Elliptical ion traps and trap arrays for quantum computation

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The properties of a rf quadrupole trap, the elliptical ion trap, are derived. Elliptical traps can confine large numbers of ions in the Lamb-Dicke regime due to a hitherto unrecognized mechanism unique to one-dimensional Coulomb crystals, implicit in the theories of Dubin and Schiffer. This follows from a linear crystal stability condition, which uniquely relates the crystal size to ellipticity, and a micromotion relation, which reveals a 1/5-root dependence on the number of trapped ions. Elliptical traps offer several advantages over linear traps in the Cirac-Zoller model of quantum computing, both for initial tests and as a potential method of creating a full-scale quantum computer. Numerical solutions of a one-electrode structure show that microscopic elliptical traps, each containing ≈ 100 ions, can be constructed at a density of 100 traps/cm^2, making possible arrays containing $>10^6$ ions in the Lamb-Dicke regime for precision spectroscopy or quantum computation. [S1050-2947(98)08008-1]

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Radio frequency ion traps based on the Mathieu equation confinement mechanism of Paul [1] have been important in several areas of physics, including laser cooling [2,3], ion crystallization [4,5], precision spectroscopy and standards [6], and most recently quantum computation [7–9]. Ion traps have so far been developed in two contrasting geometries: the circular trap [1] (Paul trap) and the linear trap [10,11]. The former has rotational symmetry and the latter translation symmetry as shown in Fig. 1. Linear traps were developed to overcome micromotion, which is inherent in Mathieu equation solutions for circular traps for more than one ion. Micromotion can prevent trapping in the Lamb-Dicke regime, which requires confinement to $\lambda/2\pi$, where λ is the wavelength of cooling laser radiation, and is often assumed for studies of quantum ion motion and computation. This paper introduces a third type of ion trap, the elliptical ion trap, which possesses neither circular nor translational symmetry but which can confine large numbers of ions in the Lamb-Dicke limit. Note that slightly elliptical traps have been used by experimenters [12–14,9] for many years to define an axis of crystallization; Dehmelt used elliptical ring electrodes in a Paul-Straubel trap as early as 1989. However, the general case of arbitrary ellipticity has not previously been discussed. Our analysis yields two unexpected results: first, that large linear crystals can be stored in traps of moderate ellipticity [Eq. (7)], and second that micromotion amplitudes depend weakly on the size of the crystal [Eq. (8)], approximately as the 1/5 root of the number of ions. This follows from the recently understood properties of Coulomb crystals, expressed in the theories [15–18] of Dubin and Schiffer. Taken together, these results show that a single elliptical trap can confine as many as 10^3 ions in the Lamb-Dicke regime. Elliptical traps can be constructed from a single flat electrode yielding a device that is simpler, smaller, and stronger than a linear trap and is suitable for microfabrication in large arrays. In this way, it is possible in principle to trap 10^6 or more ions in the Lamb-Dicke regime, to increase the signal-to-noise ratio in precision spectroscopy or for quantum computation.

The linear ion trap is the basis for the Cirac-Zoller model of quantum computation [7,8]. In this model a row of ions is confined along the z axis of the trap and laser cooled to the ground vibrational state. The lowest center-of-mass (c.m.) vibrational phonon of the ion array couples to the internal atomic states (the quantum bits or qubits) via Doppler shifted laser excitation. This phonon is used as a quantum communication channel to transmit superposition states between the atoms and carry out quantum logic. The Cirac-Zoller model contains all the interactions required for an evaluation of Shor's algorithm [19] and has been studied in such detail [20] that it is often treated as a paradigm for the practicality of quantum computing in general. Grover has developed a quantum database algorithm [21] that searches N unordered memory elements in $\sqrt{N}$ steps. Both Shor's and Grover's algorithms require large numbers of qubits to gain an advantage over classical methods. Factorization of a 400 bit number has been estimated [20] to require >2000 ions without quantum error correction (QEC) and from 10^4 to 10^6 ions

