Detecting the effects of linear acceleration on the optical response of matter

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We propose two feasible experimental studies of the interaction of light with linearly accelerating media. First, sidebands are induced in the output spectrum of a Mach-Zehnder interferometer when a dielectric in one arm is oscillated parallel to the beam. Second, linear acceleration of a dielectric in a ring cavity will induce asymmetry in the near-resonant optical response profile. Both experiments probe the interaction of photons with inertia by testing the covariant formulation of Maxwell's equations used here and so the related assumption that the local optical properties of a medium are unaffected by acceleration. The second experiment is explicitly sensitive to a Doppler shift induced when light propagates through linearly accelerating media, predicted by Tanaka [Phys. Rev. A **25**, 385 (1982)]. Observation of this Doppler shift would confirm a manifestation of the coupling between radiation and matter in a noninertial context. [S1050-2947(98)00807-5]

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I. RELATIVISTIC TESTS OF MAXWELL'S EQUATIONS

The propagation of light through moving media has long held fascination for both the experimentalist and theoretician. Its history dates from the proposals of Boscovich [1] to measure the angular aberration of light within a telescope filled with water. These (unperformed) experiments were predicted to distinguish between the emission theories (widely accepted at the time) and wave theories of light [2]. With this old controversy decided in favor of the wave theory by the work of Young and Fresnel, the latter again considered the propagation of light through moving media. From a primitive first-order form of the principle of relativity, that aberration ought not to be influenced by the presence of a medium between the source and detector, otherwise the absolute velocity of the earth would be observable [2], Fresnel [3] derived the velocity of light through moving media as $\boldsymbol{c}+\kappa\boldsymbol{v}$, where the light drag coefficient $\kappa=1-1/n^2$. Experimental verification was provided by Fizeau [4] by passing two beams of an optical interferometer through moving water.

When the medium through which light propagates is moving inertially in flat space-time, the original formulation of Maxwell's equations [5] will fully describe Fresnel drag. When this medium has a finite proper acceleration, however, the problem cannot be analyzed within Maxwell's original theory, as Maxwell's equations do not maintain their standard form under arbitrary coordinate transformations. For a consistent description of light propagating through accelerating media a covariant formulation of Maxwell's equations must be employed.

There have been several high precision light drag experiments in which dielectric samples were accelerating. Bilger and Zavodny [6] and Bilger and Stowell [7] placed a rotating fused-silica disk along one arm of a ring laser. Fresnel drag in the rotating dielectric induced a nonreciprocal change in the optical path of the laser beams which, in turn, produced a beat frequency between the opposite beams. In the later experiment, where the drag coefficient was determined to 0.04% accuracy, theory and experiment differed by two standard deviations, which was considered too small to claim a discrepancy. In the experiment of Sanders and Ezekiel [8] glass samples were oscillated along one arm of their optical cavity, a phase shift resulting in a shift in the resonance frequency of the cavity observed through optical feedback. Corrections to their theoretical analysis were given by Neutze *et al.* [9], showing this experiment to have similar accuracy to the earlier work of Bilger and Stowell. Neutze *et al.* [9] also give a review of recent drag experiments with moving matter. The observed beats between the opposite beams of a ring laser in the experiment of Kowalski *et al.* [10], while presented as a test of the propagation of light through accelerating matter, can in fact be satisfactorily interpreted as Laub drag (see, for example, Ref. [9]) at the instantaneous velocity of the sample. As an observation of Laub drag, this experiment was approximately one order of magnitude less sensitive than the similar experiment of Sanders and Ezekiel.

While all these experiments used dielectric samples moving with linear or centripetal acceleration, their design and sensitivity enabled satisfactory analyses to proceed through the assumption that the influence of acceleration on the observed phase shift was negligible. Thus the influence of light drag at the instantaneous velocity of the sample alone could yield a correct experimental fit for all recorded data. As such, these experiments *did not test Maxwell's equations beyond the realm of Maxwell's original formulation*, which fully describes the propagation of light through bodies moving inertially.

There have, however, been several observations of light propagating in a gravitational field which cannot be explained in terms of the original formulation of Maxwell's equations [11]. Dyson, Eddington, and Davidson [12] observed the bending of light as it passed near the sun in the solar eclipse of May 29, 1919. This experiment provided early verification of a prediction of general relativity and marks a milestone in its wider acceptance within the scientific community. Using radio waves emitted from a quasar as it was eclipsed by the sun, Fomalont and Sramek [13] were able to improve the accuracy of light bending experiments to $\sim 1\%$. Reasenberg *et al.* [14], by tracking a spacecraft emitting an electromagnetic signal, were able to confirm the predictions of Schwarzschild geometry to 0.2%. However, in all of these experiments light propagated through vacuum. While providing excellent verification of the covariant formulation of Maxwell's vacuum equations in Schwarzschild geometry, they did not empirically test the propagation of light through media in a gravitational context.

Similarly, Pound and Rebka's [15] experimental verification, using Mössbauer spectroscopy, of the frequency shift induced by the Earth's gravitational field, and the closely related experiment of Hay *et al.* [16], using a Mössbauer source and receiver mounted upon a rotating disk, did not test the influence of matter on the propagation of light through non-Minkowskian space-time. Had these experiments been performed with material media located between source and receiver all experimental results would have been unaffected, since gravitational redshift may be explained purely in terms of time dilation between separated clocks.

Experiments sensitive to the propagation of light through accelerating media have, however, been provided by the Sagnac effect [17]. Presenting a covariant analysis, Post [18] demonstrated that the observable fringe shift resulting from rotation of an optical interferometer is dependent on whether the optical medium is held stationary in the laboratory or rotates with the interferometer. This dependence, however, is merely a consequence of the Doppler effect due to the relative motion of source and medium, similar to the distinction between Lorentz and Laub light drag coefficients for media moving with constant velocity [9]. Despite the unrivaled precision of the state-of-the-art rotation-sensitive ring lasers [19], the Sagnac effect has only been verified to first order in the rotation rate, Ω, at least one order below that at which any influence of centripetal acceleration may be expected to arise.

