Macroscopic field superpositions from collective interactions

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We study the generation of both coherent superpositions and mixtures of field states through the interaction of an initially coherent field state with a collection of two-level atoms. The proposed scheme to build up a given superposition is based on effective dispersive interactions which arise from both the large detuning limit and the strong-field limit. The coherent superposition is obtained by interacting with a classical field after the cluster leaves the cavity followed by a measurement of the atomic population. [S1050-2947(98)01907-6]

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I. INTRODUCTION

The study of nonclassical states of the electromagnetic field has been a central problem in quantum optics for many years. Recently, some proposals to generate a variety of nonclassical states have been developed, opening a field that we know as quantum state engineering. Special classes of nonclassical states have been studied considering superpositions of coherent states (CS) on the real line and on the circle [1]. A method for a quantum engineering scheme has been proposed using an atomic beam of two-level atoms interacting with a cavity field [2]. A special interest represents generation of macroscopic superpositions of quantum states, the so-called Schrödinger cat states [3]. These states have been generated using laser-cooled trapped ions [4]. Alternative schemes for generating Schrödinger cats in microwave cavities have recently been reported both theoretically [5] and experimentally [6] for monitoring the quantum decoherence of a Schrödinger cat state.

In this article we focus our interest on the generation of superpositions of CS by considering the simultaneous interaction of a collection of two-level atoms with a quantum field. In this context we have found a possible scheme for quantum field state engineering. We basically consider two regimes where this goal can be achieved. The first case corresponds to the dispersive interaction of two-level atoms with a quantum field initially in a coherent state, i.e., in the large detuning limit. A second case corresponds to the resonant interaction of a collection of two-level atoms interacting with a strong quantum field.

This paper has been organized as follows: In Sec. II we discuss generation of the binomial mixtures of CS in the process of one atom interaction with a quantized field in a dispersive cavity and describe how to engineer a diffusion-like process into the cavity field. The following sections are devoted to different proposals for generation of coherent superpositions of field states. In Sec. III we consider a generation scheme in the dispersive regime and in Sec. IV we consider the strong field limit. A summary is drawn in Sec. V.

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II. BINOMIAL MIXTURES OF COHERENT STATES

As a starting point it is instructive to analyze the evolution of a quantum field in a cavity interacting dispersively with a beam of two-level atoms. We consider the situation when at any moment only one atom can be found inside the resonator. The mixture of field states during evolution arises due to the fact that we are not considering measurements on atoms after they leave the cavity. As is well known, the far-off-resonant limit of the evolution operator for the Jaynes-Cummings model can be approximated as follows:

$$U(\phi)=e^{-i\phi a a^{\dagger}}|1\rangle\langle 1|+e^{i\phi a^{\dagger}a}|0\rangle\langle 0|, \tag{2.1}$$

where $\phi=g^2\tau_{\rm int}/\Delta$ is an adimensional time; g is the atom-field coupling constant; $\tau_{\rm int}$ is the interaction time and Δ is the difference of frequency between the relevant atomic transition and the field mode. Finally, a and $a^{\dagger}$ are the usual field operators and the states $|1\rangle$ and $|0\rangle$ denote respectively the upper and lower atomic levels of the atomic transition. This approximation holds when the condition $(2g/\Delta)^2\langle n\rangle \ll 1$ is satisfied. The effect of the interaction is a dynamical phase shift in the field state and the atomic population remains unchanged. In Ref. [6] values of $\Delta/2\pi=70-800$ kHz, $g/2\pi=24$ kHz, and $\langle n\rangle=10$ are reported, so the previous condition is well satisfied for $\Delta/2\pi>300$ kHz, which is achieved in the experiment by adjusting the cavity mirror separation.