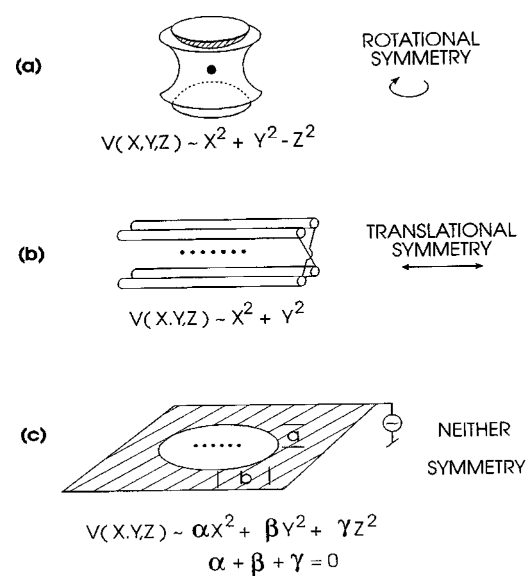

FIG. 1. (a) Original circular ion trap of Paul, (b) the linear ion trap assumed by the Cirac-Zoller model, and (c) the elliptical trap.

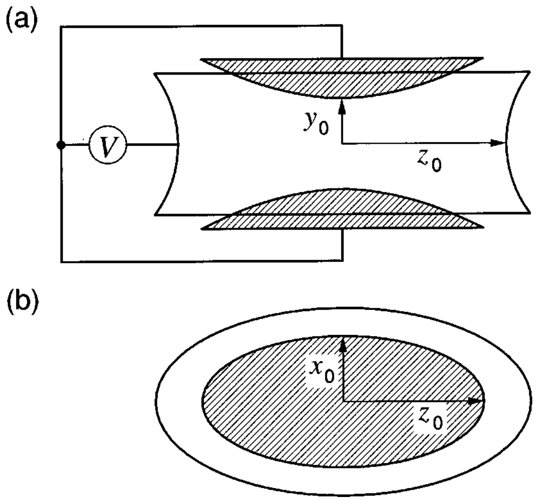

FIG. 2. Ideal elliptical ion trap, consisting of three elliptical hyperboloids. It generates the potential of Eq. (1) throughout the interior volume, since the electrodes lie on equipotential surfaces. The ellipticity $\epsilon = z_0/x_0$. In practice the simpler electrode of Fig. 5 suffices.

with QEC [22]. A full-scale implementation of either algorithm is therfore not practical in a single trap, even ignoring technical difficulties, for reasons discussed below.

The elliptical trap offers several advantages for use in the Cirac-Zoller model, both for intial experiments and for extending the model to the full-scale regime. First, it acts like a linear trap for more than 10^3 ions, so that the c.m. phonon can be used for quantum logic as before. Second, the trap requires only a single flat electrode of microscopic dimensions so that trap arrays can in principle be microfabricated at a density of $\geqslant 10^2$ traps/cm^2. Arrays of moderate size could therefore confine the 10^6 qubits needed for a full-scale device. Third, the size and geometry of the trap permit their use in testing recently proposed quantum optical interconnects [23–26], since the trap is small enough to be enclosed by an optical cavity that can satisfy the ''strong-coupling'' condition of cavity quantum electrodynamics (CQED). Long-term confinement to the Lamb-Dicke regime is important for tests of quantum optical interconnects and of CQED in general. Current experiments have been limited to beams of neutral atoms that transit the cavity in milliseconds to microseconds [27]. Ion traps have so far not been used despite their unlimited confinement time in part because the mirror spacing for a strong-coupled cavity is substantially smaller than the smallest linear ion trap. The two-dimensional structure of elliptical traps overcomes this problem, as discussed in more detail below. Fourth, trap arrays provide a specific physical model of the ''nodes'' of a distributed quantum network [23,24], in which each microscopic elliptical trap represents a separate Cirac-Zoller processor.

The elliptical ion trap, shown in Fig. 2, is a generalized ion trap that includes both the Paul trap [1] and the linear trap [10] as special cases. It is defined by the potential

$$V(x,y,z) = [V_{dc} + V_0\cos(\Omega t)]\left(\frac{x^2}{x_0{}^2} - \frac{y^2}{y_0{}^2} + \frac{z^2}{z_0{}^2}\right). \quad (1)$$