Also within a rotational context, Mashhoon [20] has suggested that the intrinsic spin of any particle will couple to rotation. As a consequence, it becomes impossible to equate the measurement of frequency by an observer O, who moves with constant rotation, with the same measurement performed by an inertially moving observer instantaneously comoving with O. This discrepancy was considered to constitute an extension of the standard measurement hypothesis underpinning relativity theory (that accelerated clocks tick at the rate of a comoving unaccelerated clock) [21]. Experimental support for spin-rotation coupling has been reported as the apparent coupling of the nuclear spin of mercury with the rotation of the Earth [22,23].

In a number of related experiments [24] it has been observed that circularly polarized light passing through a rotating half-wave retardation plate (HWP) both has its circular polarization reversed *and becomes shifted in frequency by twice the rotation rate of the* HWP: downshifted when this rotates with the same sense as the optical electric field and upshifted when these are opposite. Pippard [25] has presented a consistent analysis of these observations. Their significance lies in the fact that the extension of Maxwell's equations to noninertial systems alone provides *a mechanism through which energy may be donated to the electromagnetic field due to matter's inertial resistance to acceleration*. Furthermore, observation of this ''angular Doppler effect'' experimentally confirms Mashhoon's spin-rotation coupling for photons, although this was not previously recognized as such. This connection may be seen by considering an optical Mach-Zehnder interferometer with two HWP's located along one arm, the entire interferometer being set in rotation about this arm. From spin-rotation coupling Mashhoon [26] predicted a phase shift would result if circularly polarized light were injected into this interferometer. Viewed from an inertial frame of reference this phase shift may also be derived from the Doppler shift in frequency suffered by circularly polarized light upon passing through each rotating HWP.

A linear analog of the angular Doppler effect arises from Tanaka's [27] prediction that linearly polarized light will suffer a change of frequency when propagating through linearly accelerating media, where both the acceleration and wave vector were assumed to be parallel. In this article we present two experimental proposals, falling within the scope of current technology, which will test the influence of linear acceleration on Fresnel drag. The latter of these proposals, in particular, is considered of fundamental interest, because it is explicitly sensitive to the consequence that a covariant extension of Maxwell's equations provides a mechanism through which *photons can exchange energy with linearly accelerating matter.*

II. COVARIANT FORMULATION OF MAXWELL'S EQUATIONS

A simple prescription for obtaining Maxwell's equations under an arbitrary coordinate transformation is to replace all partial derivatives (commas) in the special relativistic tensorial description with covariant derivatives (semicolons), and further let $\eta_{\mu\nu}\rightarrow g_{\mu\nu}$, the metric tensor of an arbitrary space-time. As presented in detail by Schleich and Scully [11], this prescription enables the covariant form of Maxwell's equations to be written

$$F_{\mu\nu}=A_{\mu,\nu}-A_{\nu,\mu},$$

$$F_{\mu\nu,\rho}+F_{\nu\rho,\mu}+F_{\rho\mu,\nu}=0, \tag{1}$$

$$\frac{1}{\sqrt{-g}}(\sqrt{-g}F^{\mu\nu})_{,\nu}=-\mu_0 J^{\mu}, \tag{2}$$

where $g\equiv\det g_{\mu\nu}$ and only the contravariant field equation has been affected by the change to arbitrary coordinates, the covariant derivatives of the covariant field equations simplifying to contain only partial derivatives.

When there are no free charges but a material medium is present, current densities described by J^{μ} arise only from electric and magnetic displacement currents. It therefore proves convenient to incorporate these displacement currents and the influence of the metric tensor into a simplified contravariant electromagnetic tensor density, $G^{\mu\nu}$ [28], and thereby rewrite Eq. (2) as

$$G^{\mu\nu}_{,\nu}=0. \tag{3}$$

With the components of $G^{\mu\nu}$ defined as

$$G^{\mu\nu}=\begin{pmatrix} 0 & cD_x & cD_y & cD_z \\ -cD_z & 0 & H_z & -H_y \\ -cD_y & -H_z & 0 & H_x \\ -cD_z & H_y & -H_x & 0 \end{pmatrix},$$

Eqs. (1) and (3) may be combined to yield Maxwell's covariant equations in familiar vector notation:

$$\nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}, \tag{4}$$

$$\nabla\cdot\boldsymbol{B}=0, \tag{5}$$

$$\nabla\times\boldsymbol{H}=\frac{\partial}{\partial t}\boldsymbol{D}, \tag{6}$$

$$\nabla\cdot\boldsymbol{D}=0. \tag{7}$$

While these equations have a form identical to Maxwell's equations in an inertial frame of reference, this similarity is misleading as the electric $\boldsymbol{E}$ and magnetic $\boldsymbol{B}$ field vectors are not simply proportional to the generalized electric displacement $\boldsymbol{D}$ and magnetic intensity $\boldsymbol{H}$. Instead, in obtaining Eq. (3) from Eq. (2) we require their constitutive relations to be dependent on both the dispersive properties of the medium *and the space-time metric* with a physically motivated prescription needing to be chosen to specify these relations.