Let us consider a lossless cavity field initially prepared in a coherent state: $\rho=|\alpha\rangle\langle\alpha|$ (for simplicity, α is chosen to be real). The atoms in the beam are initially prepared in the state $|\psi\rangle=d_1|1\rangle+d_0|0\rangle$. After the interaction with the first atom, the initial field state evolves into a superposition of two CS. The reduced field density matrix after the interaction (without measurement on atomic variables) takes the form

$$\rho_f(\phi)=d_0|\alpha e^{i\phi}\rangle\langle\alpha e^{i\phi}|+d_1|\alpha e^{-i\phi}\rangle\langle\alpha e^{-i\phi}|. \tag{2.2}$$

If we look at the phase space, the corresponding quasiprobability $Q(\beta)=\langle\beta|\rho_f(\phi)|\beta\rangle$ consists of two peaks centered on the circle of radius α and separated by the angle 2ϕ. The distance between peaks depends on the value of ϕ and is of the order $l\sim 2\phi\alpha$. If this distance is larger than the spread of the initial CS the peaks are well distinguishable, i.e., $l\sim 2$.

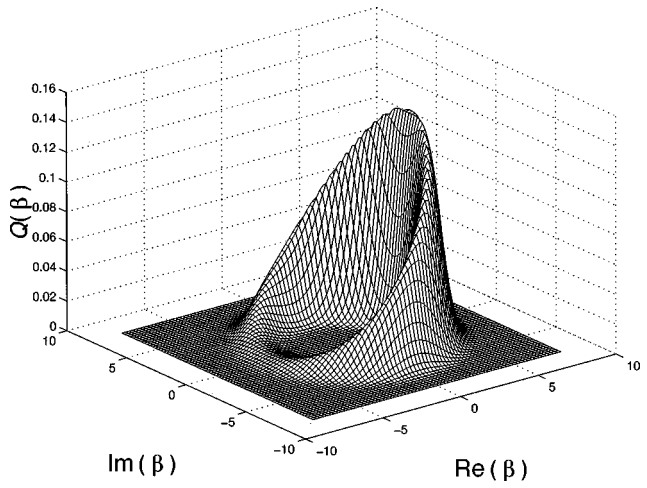

FIG. 1. Field Q function after $N=300$ atoms have passed through the cavity, for an initial coherent state with $|\alpha|^2=20$, $\phi=\pi/50$.

The passage of the next atom generates the splitting of each peak, giving rise to a new peak. The case of two atoms has been used to study the effect of progressive decoherence of components of field Schrödinger cats [5,6]. If $\phi=\pi/m$ and m is an integer number, only a finite number of peaks are generated. m peaks will be created after the passage of $m-1$ atoms and then the number of peaks is conserved. Of course not all the peaks will have the same size. Due to the specific process of splitting, the peaks tend to accumulate in the vicinity of the original peak, coherent state. This leads to the peculiar (diffuse) form of the Q function (as well as the phase distribution function) after the passage of many atoms.

Formally, we can write a general expression for the field density matrix after the passage of N atoms:

$$\rho_f^{(N)}=\mathrm{Tr}_{\mathrm{atom}}[U(\phi)\rho_f^{(N-1)}\otimes\rho_{\mathrm{atom}}U^{\dagger}(\phi)]. \tag{2.3}$$

After a brief calculation the field density matrix can be represented as follows:

$$\rho_f^{(N)}=\sum_{k=0}^{N} C_k|\alpha_k\rangle\langle\alpha_k|, \tag{2.4}$$

where $\alpha_k=\alpha\exp[2i\phi(k-N/2)]$. The probability distribution C_k is binomial in terms of initial atomic parameters and has the following form:

$$C_k=d_0^{N-k}d_1^k\binom{N}{k}. \tag{2.5}$$

A simple consequence of this statistical mixture is that the photon number fluctuations remain the same as in the initial coherent state. In the case of equal initial atomic population Eq. (2.4) corresponds to a binomial mixture of CS. In Fig. 1 we can observe the binomial splitting of the field for $N=300$ for small angle ϕ. This corresponds to a completely diffused binomial state, where the angle ϕ is too small to observe the individual peaks related to each field component in Eq. (2.4).