The axes are named in agreement with the linear trap convention rather than the Paul trap convention. This is the most general quadratic potential satisfying $\nabla^2 V = 0$, which requires $1/x_0^2 + 1/z_0^2 = 1/y_0^2$. Such a potential can be created by hyperboloidal electrodes that form ellipses in the x-z plane, where x_0 and z_0 are the minor and major axes of the ellipse at $y=0$, while y_0 is the distance to the endcap electrode at $x=z=0$. Solution of Newton's law for the above potential leads to three Mathieu equations [28,29] for the trapped ion's coordinates $u = x,y,z$,

$$\frac{d^2u}{d\tau^2} + \alpha_u[a_u - 2q_u\cos(2\tau)]u = 0, \quad (2)$$

where $\alpha_u = 1$ for $u = x,z$ and $\alpha_u = -1$ for $u = y$, the normalized time $\tau = \Omega t/2$, the Mathieu parameter (a dimensionless trap strength) is

$$q_u = -\frac{4eV_0}{m\Omega^2 u_0{}^2}, \quad (3)$$

m is the mass of the particle, $\Omega/2\pi$ the rf drive frequency, and $u_0 = x_0, y_0, z_0$. An identical equation holds for a_u with V_0 replaced by $2V_{dc}$. $\nabla^2 V = 0$ implies the relation $q_x + q_z = q_y$ and $a_x + a_z = a_y$. Define the ellipticity $\epsilon = z_0/x_0$ to be the ratio of the major to minor axes of the ellipse, which yields $q_z = q_y/(\epsilon^2+1)$ and $q_x = q_y\epsilon^2/(\epsilon^2+1)$. In the low q limit ($q<0.2$) where most traps are operated [29,30] the trap secular frequencies $\omega_u \to q_u\Omega/2\sqrt{2}$ and therefore obey

$$\omega_z = \omega_y \frac{1}{\epsilon^2+1} \quad (4)$$

and

$$\omega_x = \omega_y \frac{\epsilon^2}{\epsilon^2+1}. \quad (5)$$

The Paul trap is a special case of $\epsilon = 1$ where $\omega_x = \omega_z = \omega_y/2$, while the linear trap is a limiting case of $\epsilon \to \infty$ where $\omega_z \to 0$ and $\omega_x \to \omega_y$. Although it is well known that the linear trap is a limiting case of a stretched circular trap [10], and although the general form of Eq. (1) was discussed by Paul [28] many years ago, the regime $1 < \epsilon < \infty$ has not previously been considered in detail.

To play the role of a linear trap in the Cirac-Zoller model, an elliptical trap must first confine the ions on the z axis so that the c.m. phonon can act as a quantum communications channel, and second have sufficiently small micromotion so that the resulting Doppler shifts do not decouple the ions from the lasers. Schiffer [17,18], Dubin [15,16], and others have shown through analytic and numerical studies of ion crystals in asymmetric harmonic potentials that the ions will stay on the z axis providing

$$d > d_0 \equiv \left(\frac{\alpha e^2}{m\omega_x^2}\right)^{1/3}, \quad (6)$$

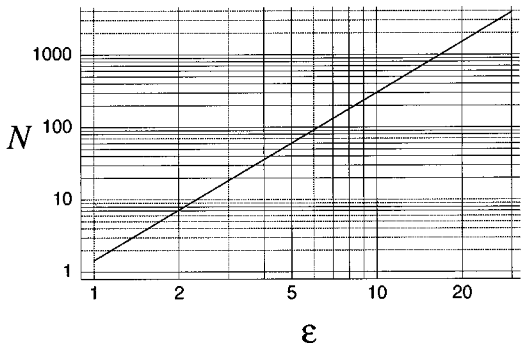

FIG. 3. Plot of Eq. (7), showing $N(\epsilon)$, the maximum number of ions that can be confined on the z axis before the zig-zag transition, as a function of the ellipticity ϵ. Note that $N(\epsilon)$ is independent of all other trap parameters.