In an inertial frame with stationary, linear, and homogeneous media clearly

$$\boldsymbol{D}=\epsilon\epsilon_0\boldsymbol{E}, \tag{8}$$

$$\boldsymbol{B}=\mu\mu_0\boldsymbol{H}, \tag{9}$$

where ϵ_0 and μ_0 are, respectively, the permittivity and permeability of the vacuum and ϵ and μ are the relative permittivity and permeability of the medium. In generalizing this result to include media with arbitrary acceleration, Heer [29] imposed the physical assumption that the above constitutive relations hold in the inertial frame of reference instantaneously comoving with the accelerating media. This assumption is tantamount to assuming that *the optical properties of accelerating media are locally unaffected by acceleration*. While one may expect this to be only approximate in conditions of extreme acceleration, we accept this prescription throughout this work. This assumption enables us to examine whether the natural covariant extrapolation of Maxwell's equations to linear accelerating systems generates new and experimentally testable effects, independent of the influence acceleration may have at an atomic level.

Adopting the notation of Tanaka [30], the constitutive relations are found through use of the transformation properties of $F_{\mu\nu}$ and $G^{\mu\nu}$. Under arbitrary coordinate transformations $x^\mu\rightarrow x^\mu(x^{\nu'})$ these transform according to

$$F_{\mu'\nu'}=\frac{\partial x^\alpha}{\partial x^{\mu'}}\frac{\partial x^\beta}{\partial x^{\nu'}}F_{\alpha\beta},$$

$$\left|\det\left(\frac{\partial x^\beta}{\partial x^{\nu'}}\right)\right|G^{\mu'\nu'}=\frac{\partial x^{\mu'}}{\partial x^\alpha}\frac{\partial x^{\nu'}}{\partial x^\beta}G^{\alpha\beta}.$$

Making a coordinate transformation from the inertial frame instantaneously comoving with an accelerating dielectric into the accelerating frame of the dielectric, Eqs. (8) and (9) may be written in the accelerating coordinates as

$$G_{0k}=\sqrt{\frac{-g\epsilon_0}{\mu_0}}\epsilon F_{0k}, \tag{10}$$

$$\mu G^{kl}=\sqrt{\frac{-g\epsilon_0}{\mu_0}}F^{kl}, \tag{11}$$

where spatial indices k and l run from 1 to 3.

Several authors have studied the propagation of electromagnetic waves in linearly accelerating media using the above formulation of Maxwell's equations. Anderson and Ryon [28,31] and Mo [32] both derived a wave equation for the propagation of light through a dispersionless dielectric with constant acceleration and obtained approximate solutions. Tanaka [30] found an analytic solution using Fermi coordinates, demonstrating that the relativistic energy and phase velocity addition laws are equivalent to the Einstein velocity addition laws for inertial media instantaneously comoving with the accelerating sample. Similarly, at normal incidence, light reflected from an accelerating dielectric surface suffers the same Doppler perturbations as that reflecting from an identical inertial dielectric boundary instantaneously comoving with this accelerating surface. In a later publication Tanaka [27] predicted that light, upon transmission through an accelerating dielectric, should be shifted in frequency by an amount proportional to the acceleration of the glass sample, its thickness, and the difference of its refractive index from unity. It is this conclusion, following directly from the covariant formulations of Maxwell's equations, that we propose can be tested within current technology.

III. TRANSMISSION OF LIGHT THROUGH A LINEARLY ACCELERATING SAMPLE

A linearly accelerating frame of reference S with spatial geometry identical to that of the instantaneously comoving inertial frame has the line element [33]

$$ds^2=-\left(1+\frac{ax}{c^2}\right)^2c^2dt^2+dx^2+dy^2+dz^2. \tag{12}$$

That this line element represents a frame of reference with linear acceleration follows from the coordinate transformation relating the (upper case) coordinates of the inertial laboratory frame I to the (lower case) coordinates of S:

$$T=\frac{c}{a}\left(1+\frac{ax}{c^2}\right)\sinh\left(\frac{at}{c}\right),$$

$$X=\frac{c^2}{a}\left(1+\frac{ax}{c^2}\right)\cosh\left(\frac{at}{c}\right)-\frac{c^2}{a},$$

$$Y=y,$$

$$Z=z,$$

with each point of S following hyperbolic motion in I.

We suppose that the proper length of a dielectric sample, with index of refraction n, is unaffected by acceleration. Taking the sample's parallel faces as lying on the planes $x=0$ and $x=h$ in S we assume, for simplicity of analysis, that electromagnetic waves incident upon the dielectric surface propagate parallel to the direction of acceleration, $\boldsymbol{k}\|\hat{\boldsymbol{x}}$. Thus $E_x=0$; we choose the electric field to be linearly polarized, and the $\hat{\boldsymbol{y}}$ axis to be parallel to the electric field. From the covariant constitutive relations, Eqs. (10) and (11), and the metric components of Eq. (12), the nonzero components of the vectors $\boldsymbol{D}$ and $\boldsymbol{H}$ are necessarily

$$D_y=\epsilon\epsilon_0 E_y\left(1+\frac{ax}{c^2}\right)^{-1},$$

$$H_z=\frac{1}{\mu\mu_0}B_z\left(1+\frac{ax}{c^2}\right).$$

Taking the curl of Eq. (4) and using Eqs. (4)–(7) and the expressions above to eliminate D_y, H_z, B_z, the wave equation [Eq. (14) of Tanaka [27]]

$$\left(1+\frac{ax}{c^2}\right)^2\frac{\partial^2 E_y}{\partial x^2}+\frac{a}{c^2}\left(1+\frac{ax}{c^2}\right)\frac{\partial E_y}{\partial x}-\frac{n^2}{c^2}\frac{\partial^2 E_y}{\partial t^2}=0 \tag{13}$$

is recovered, where $\epsilon_0\mu_0=1/c^2$ and $\epsilon\mu=n^2$.