If parameter ϕ/π is not an integer (rational) number, the peaks will densely fill the circle after the passage of approximately $N\sim\pi\alpha$ atoms. In both cases, if ϕ/π is not too large, a discrete splitting at the beginning of the process (after interaction with a few atoms) turns into a diffusionlike process after many atoms have passed. For small values of the phase $\phi\sim 1/\alpha$, the peak splitting is *almost* indistinguishable. In this case the atom acts on the field as a kind of diffuser since the beginning of the process. It is well known that a diffusion process takes place only when an irreversible process is present. For example, in the case of the laser spontaneous emission plays a fundamental role in diffusion of the phase. In this case we do not have spontaneous emission, but we lose the information about the atomic state after the atoms have crossed the cavity.

The above described effective diffusion can be better explained in the limit of a small phase ϕ. The passage of successive atoms with small ϕ introduces a slow phase spreading of the field rather than a real splitting. In order to understand this point let us consider a large number of atoms passed through the cavity, so that the binomial distribution (the case of equal initial atomic populations) can be approximated by a Gaussian distribution

$$C_k\simeq\frac{1}{\sqrt{\pi N/2}}\exp\left(-\frac{(k-N/2)^2}{N/2}\right). \tag{2.6}$$

Introducing this distribution in the expression for the density matrix and carrying out the averaging we get a reduced expression for the field density matrix given by

$$\rho_f=\sum_{l,j=0}^{\infty} e^{-\alpha^2}\frac{\alpha^{l+j}}{\sqrt{l!j!}}e^{i\xi_0(l-j)-(N/2)\phi^2(l-j)^2}|l\rangle\langle j|, \tag{2.7}$$

where the number of atoms N represents an adimensional time scale of field evolution. We can rewrite this scale by using $N=rT$, where r is the average atomic injection rate and T is the total time since the first up to Nth atom in the atomic beam. The structure of this expression for the density matrix is closely related to that for the evolution of the laser field after the stationary regime of photon number has been reached. In the laser case the evolution of the nondiagonal matrix elements satisfies [7]

$$\frac{d}{dt}\rho_{l,j}=-\frac{D_l}{2}(l-j)^2\rho_{l,j}, \tag{2.8}$$

where D_l is the well-known Schalow-Townes laser diffusion coefficient, which is expressed as

$$D_l=\frac{A+\gamma}{4\bar{n}}. \tag{2.9}$$

Considering the initial condition of the laser as a coherent state with an average photon number $\bar{n}$, the later evolution can be written as

$$\rho_{\mathrm{laser}}=\sum_{l,j=0}^{\infty} e^{-\bar{n}}\frac{\bar{n}^{(l+j)/2}}{\sqrt{l!j!}}e^{i\theta_0(l-j)-(DT/2)(l-j)^2}|l\rangle\langle j|. \tag{2.10}$$

By a direct comparison of this expression with the present scheme, Eq. (2.7), we recognize the existence of a phase diffusion coefficient for the field inside the cavity as

$$D = r\phi^2. \tag{2.11}$$

As we have noted, for small values of ϕ each successive atom introduces a weak phase spreading of the field rather than a real splitting. We studied, for example, the evolution of the phase distribution function $P(\theta) = \langle \theta | \rho_f | \theta \rangle$, where $|\theta\rangle$ is a phase state of the field. Then, we get from Eq. (2.4) the following expression for the phase distribution function after pass of N atoms:

$$P^{(N)}(\theta) = \sum_{k=0}^{N} C_k P_{\mathrm{coh}}(\theta - 2\phi(k - N/2)), \tag{2.12}$$

where $P_{\mathrm{coh}}(\theta)$ is the coherent state distribution function. In the case of relatively intense coherent states the phase probability distribution can be well approximated by the periodic Gaussian

$$P_{\mathrm{coh}}(\theta) \approx \sum_{n=-\infty}^{\infty} \alpha\sqrt{2/\pi}\exp[-2\alpha^2(\theta - 2\pi n)^2]. \tag{2.13}$$

Applying the Poisson summation formula

$$\sum_{n=-\infty}^{\infty} f_n = \sum_{m=-\infty}^{\infty} \int_{\infty} dx f(x) e^{2\pi i m x}$$

we can rewrite the expression for $P_{\mathrm{coh}}(\theta)$ in the following form:

$$P_{\mathrm{coh}}(\theta) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{-im\theta} e^{-m^2/8\bar{n}}. \tag{2.14}$$