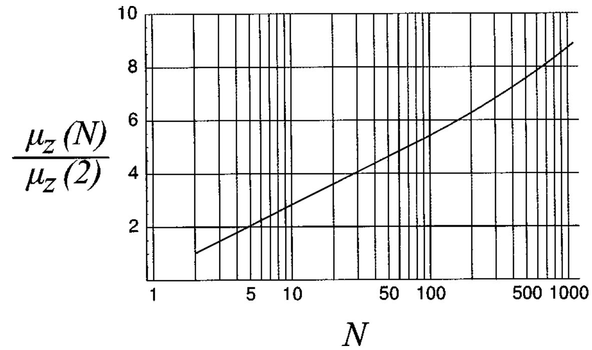

FIG. 4. Maximum micromotion amplitude μ_z in a linear crystal of N ions, Eq. (8). Note the weak dependence on N, which approximates $N^{1/5}$ at large N, due to the confinement properties of one-dimensional Coulomb crystals.

where d is the ion-ion separation at the center of the trap and $\alpha = 7\zeta(3)/2 = 4.207$, where ζ is the Riemann ζ function. For $d < d_0$ the ions undergo what is called the ''zig-zag transition,'' where alternate ions move off the z axis in opposite directions. Note that Eq. (6) does not depend directly on the axial trap strength ω_z, but only on the transverse trap strength ω_x. No exact analytic expression for d exists for all N but several accurate approximations have been derived. Equating Steane's [8] result $d = 2.0 s_0 N^{-0.57}$ to d_0 in Eq. (6), where the scale length $s_0^3 = e^2/m\omega_z^2$, yields the relation

$$N(\epsilon) = 1.44\epsilon^{2.34}, \tag{7}$$

which determines the capacity of a trap of given ellipticity ϵ to hold a number $N(\epsilon)$ of ions on the z axis. See Fig. 3. For $N > N(\epsilon)$, the ions assume the zig-zag structure while for $N < N(\epsilon)$, the ions are farther apart than the minimum value d_0. Note that Eq. (7) is independent of all trap parameters except the ellipticity.

The second requirement is that the elliptical trap confine large crystals in the Lamb-Dicke regime, that is, that the micromotion amplitude is less than $\lambda/2\pi$, where λ is the wavelength of the laser light. It has been assumed previously that only linear ion traps, which have no rf field along the z axis, can provide small enough micromotion for quantum computation. However, a detailed calculation of micromotion amplitudes shows that a properly designed elliptical trap can limit micromotion oscillations to a few tens of nm, even for large ion crystals containing more than 1000 ions and even though rf fields are used for all confinement. Micromotion [29,3] is the oscillatory motion of the ions at the rf drive frequency Ω and in the low q limit is given by $\mu_z(t) = \overline{z} q_z \cos(\Omega t)/2$, where $\overline{z}$ is the time-averaged (secular) position of the ion. The largest micromotion occurs at the ends where $\overline{z} = l_z/2$, where l_z is the total length of the ion chain. The theories of Dubin and Schiffer assume a static harmonic potential (the pseudopotential approximation) that yields accurate values of l_z but gives no information about micromotion. These results may be summarized by the approximation $l_z = d(N-1)^{1.053}$, which agrees within 10% with numerical results [15,31]. The micromotion may then be estimated using Steane's approximation for d yielding $\mu_z(N) = q_z s_0 (N-1)^{1.053}/2N^{0.57}$, which approximates $q_z s_0 \sqrt{N}/2$ as $N \to \infty$. However, a much lower value of μ_z can be attained by matching the ellipticity to N. Consider a series of traps that are identical except for their ellipticity and assume that each trap contains $N(\epsilon)$ ions in accordance with Eq. (7). This minimizes micromotion for a given N by using the weakest axial confinement, which will keep the ions on the z axis. Using Eqs. (4), (5), (6), and (7) yields

$$\mu_z(N) = \mu_z(2)\frac{1.985(N-1)^{1.053}}{N^{0.569}(1+0.733N^{0.854})^{1/3}}, \tag{8}$$

where $\mu_z(2)$ is the micromotion of two ions in a trap with $\epsilon = 1$. In the large $N (N \geq 20)$ limit $\mu_z(N) \to 2.20 N^{0.20} \mu_z(2)$, that is, it depends approximately on the fifth root of N. This weak dependence is shown in Fig. 4 and permits trap parameters to be chosen so that micromotion is of little significance throughout the range $N = 2-1000$, as shown in an example below.

This result is unexpected since in three dimensions the micromotion amplitude increases approximately as $N^{1/3}$, due to the constant ion density [10]. Why does one-dimensional confinement result in a weaker power law of $N^{1/5}$? The answer is that in one dimension the density increases with N, due to more effective cancellation of the Coulomb repulsion of neighboring ions. A related result is that traps filled according to Eq. (7) all have the same value of $d = d_0$, despite the reduction of the axial trap force with increasing ϵ.