In I a y-polarized electromagnetic wave in vacuum may most generally be described by the electric field $\mathcal{E}_Y^{\mathrm{vac}}=f(\Phi_{\mathrm{vac}})$ and magnetic field $\mathcal{B}_Z^{\mathrm{vac}}=\mathcal{E}_Y^{\mathrm{vac}}/c$, where $f(\Phi_{\mathrm{vac}})$ is an arbitrary complex wave function satisfying Maxwell's vacuum wave equation, and the vacuum phase $\Phi_{\mathrm{vac}}=-\omega_0(T-X/c)$. From the properties of $F_{\mu\nu}$ under a change of coordinates,

$$\mathcal{E}_y^{\mathrm{vac}}=\left(1+\frac{ax}{c^2}\right)\exp\left(-\frac{at}{c}\right)f(\Phi_{\mathrm{vac}}) \tag{14}$$

represents the general solution to the vacuum wave equation in S, Eq. (13) in the limit $n\to 1$, where the invariant phase is

$$\Phi_{\mathrm{vac}}=\frac{\omega_0 c}{a}\left\{\left(1+\frac{ax}{c^2}\right)\exp\left(-\frac{at}{c}\right)-1\right\}. \tag{15}$$

A general analytical solution of Eq. (13) within the dielectric may be obtained by noting n appears only as a factor scaling the temporal partial derivative of E_y. Therefore replacing t by t/n in Eqs. (14) and (15) gives

$$\mathcal{E}_y^{d}=\left(1+\frac{ax}{c^2}\right)\exp\left(-\frac{at}{nc}\right)f(\Phi_d), \tag{16}$$

with the complex phase becoming

$$\Phi_d=\frac{\omega_0 c}{a}\left\{\left(1+\frac{ax}{c^2}\right)\exp\left(-\frac{at}{nc}\right)-1\right\}. \tag{17}$$

Consider now a harmonic and monochromatic incident light source: $E_Y^{\mathrm{in}}=E_0\exp i\omega_0(T-X/c)$. We assume that the amplitude of light transmitted at the interfaces of the medium is reduced by a factor $r(<1)$; this may be maximized $(1-r\sim 100$ ppm) by, for example, the use of faces cut at the Brewster angle and polished to $\lambda/100$. [7] However, for simplicity of analysis in this paper we assume that the angles of incidence are $0°$ at the media boundaries; multilayer antireflection coatings can in principle achieve a comparable transmission.

The electric field within the dielectric is found by matching the general solution in S, Eq. (16), and its phase, Eq. (17), to the electric field incident upon the dielectric face at $x=0$ when expressed in the lower case coordinates of S. With the transmitted electric field amplitude reduced by a factor of r we find within the dielectric

$$E_y^{d}=E_0 r\left(1+\frac{ax}{c^2}\right)^n\exp\left(-\frac{at}{c}\right)\times\exp\left[i\left\{\frac{\omega_0 c}{a}\left(1+\frac{ax}{c^2}\right)^n\exp\left(-\frac{at}{c}\right)-\frac{\omega_0 c}{a}\right\}\right]. \tag{18}$$

In S this wave emerges from the dielectric at $x=h$. Again matching the general solution in vacuum, Eq. (14), and its phase, Eq. (15), to Eq. (18) at the second dielectric-vacuum interface yields

$$E_y^{\mathrm{out}}=E_0 r^2\left(1+\frac{ah}{c^2}\right)^{n-1}\left(1+\frac{ax}{c^2}\right)\exp\left(-\frac{at}{c}\right)\times\exp\left[i\left\{\frac{\omega_0 c}{a}\left(1+\frac{ah}{c^2}\right)^{n-1}\left(1+\frac{ax}{c^2}\right)\times\exp\left(-\frac{at}{c}\right)-\frac{\omega_0 c}{a}\right\}\right].$$

Upon a coordinate transformation from S back to the laboratory frame I,

$$E_Y^{\mathrm{out}}=E_0 r^2\left(1+\frac{ah}{c^2}\right)^{n-1}\exp[i\{-\omega'(T-X/c)+\Delta\phi\}], \tag{19}$$

where the emerging beam is shifted in frequency:

$$\omega' = \omega_0 \left(1 + \frac{ah}{c^2} \right)^{n-1} \approx \omega_0 \left(1 + (n-1) \frac{ah}{c^2} \right), \tag{20}$$

and the phase is perturbed by

$$\Delta\phi = \frac{c}{a}(\omega' - \omega_0). \tag{21}$$

Equation (20), derived by Tanaka [27], shows that even with the assumption that the local optical properties of matter are unaffected by acceleration, light propagating through a linearly accelerating dielectric will suffer a shift in frequency dependent on the acceleration and refractive index of the sample but independent of its velocity. Therefore, the minimal covariant extension of Maxwell's equations to linearly accelerating systems predicts *light will couple to inertia through mass-energy exchange with linearly accelerating matter: photons propagating parallel to the direction of acceleration are raised in frequency upon their transit.*

Intuitive understanding of this result follows as, quantum mechanically, the refractive index of any material differs from that of the vacuum due to photons being absorbed into virtual atomic states and later reemitted (e.g., [34]). If the material is accelerated during the lifetime of a virtual state, energy and momentum conservation results in a shift in energy of the emitted photon relative to that absorbed, arising from the changes to the velocity of an atom between these two events. Algebraically this argument yields Eq. (20) (at first order in ah/c^2). It is interesting to note that, from $E = mc^2$, this transient population of higher energy virtual atomic states increases the mass of the dielectric relative to when no light propagates through it. As such its resistance to acceleration (inertial mass) is increased and energy may thereby be imparted to light while in transit.