In the limit of a large number of atoms $N \gg 1$ in Eq. (2.4), we approximate the binomial distribution by its Gaussian limit, Eq. (2.6). For small values of the phase $\phi \ll 1/\alpha$ we can substitute the sum in Eq. (2.12) by the appropriate integral. Evaluating this integral with the distribution C_k given by Eq. (2.6) we obtain the following expression for $P^{(N)}(\theta)$:

$$P^{(N)}(\theta) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{-im\theta} e^{-(m^2/2)\sigma^2}. \tag{2.15}$$

with

$$\sigma^2 = 1/4\alpha^2 + N\phi^2.$$

In the representation (2.15) the explicit periodicity of the phase distribution function is conserved and it is easy to find the mean value and variance of the sin θ function as

$$\langle \sin\theta \rangle = \int_{-\pi}^{\pi} d\theta (\sin\theta) P^{(N)}(\theta) = 0, \tag{2.16}$$

$$\sigma_{\sin\Theta}(t) = \langle \sin^2\theta \rangle - \langle \sin\theta \rangle^2 = \frac{1 - e^{-2\sigma^2}}{2}.$$

Applying the Poisson summation formula to Eq. (2.15) we get the equivalent representation for $P^{(N)}(\theta)$,

$$P(\theta) = \frac{1}{(2\pi\sigma^2)^{1/2}} \sum_{k=-\infty}^{\infty} e^{-(1/2\sigma^2)(2k\pi - \theta)^2}, \tag{2.17}$$

which has the form of a periodic Gaussian. The dispersion of each Gaussian σ^2 linearly grows with the number of passing atoms. The above expression well describes the phase diffusion process. As one can see from Eq. (2.16) the phase dispersion tends to one-half: $\sigma_{\sin\Theta}(t) \to 1/2$ as the number of atoms goes to infinity. This means that the phase distribution tends to be random in the limit of a large number of atoms.

In the same approach an approximate expression for the Q function $Q = \langle \beta | \rho_f | \beta \rangle$ can be found:

$$Q(\beta = re^{i\psi}) = \frac{e^{-(\alpha - r)^2}}{\sqrt{2r\alpha\sigma^2}} \sum_{m=-\infty}^{+\infty} \exp\left[-\frac{(\psi - 2\pi m)^2}{2\sigma^2} \right].$$

Here each term is a Gaussian function in phase variable, centered at $\psi = 0$ with an intensity dependent variance σ^2 that describes a phase spreading of the wave packet.

We can include the effect of cavity losses by making the replacement $\alpha \to \alpha e^{-\gamma N/2r}$ in Eq. (2.7), which corresponds to assuming a zero-temperature thermal bath and that the interaction time is small enough compared with the time between consecutive atoms in order to neglect the dissipation during the gain process. Besides, we remark that this replacement is a consequence that there are no pump atoms in this model, thus the cavity losses establish that the average of the field energy is only decaying.

This result shows how we can engineer a diffusionlike behavior onto the phase of a quantum field by introducing irreversibility while ignoring the state of the two-level atoms after the interaction. This is the simplest picture we can implement by using a beam of independent two-level atoms, ignoring the final state of individual atoms after the interaction with the cavity field.

III. DICKE MODEL IN THE LARGE DETUNING LIMIT

If in the situation discussed in the previous section atoms pass through a Ramsey zone after their interaction with the field followed by a detection of atomic population, the field state turns into a coherent superposition of CS. This is the mechanism used in Ref. [6] and previously discussed in a number of works to study decoherence [8]. If a measurement of atomic population is performed for all the atoms of the beam, without the interaction with the Ramsey zone after the cavity, a cavity quantum electrodynamic (QED) version of the quantum random walk process is obtained. In this case the stochastic variable is the phase of the CS inside the cavity. In this section we consider the possibility of using a collection of two-level atoms to produce a coherent superposition of CS.