An elliptical trap can be constructed from single flat electrode, for example, an elliptical aperture in a conducting surface as shown in Fig. 5, which is straightforward to micro-

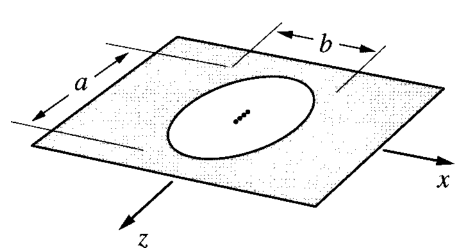

FIG. 5. ''Aperture trap'' consisting of an elliptical hole in a conducting surface that generates the potential of Eq. (1) near the z axis. For the case considered in the text, trap arrays can be generated at a density of ≥ 100 traps/cm^2.

fabricate in arrays by photolithography. This is in contrast to a linear trap, which consists of a three-dimensional array of four segmented rods [11]. The elliptical aperture generates a potential that reduces to Eq. (1) on the z axis, where the ion crystal resides. The theory of such ''aperture traps'' has been discussed in detail [13] for the circular case and they have already been used in quantum optics [14] and quantum logic experiments [9]. In the circular case the trap parameters can be derived as coefficients in the Legendre expansion of the potential [13], but for $\epsilon > 1$ the corresponding power series contains ellipsoidal harmonics, which are rarely used. A three-dimensional numerical routine has therefore been used to estimate the three relevant parameters of the potential: the efficiency, the ellipticity, and the anharmonic terms. Potentials have been computed for apertures with ellipticities of 1, 3, 5, and 10, corresponding to $N(\epsilon)$ of 2, 18, 60, and 300 in Eq. (7). The efficiency is defined as $E = V_0/V_t$, where V_t is the potential applied to the conducting surface. The routine yields $0.05 < E < 0.10$ depending on the thickness of the electrodes. The ellipticity of the potential is found to be equal to the ellipticity of the aperture within the accuracy of 20% and the anharmonic terms were found to be comparable to the circular case [13]. Specifically for a 100 μm by 300 μm aperture, the z axis fourth order (anharmonic) term was less than 1% of the second-order term along a central 100 μm region of the z-axis. It is not apparent from numerical work whether an elliptical aperture produces the most accurate approximation to Eq. (1), since simpler apertures such as rectangles produce similar fields.

The practicality of elliptical traps can be demonstrated by considering an example based on recent experiments. Consider first an $\epsilon = 1$ trap of 100 μm radius, similar to Refs. [14,9]. To separately excite each ion for quantum logic, the ion-ion spacing d_0 must be greater than λ. Choosing $d_0 = 4$ μm yields an axial oscillation frequency $\omega_z/2\pi \approx 1$ MHz at an applied potential $V_t = 1000$ V rms (efficiency $E = 0.08$) for the ion Ba 137, currently under study in our laboratory. The Mathieu parameter $q_z = 2\sqrt{2}\omega_z/\Omega = 0.01$ for $\Omega/2\pi = 282$ MHz. This yields a micromotion $\mu_z = 10$ nm for a two-ion crystal. Now assume that for quantum computing we wish to confine 64 ions. Equation (7) yields an optimum ellipticity $\epsilon = 5.2$ that determines $q_z = 7.1 \times 10^{-4}$. The crystal length $l_z/2 \approx 32 d_0 = 128$ μm, which gives $\mu_z = 45$ nm at the ends of the crystal and proportionately less near the center. The Doppler sidebands will therefore have an intensity of $J_1^2(\beta) \leq 0.06$, where $\beta = 2\pi\mu_z/\lambda$. This establishes that the Lamb-Dicke limit is reached for all ions in the trap. Note that a larger μ_z has been used in a recent experiment, which observed interference and superradiance of two trapped ions [14]. Such small micromotion amplitudes appear unrelated to phonon heating rates in circular microtraps [9], so that elliptical and linear traps should have similar rates.