IV. A PASSIVE INTERFEROMETER CONTAINING AN OSCILLATING GLASS SAMPLE

Several Laub-drag-type experiments have been performed to date using samples with varying degrees of acceleration. In the work of Zeeman *et al.* [35] and Sanders and Ezekiel [8] dielectric rods were oscillated in a direction parallel to the incident beam, but all data were recorded when the sample had maximum velocity and therefore vanishing acceleration. Kowalski *et al.* [10], who accelerated a dielectric sample at ~ 5 cm/s^2 along one arm of a ring laser, were able to obtain good agreement with experiment by disregarding the influence of acceleration and simply calculating the instantaneous Laub drag coefficient. Bilger and Stowell [7], in rotating a fused-silica disk, studied the light drag under conditions of high acceleration (albeit using an experimental configuration different from that discussed below). With rotation rates up to ~ 60 Hz and the counterpropagating beams of their ring laser passing through the disk up to ~ 1.6 cm off the rotation axis, the local centripetal acceleration of the atoms interacting with light ranged up to ~ 2000 m/s^2. Their analysis, based purely on the local instantaneous velocity of their sample, proved sufficient to obtain close agreement with their observations. As such their work validated the assumption that the optical properties of media are locally unaffected by acceleration, made in recovering Eqs. (10) and (11) from Eqs. (8) and (9) and consequently the wave equation (13), over a wide range of acceleration.

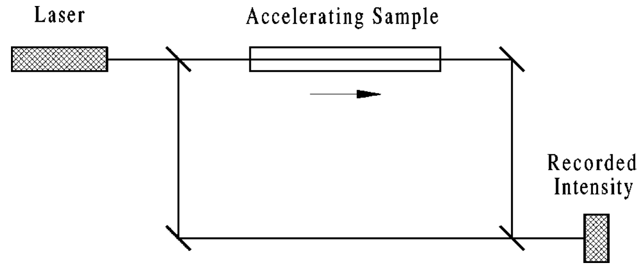

FIG. 1. A dielectric sample is linearly accelerated along one arm of an optical Mach-Zehnder interferometer and the output intensity recorded as a function of time.

In imparting velocity to their optical samples relative to their interferometer-ring laser, the novel effect observed within all these experiments was that the phase velocity of light was increased (or decreased) due to this motion: following almost trivially from optics in the rest frame of the sample and special relativity. In making the step to linearly accelerated motion the experimental feature we aim to observe is that *the energy of a photon will be increased (or decreased) upon interaction with linearly accelerating media*, Eq. (20): this energy coupling following simply from the natural extension of Maxwell's equations to the linearly accelerating frame of the sample.

A gain in frequency of one beam relative to another is most easily resolved by recombining the two beams and observing a beat frequency. Consider the passive Mach-Zehnder optical interferometer illustrated in Fig. 1. An injected beam is split, one part traversing a linearly accelerating dielectric before its recombination with the other prior to detection. Expansion of the phase of Eq. (19) to first order in ah/c^2 gives the measurable (time-dependent) phase perturbation as

$$\Delta\Phi(T) = \frac{\omega_0 h}{c}(n-1)\left\{ 1 - \frac{aT}{c} + \frac{a}{c^2}\left(B + \frac{1}{2}(n-2)h \right) \right\}, \tag{22}$$

where B represents the proper distance along the optical path from the origin (location of the first glass face when $T=0$) to the point of beam recombination. Physically, the first term arises from the presence of a stationary dielectric and the second term is exactly the Laub drag phase shift [9], where aT is equated with the instantaneous velocity of the dielectric sample, accelerated from rest when $T=0$. The final term, an additional perturbation to the phase stemming from the Doppler shift Eq. (20), is insignificant in comparison with the Laub drag term and has not been observed to date. Kowalski has suggested that a number of related phase shifts may be experimentally observable using light [36] or neutron [37,38] interferometry, the latter two articles predicting violations of the principle of equivalence. Comparing Eq. (20) with Eq. (16) of Ref. [40] we note Kowalski's transit time analysis with the approximation Eq. (11) of [37] led him to falsely conclude that this frequency shift was dependent on the sample's velocity (the opposite sign being a typographical error). Had his formalism been valid it would hold equally well for photons as for neutrons: however, our covariant analysis shows that the principle of equivalence holds for such effects when photons are involved.

Should it be possible to maintain constant linear acceleration of the sample, one would expect, from the time dependence of the above phase shift, to observe the intensity varying sinusoidally with its frequency given by the perturbation to Eq. (20). Experimental constraints, however, would force one to oscillate a glass rod with angular frequency Ω along one arm using a stabilized laser, say the Newport NL-1, as the input beam. Sinusoidal acceleration would modulate the frequency of the beam traversing the sample through Eq. (20). This experimental configuration is extremely similar to that of Zeeman *et al.* [35] and resembles Sanders and Ezekiel's [8] arrangement, but rather than window the detector response or convert this phase shift to a beating using an optical feedback mechanism, we propose to record the photon intensity transmitted through our beam recombiner as a function of time. This transmission will vary due to the time dependence of the relative phase of opposite beams. Introducing a constant phase difference ϕ by, for example, mounting one mirror on a piezoelectric crystal, the recorded signal can be expected as

$$V_B(T)=V_0|\exp-i\omega_0T+\alpha\exp-i\{\omega_0T+\beta\cos\Omega T+\phi\}|^2$$
$$=V_0\{1+\alpha^2+2\alpha\cos(\beta\cos\Omega T+\phi)\},$$

where

$$\beta=\frac{\Delta\omega}{\Omega}=\frac{\Omega A_0}{c}\frac{\omega_0 h}{c}(n-1)$$

is the amplitude (in radians) of the phase excursion induced by the frequency modulation, A_0 is the amplitude of sample oscillation, and α is the relative amplitude of opposite beams when recombined.