A collection of two-level atoms interacting with a quantum field is described by the Dicke Hamiltonian [9], which can be conveniently written as follows ($\hbar = 1$):

$$H = \Delta S_z + g(aS_+ + a^\dagger S_-), \tag{3.1}$$

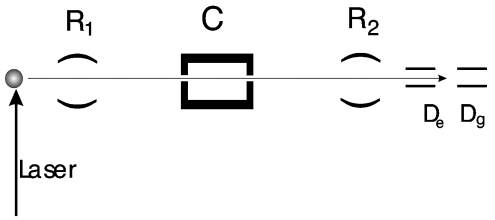

FIG. 2. Schematic diagram for generating coherent superpositions of CS. R_1 denotes the Ramsey zone, which interacts with the collection of atoms for preparing an atomic state $|\underline{p}\rangle$. The cavity C contains the initial coherent state $|\alpha\rangle$. The second Ramsey zone R_2 allows an additional rotation of atomic states after the interaction with the field. Finally, the detection zone D gives information of the final number of excited atoms.

where we have omitted the constant of motion term $\omega_c(a^\dagger a+S_z)$, which corresponds to describing the system in a rotating frame at frequency of the cavity, ω_c. The collective atomic operators are defined as usual:

$$S_j=\sum_{k=1}^{A}\sigma_j^{(k)},\quad j=z,+,- \tag{3.2}$$

and obey the su(2) commutation relations

$$[S_+,S_-]=2S_z,\quad [S_z,S_\pm]=\pm S_\pm .$$

The operators $\sigma_z^{(k)}$ and $\sigma_+^{(k)}$ ($\sigma_-^{(k)}$) are the difference of population and raising (lowering) operators of the kth atom, respectively. A schematic view of the proposal is depicted in Fig. 2. In the space of symmetrical initial states the atomic operators form an $(A+1)$-dimensional representation of su(2) algebra. In this basis collective operators act as

$$\begin{aligned} S_z|k\rangle&=(k-A/2)|k\rangle,\\ S_+|k\rangle&=\sqrt{(A-k)(k+1)}\,|k+1\rangle,\\ S_-|k\rangle&=\sqrt{k(A-k+1)}\,|k-1\rangle, \end{aligned} \tag{3.3}$$

where $|k\rangle$ is the symmetric state with k excited atoms and $k=0,1,\ldots,A$. We shall refer to them as bare states. In the one-atom case these states are the atomic levels $|0\rangle$ and $|1\rangle$.

In the limit of large detuning $\Delta\gg g\alpha$ we can adiabatically eliminate the transitions among different eigenstates of S_z, thus achieving a dispersive evolution similar to that previously given in Eq. (2.1). In this limit we obtain an effective evolution operator from a perturbation theory and its derivation is included in Appendix A. The result for a collection of A atoms is given by

$$U(\phi)=\exp\{i\phi[S^2-S_z^2+(2a^\dagger a+1)S_z]\}, \tag{3.4}$$

where

$$S^2|k\rangle=\frac{A}{2}\left(\frac{A}{2}+1\right)|k\rangle.$$

A similar effective Hamiltonian was derived in a recent work by Agarwal *et al.* [10] to study the generation of atomic Schrödinger-cat states, where the parameter $\phi=g^2\Delta/(\kappa^2+\Delta^2)$; κ is the cavity decay rate and $a^\dagger a\to\overline{n}$ (because of a trace over field variables). An alternative proposal for atomic Schrödinger-cat generation is studied in Ref. [11]. In the two-atom case the evolution operator (3.4) explicitly reads as follows:

$$U(\phi)=e^{2i\phi(a^\dagger a+1)}|2\rangle\langle 2|+e^{2i\phi}|1\rangle\langle 1|+e^{-2i\phi a^\dagger a}|0\rangle\langle 0|. \tag{3.5}$$

