a larger Hilbert space, we are able to solve an otherwise difficult problem. Our method should therefore allow a fairly easy realization of quantum dense coding in which a full two bits of information (corresponding to the four polarization Bell states) are encoded using only the two-state polarization subspace of one of the photons. While 100% efficient teleportation of an arbitrary state (of polarization) does not appear to be possible with these techniques (due to a lack of the additional entanglement between noncorrelated photons), they may be useful, for instance, in extending the capabilities of all-optical emulations of quantum computers, which rely on multiple degrees of freedom of a *single* particle to allow implementation of any quantum circuit using only linear passive elements [25].

Note added in proof. We have recently learned that both Lev Vaidman and Norbert Lütkenhaus have, in fact, proved that the four Bell states *cannot* be distinguished using only linear optics; there is no contradiction with our result, however, as they do not allow multiple entanglements [for which the states (1) do not then span the enlarged Hilbert space of the two particles].

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