Fourier analysis of the time-dependent output, as is now standard in ring-laser spectroscopy [39], provides a simple method of signal analysis. Sideband satellites on either side of zero frequency (in the absence of a carrier) with separation Ω will appear in the Fourier transformed frequency spectrum. A resolvable signal requires only that the short term longitudinal light-phase coherence is maintained over the difference in time of transit between the opposite beams, $\sim(n-1)h/c\sim2$ ns, which would easily be achieved using a Newport NL-1 laser with its long term frequency drift stabilized to within ±300 kHz. When combining two independent lasing modes of a ring laser, a noise floor some 65 dB below the Sagnac signal has been recorded with the Canterbury ring laser [19]. As such it is reasonable to anticipate that this noise floor can, at least, be matched in the proposed experiment.

In measuring the Laub drag coefficient with a passive interferometer, Zeeman *et al.* [35] oscillated samples of quartz and flint glass ($n\sim1.5$) of lengths up to 1.4 m imparting a maximal velocity ~10 m/s. With these experimental parameters and $\lambda=633$ nm, the maximal phase excursion β ranges up to ~0.25. Figure 2 gives a numerical simulation of the frequency spectrum thereby predicted with the sample oscillating at 9.8 Hz and $\phi=\pi/4$. Sideband structure is clearly visible. By simultaneously increasing the frequency and decreasing the amplitude of oscillation, one may hold β (and therefore the sample's velocity domain) constant yet scan a range of maximal sample accelerations. Should the local optical properties of the dielectric medium be affected by acceleration, the time-dependent frequency modulation would become anharmonic, with variations in the heights of each sideband relative to the carrier thereby arising upon changing the sample's frequency of oscillation. We note that this approach naturally calibrates for the influence of dispersion within the sample, which was not included in the above analysis.

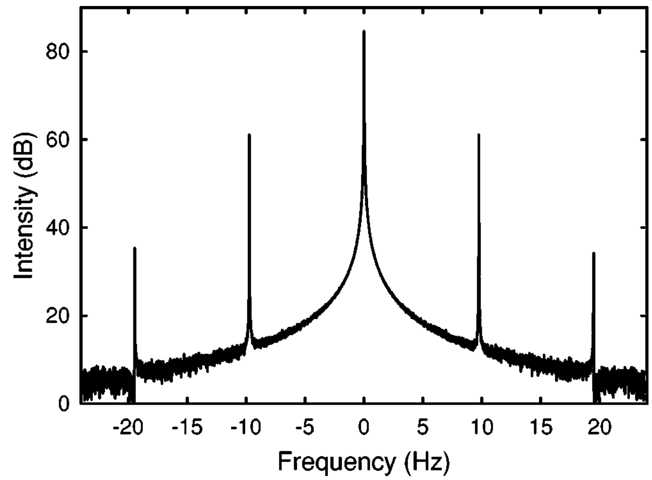

FIG. 2. Numerical simulation of the Fourier-transformed interferogram resulting from oscillating a sample along one arm of the interferometer of Fig. 1. The sample refractive index $n=1.5$, length $h=1.4$ m, and its maximum velocity is 10 m s^{-1} (parameters realized in the experiments of Zeeman *et al.*). Sideband satellites separated by the oscillation frequency ($\Omega/2\pi=9.8$ Hz in this example) are clearly visible.

Care should be taken to avoid optical feedback from light scattered from the moving dielectric surface, modulated in frequency via the Doppler effect. Tilting the sample relative to the optical axis should suffice [8]. Mechanical feedback when oscillating the sample could also cause spurious structure in the Fourier resolved spectra, although this was not a limiting factor in the work of Zeeman *et al.*

It is tempting to regard a successful observation of sideband structure in the above experiment as verification of the acceleration-induced Doppler shift, Eq. (20). However, an analysis based upon Laub drag within the sample assumed, at each instant, to be moving inertially would also recover sideband structure in the Fourier-transformed interferogram, arising from the velocity dependence of the Laub drag phase shift. Therefore, as with the experiments of Bilger and Stowell [7] and Kowalski *et al.* [10], the above apparatus would test the assumption that the optical properties of matter are unaffected by acceleration but could not verify any novel consequences of Maxwell's equations for linearly accelerated systems beyond this assumption.

V. RESONANT CAVITY CONTAINING A LINEARLY ACCELERATING GLASS SAMPLE

Optical ringing may occur whenever the optical perimeter of a cavity, or the frequency of the injected light, are functions of time. The phenomenon itself results from beating

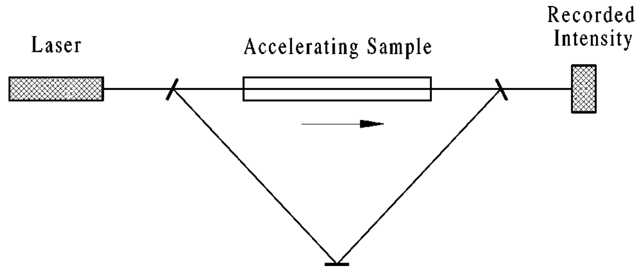

FIG. 3. Along one arm of a triangular resonance cavity a dielectric sample is linearly accelerated. The intensity of light transmitted through the second mirror is recorded as a function of time. This light consists of an infinite sum of contributions from waves having completed l circuits of the optical cavity; see Eq. (24).

between the energy stored within a cavity and that being injected and it may be used as a very sensitive technique to detect perturbations to the frequency of light. Ioannidis *et al.* [40] observed ringing in an optical fiber ring resonator, a variable optical path length being obtained through wrapping a length of optical fiber around a piezoelectric cylinder. An analogous observation was made by Li *et al.* [41] who, upon exciting a high finesse ring cavity with a frequency-modulated external laser, determined the finesse of this cavity through the observed ringing profile. An asymmetric response profile was also observed by Li and Stedman [42] when the scanning rate of a scanning Fabry-Pérot interferometer multiplied by the decay time of the interferometer squared was the order of unity. Further studies of optical ringing within a Fabry-Pérot cavity have recently been presented by Poirson *et al.* [43], and Neutze [44] has predicted that ringing again will arise when an optical ring interferometer with angular acceleration is excited by an external monochromatic signal.