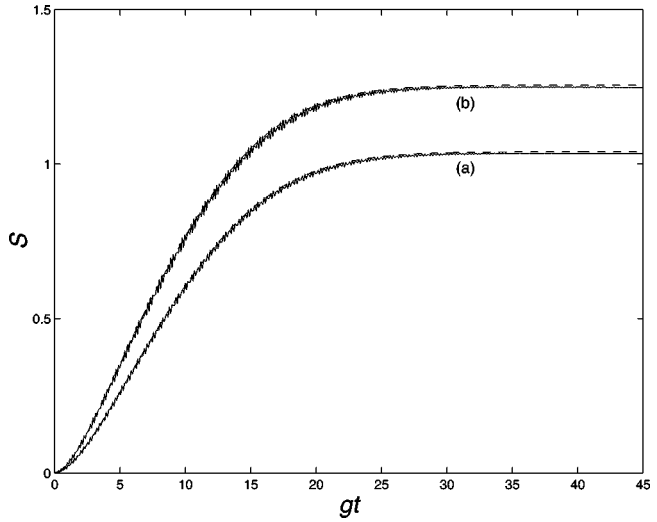

FIG. 3. Field entropy as a function of time for (a) two-atom and (b) three-atom cases from both the numerical evolution (solid line) and the adiabatic approximation (dashed line). The field was assumed to be initially in a coherent state with $\overline{n}=16$ and the atoms in a $|\underline{0}\rangle$ state. We have chosen $\Delta/g=100$.

The quality of the adiabatic approximation can be studied by comparing the predicted evolution by Eq. (3.4) with the corresponding exact (numerical) evolution of the model. In Fig. 3 we plot the field entropy, $S=-\mathrm{Tr}(\rho_f\ln\rho_f)$, for the case of a field initially in a coherent state and a collection of two atoms (three atoms) prepared in an eigenstate of the atomic operator S_x:

$$\begin{aligned} S_x|\underline{p}\rangle&=\lambda_p|\underline{p}\rangle,\\ \lambda_p&=p-A/2,\quad p=0,\ldots,A. \end{aligned} \tag{3.6}$$

The states $|\underline{p}\rangle$ correspond to the case when the collection of atoms initially prepared in their lower state interacts with a resonant classical field of zero phase. This corresponds to the excitation of the collection of atoms up to their ground state by a laser followed by a passage though a Ramsey zone before going into the resonator. The states $|\underline{p}\rangle$ form an alternative basis of the symmetric atomic subspace. They can be written in the bare atomic basis as follows:

$$|\underline{p}\rangle=\sum_{k=0}^{A}C_p^k|k\rangle, \tag{3.7}$$

where the C_p^k coefficients can be expressed in terms of Wigner d functions as $C_p^k=\langle k|\underline{p}\rangle=i^{p-k}d_{pk}^A(-\pi/2)=\overline{C}_k^p$ [12]. The exact and approximated dynamics shown in Fig. 3 exhibit an excellent agreement even for long times. The

small oscillations in the exact entropy are not described by the approximation because we have neglected transitions between eigenstates of the operator S_z.

A. Coherent superpositions

In what follows we want to build up a coherent superposition of CS. This depends on what we do to the collection of atoms when it leaves the cavity. We assume the collection of atoms initially prepared in an arbitrary superposition in the bare basis, which contains more than one element of the basis $\{|k\rangle\}$. In this case, the atoms-field state after the interaction is

$$|\Psi(\phi)\rangle = U(\phi)|\alpha\rangle \otimes |\psi_{\text{atoms}}\rangle, \tag{3.8}$$

where $|\psi_{\text{atoms}}\rangle = \Sigma_k^A c_k |k\rangle$. Using Eq. (3.4) we can write explicitly the previous expression as

$$|\Psi(\phi)\rangle = \sum_{k=0}^{A} c_k e^{-i\phi\nu_k} |\alpha_k\rangle \otimes |k\rangle, \tag{3.9}$$