The above 64-ion trap occupies an area of $< 2 \times 10^{-3}$ cm^2 and may be placed in an array at a density of ≈ 100 traps/cm^2, yielding a density of 6400 ions/cm^2. A 30-cm-diam disk could therefore support 64 000 traps holding 4×10^6 ions, all in the Lamb-Dicke limit. Such a disk driven at 1000 V rms at 282 MHz will dissipate only 3 W, assuming $Q = 5000$. The laser power required to excite every ion in such an array is also nominal, since recent experiments have saturated single ions with $\approx 10^{-7}$ W of resonant light. This array could be used to increase the signal-to-noise ratio of a single atom frequency standard [32] by a factor of $\sqrt{N} = 2000$.

To be useful for quantum computing, the trap array must have a method of sharing entanglement between separate traps. Quantum optical interconnects that have been proposed for parallel and distributed computing [23–26] could in principle serve this purpose. These methods typically assume that the atoms are at rest in an optical cavity that obeys the ''strong coupling'' condition of cavity quantum electrodynamics [27], which requires that the cavity mode volume V_m obey $V_m < V_r$, where $V_r = \sigma_0 c/\Gamma$ is the atomic radiative volume, $\sigma_0 = 3\lambda^2/2\pi$, Γ is the atom's spontaneous emission decay rate, and c is the speed of light. Since $V_r \approx (100\ \mu\text{m})^3$ for ions of interest, cavity lengths ≈ 1 mm. No such method has yet been tested. Elliptical aperture traps are consistent with quantum optical interconnects since their flat structure permits dielectric mirrors to be brought within several hundred micrometers of the trap center. The effect of such mirrors has been studied by including disks of quartz 1 mm diam and 100 μm thickness in the numerical solutions for the trap fields. A pair of disks spaced ± 300 μm from the center of a 100 μm by 300 μm radius elliptical trap produced only a 15% change in the trap fields while preserving their symmetry. The presence of these dielectrics will not affect the ion crystal structure because of the relatively large spacing and small size of the image charges.

The combination of quantum optical interconnects with ion trap arrays provides a specific physical model of a scalable quantum computer. The original Cirac-Zoller model, in which all the ions are placed in one trap, cannot accommodate the 10^6 ions needed for a full-scale quantum computation, even in principle. One limit is that the c.m. phonon frequency $\omega_z \propto 1/\sqrt{N}$, so that a trap containing 10^6 ions would have $\omega_z \leq 100$ Hz. Evaluating the 10^{11} operations required for Shor's algorithm would therefore take over 10^9 sec (30 years) without error correction. The trap array model proposed here contains M separate elliptical ion traps each containing N ions, with M or more quantum optical interconnects being used to transmit entanglement between the traps. This model remains highly speculative since such interconnects have not yet been realized in practice. However, the trap arrays themselves with $M = 64\,000$ and $N = 64$ are relatively low risk, as discussed above. Since each trap contains < 1000 ions, the phonon frequency remains large, in the above example, $\omega_z = 125$ KHz for $N = 64$. This is 10^3 faster than the single-trap model. Moreover, the array possesses both classical as well as quantum-mechanical parallelism, since there are M independent Cirac-Zoller processors operating simultaneously, which can be placed in entangled states when necessary by the interconnects. It is interesting to ask how much this M-fold parallelism would reduce the execution time and number of gate operations estimated in the original Cirac-Zoller model for the modular exponentiation [20] in Shor's algorithm.

In conclusion, the elliptical ion trap presents a solution to the problem of confining large numbers of ions in the Lamb-Dicke limit, as is required for precision spectroscopy, trapped ion frequency standards, quantum computation, or other studies of quantum-mechanical ion motion. It was previously thought that only linear traps, which use a dc axial field requiring a complex three-dimensional electrode structure, could achieve this. The linear crystal stability condition, Eq. (7), and the micromotion relation, Eq. (8), are the two main results that define the Lamb-Dicke confinement properties of elliptical traps. The former shows that traps of moderate ellipticity can confine a useful number of ions; for example, $\epsilon = 5$ traps 60 ions, sufficient for all current tests of quantum computing. The latter shows that micromotion grows very slowly with ion number N, approximately as the 1/5 root, so that 1000 ions have less than 10 times the micromotion of two ions. Proper choice of the Mathieu parameter q can therefore put all ions in the Lamb-Dicke regime. The simple one-conductor electrode structure permits elliptical traps to be smaller and stronger than linear traps (note the $\omega_z = 1$ MHz derived above), and to be microfabricated in large arrays.

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