When a glass sample accelerates within an optical cavity, its optical perimeter varies and energy is donated to the electromagnetic field via Eq. (20). Our analysis below of the experimental arrangement illustrated in Fig. 3, where an injected beam is repeatedly reflected through a dielectric sample accelerating along one arm of a high quality optical ring cavity, also predicts an asymmetry in the recorded response profile. Using a stabilized laser (again, for example, the Newport NL-1) we inject monochromatic light into the cavity through the first mirror, M_1. Behind the second mirror, M_2, we place a detector at $X=B$ which records, as a function of time, the light intensity transmitted through M_2. A third mirror, M_3, serves only to close the optical cavity, having an optical perimeter P in the absence of the glass sample. As this cavity is planar its resonant modes will be linearly polarized [45], the direction of polarization again fixing $\hat{\boldsymbol{y}}$.

At the detector the electric field consists of an infinite sum of partial contributions, $E_l(T,B)$, each having traversed the ring perimeter l times and therefore having passed through the accelerating glass sample $l+1$ times. Upon each transit of the sample the electric field suffers a drop in amplitude of r^2; and each transit of the ring perimeter further decreases the amplitude by s^3, arising from reflection from three mirrors. Finally, transmission losses through M_1 and M_2 reduce the amplitude by $(1-s^2)$. Thus, at B,

$$E_l(T,B)=E_0(1-s^2)r^2(s^3r^2)^l\exp(i\{\omega_0[-T+B/c] + \Delta\Phi(T)+\Phi_l\}),$$

where $\Delta\Phi(T)$ is defined by Eq. (22) and Φ_l is the phase contribution due to the electric field component $E_l(T,B)$ having traversed the ring perimeter a further l times before reaching the detector. When j passes through the accelerating sample light has frequency $\omega_j=\omega_0[1+jah(n-1)/c^2]$ due to the cumulative effect of the Doppler shift, Eq. (20). If we take a snapshot at time T we find light of frequency ω_j produces a phase change $\phi_j=\omega_j P'/c+[\omega_0(n-1)ah/c^2][-T+(M_2-h+nh/2)/c]$ upon completing one full circuit of the optical cavity and returning to M_2, where $P'\equiv P+(n-1)h$ is the optical perimeter of the cavity with the sample enclosed. Summing over l transits of the optical perimeter, and including the influence of this light having frequency ω_l in propagating from M_2 to B before reaching the detector, we find

$$\begin{aligned}\Phi_l&=\sum_{j=1}^{l}\phi_j+(\omega_l-\omega_0)(B-M_2)/c\\&=\frac{\omega_0}{c}\left\{l\left(P'+\frac{ah}{c^2}(n-1)(-cT+B')\right)\right.\\&\qquad\left.+l^2\frac{ah}{2c^2}(n-1)P'\right\},\end{aligned}\tag{23}$$

where $B'\equiv B+P/2-3h/2$. The final term, quadratic in l, arises directly from the cumulative influence on the phase of the Doppler shift, Eq. (20), for light having made l complete transits of the ring perimeter. Mathematically this quadratic term is central to our prediction that optical ringing will occur. The electric field reaching the detector is therefore

$$\begin{aligned}E_{\mathrm{out}}(T,B)&=(1-s^2)r^2E_0\ \exp(i\{\omega_0[-T+B/c]\\&\quad+\Delta\Phi(T)\})\sum_{l=0}^{\infty}(s^3r^2)^l\exp\{i\Phi_l\}.\end{aligned}\tag{24}$$

The summation within Eq. (24) is of the same form as that which arises in the analysis of the optical response of a scanning Fabry-Pérot interferometer [42], an analytic expression for the output intensity being obtained through its algebraic equivalence with the intensity profile of a Fabry-Pérot interferometer with a frequency-modulated input signal. If optical resonance is assumed to occur when the dielectric is stationary, then, by analogy with the expressions of Appendix C of Li and Stedman [42], the output intensity of this resonant cavity containing a linearly accelerating dielectric slab is

$$\frac{I_{\mathrm{out}}}{I_0}=\frac{|A_{\mathrm{out}}|^2}{|A_0|^2}=\left(\frac{1-s^2}{1-s^3r^2}\right)^2\frac{\pi}{8\eta}|w(\zeta)|^2,\tag{25}$$

where [46]

$$w(\zeta)=\frac{i}{\pi}\int_{-\infty}^{\infty}d\nu\frac{\exp(-\nu^2)}{\zeta-\nu}=\exp(-\zeta^2)\mathrm{erfc}\times(-i\zeta)(\mathrm{Im}\zeta>0),$$

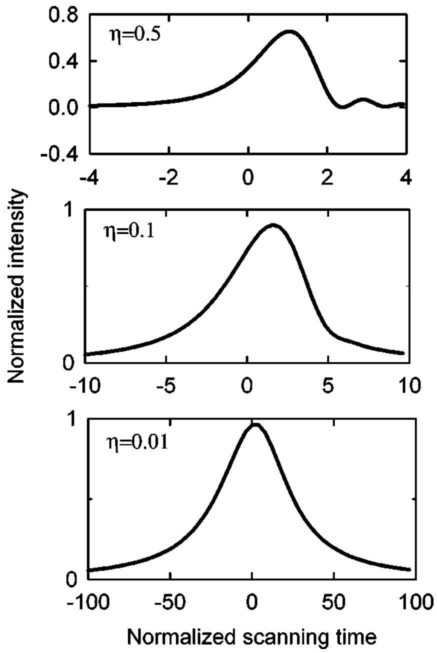

FIG. 4. Predicted intensity profiles for the resonant cavity containing a linearly accelerating dielectric sample illustrated in Fig. 3, calculated by numerical evaluation of Eq. (25) at various values of the asymmetry parameter η: (a) $\eta=0.5$, (b) $\eta=0.1$, and (c) $\eta=0.01$. The normalized time $\tau\equiv\Gamma T$. With $\eta\geqslant 0.1$, asymmetry in the optical output is clearly visible.

$\mathrm{erfc}(-i\zeta)$ being the complex error function. The asymmetry parameter $\eta\equiv\gamma/\Gamma^2$, where $\gamma\equiv-\omega_0(n-1)ah/cP'$ and $|\gamma|$ is the change in frequency from each traversal of the ring perimeter divided by the duration of each traversal and $\Gamma=2c(1-s^3r^2)/(P's^3r^2)$ is the power rate of decay of the cavity. The variable $\zeta\equiv\pm\exp(-i\pi/4)\{-i\Gamma+2|\gamma|(T-B'/c)\}/\sqrt{8|\gamma|}$, the sign being positive when $|\gamma|(T-B'/c)>\Gamma$, and was chosen so as to keep the imaginary part of ζ positive. The factor $(1-s^2)^2/(1-s^3r^2)^2\approx[2(1-s)/\{3(1-s)+2(1-r)\}]^2$ in Eq. (25) expresses the fact that this cavity has transmission losses from three mirrors and two surfaces of the dielectric, whereas a Fabry-Pérot interferometer [44] suffers losses from only two mirrors.

Plots of I_{out}/I_0 versus the normalized time $\tau=\Gamma T$ for $\eta=0.01$, 0.10, and 0.50 are given in Fig. 4. With $\eta\sim0.10$ the asymmetry becomes easy to resolve experimentally.

A triangular ring cavity constructed from (commercially available) mirrors with $1-s\sim10$ ppm and each arm 2 m in length will have $\Gamma\sim2.4$ kHz. Placing a dielectric sample with $1-r\sim100$ ppm (transmission maximized using multilayer antireflection coatings or cutting and polishing the faces at the Brewster angle) within this cavity, however, causes the ring-down $\tau_c=1/\Gamma$ to be dramatically reduced, with Γ becoming ~20 kHz. As in the preceding section, we note that Zeeman *et al.* mechanically accelerated 1.4 m long glass rods ($n\sim1.5$) to velocities of ~10 m/s. Using these experimental parameters and assuming these samples may be oscillated at ~10 Hz, an acceleration-induced frequency shift of ~2 Hz is predicted from Eq. (20) when the sample is at its turning points. This gives $|\gamma|=c\Delta\omega/P'\sim10^8$ and hence $|\eta|=|\gamma|/\Gamma^2\sim0.2$ with the resulting asymmetry in the measured optical output, as illustrated in Fig. 4, clearly observable.

With the required acceleration approximately proportional to the minimum of $(1-s)^3$ or $(1-r)^2$, improvements in transmission losses at the dielectric boundary (should custom-antireflection coatings approach 1 ppm loss mirrors now commercially available) would considerably simplify this experiment, although losses within the sample itself would then become significant. A successful experiment further requires phase correlations are maintained on a time scale $\sim1/\Gamma$. Frequency drift of the input laser of the order of kilohertz could therefore induce experimentally significant phase fluctuations, but could be calibrated by simultaneously injecting the laser beam into a similar cavity, again using optical resonance to monitor such drift [41]. Similarly, it would be relatively straightforward to calibrate experimentally such velocity-dependent features as optical dispersion within the sample, Doppler-induced variations in the transmission through the multilayer antireflection coatings, or misalignment of the cavity. While these may also contribute asymmetry in the observed resonance profile, all such features would appear in the low acceleration limit and could be accounted for by careful measurement of the resonance profile across a range of accelerations.

Unlike the experiment proposed in the preceding section, *an analysis based upon Laub drag at the instantaneous velocity of the sample would not predict any asymmetry in the optical response of this cavity*. This is because such an analysis provides no mechanism for the Doppler shift, Eq. (20), which was central in deriving the term quadratic in l of Eq. (23) from which the predicted asymmetry arose. Physically, optical ringing represents the beating of energy stored within a cavity against the input signal as the entire apparatus passes through resonance. It is the sample's linear acceleration which provides the energy to shift the frequency of light quanta upon each transit through the sample, thereby inducing this beating. An analysis based upon the fully covariant formulation of Maxwell's equations underpins this potentially observable phenomenon. In this manner the proposed experiment takes one step beyond the experiments of Zeeman *et al.*, Bilger *et al.*, and others, being explicitly sensitive to *the energy exchange induced by linear acceleration of media* and thereby testing a prediction of the covariant form of Maxwell's equations.

VI. CONCLUSION

It is intriguing that the energy of a photon propagating parallel to the direction in which a transparent sample linearly accelerates will be increased due to this acceleration. Experimental verification of this prediction, as with the observation of energy coupling between circularly polarized light and a rotating half-wave plate [24] would both extend empirical support for Maxwell's theory to include linearly accelerated motion and challenge the search for similar phenomena within a gravitational context. As the mathematical formulation of a covariant theory of electrodynamics is closely related to the foundations of general relativity, empirical verification of any assumptions within this framework is of fundamental interest.

Had the experimental apparatus of Fig. 4 been aligned vertically in the earth's gravitational field and the dielectric allowed to freely fall, optical ringing would again have been predicted [47]. However, our description of the propagation of light through the sample is quite different in this context: the worldlines of the atoms within the dielectric themselves define locally inertial frames of reference and it is the interferometer which suffers an apparent acceleration. As such photons, although retarded in velocity, still follow space-time geodesics through the dielectric. When an optical sample accelerates relative to an inertial frame of reference, by contrast, a photon traversing this sample actually appears to have proper acceleration and its worldline can no longer be described as a geodesic of the space-time.

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