where $\nu_k = k(A+1-k)$ and $\alpha_k = \alpha e^{i\phi(2k-A)}$. Now, the atoms go through Ramsey zone R_2. Each $|k\rangle$ state goes to a new superposition. The effect of the Ramsey zone on the atomic states is $|k\rangle \rightarrow \Sigma_{l=0}^A c_k^l(\Omega\tau)|l\rangle$, with $c_k^l(\Omega\tau) = \Sigma_{q=0}^A \overline{C}_k^q C_q^l \exp(i\Omega\tau\lambda_q)$, where Ω is the Rabi frequency of the classical field and τ is the interaction time with this field. The next step is to test the number of excited (unexcited) atoms, which will project the field to a specific superposition. Assuming that the measurement process indicates that l atoms were in their excited states, the field state is given by the expression

$$|\Psi_l^{(A)}(t)\rangle_{\text{field}} = \frac{1}{\sqrt{\mathcal{N}}} \sum_{k=0}^{A} c_k^l(\Omega\tau) c_k e^{-i\phi\nu_k} |\alpha_k\rangle, \tag{3.10}$$

where $\mathcal{N}$ denotes a normalization constant. We obtain in this way a coherent superposition of A CS on the circle of radius α. In the above calculation we have neglected contributions to the phase of terms of the order $Ag^2\alpha^3/\Delta^2 \ll 1$, which correspond to the next order contribution term in Eq. (A4). We have considered times of the order $t^* \sim \Delta/(g^2\alpha)$ for which a substantial splitting of CS can be observed. In Fig. 4 we plot the Q function of the state $|\Psi_l^{(A)}(t)\rangle_{\text{field}}$ for the case of two atoms initially prepared in the $|\underline{p=0}\rangle$ state and measuring a final state $|l=0\rangle$, i.e., $|\Psi_{0,\underline{0}}^{(2)}(t)\rangle_{\text{field}}$ state. We observe that the field was split into a superposition of three well-resolved CS.

If we consider interaction times of the order of t^* the field CS $|\alpha_k\rangle$ can be considered as a macroscopic basis of an $(A+1)$-dimensional Hilbert space. In the next subsection we exploit this property to establish nonlocal correlations between two cavities.

B. Nonlocal correlation between cavities

A direct extension of the atom-field interaction in the large detuning limit is the generation of strongly entangled states between distant cavities. The coherent states $|\alpha_k\rangle$ obtained in the previous subsection form an orthogonal field macroscopic basis. This is equivalent to saying that

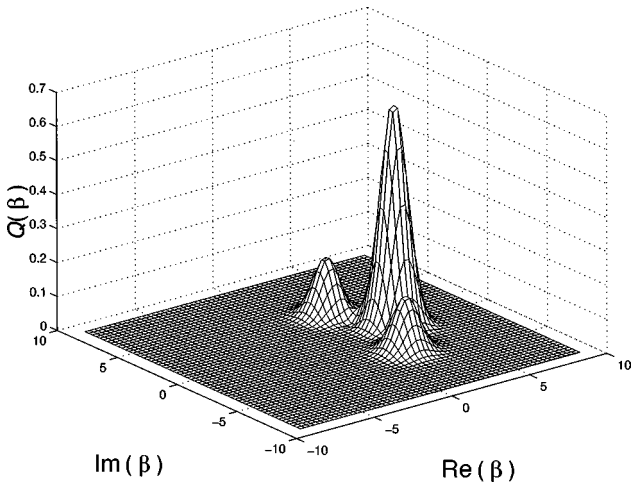

FIG. 4. Field Q function of the state $|\Psi_l^{(A)}(t)\rangle_{\text{field}}$ for the case of two atoms initially prepared in the $|\underline{p=0}\rangle$ state and measuring a final $|l=0\rangle$ atomic state, i.e., $|\Psi_0^{(2)}(t)\rangle_{\text{field}}$ state. The interaction time was $gt=50$, which is the minimum required to observe a whole splitting. The effective interaction time in the Ramsey zone was $\Omega\tau=\pi/4$. The other parameters are as in Fig 3.

$\langle\alpha_k|\alpha_{k'}\rangle \simeq \delta_{k,k'}$. This is achieved when the interaction parameter ϕ holds the condition $\phi \sim 1/2\alpha$. In this sense CS $|\alpha_k\rangle$ give origin to $(A+1)$-dimensional space. Establishing strong entanglement between cavities can be useful for a teleportation [13] proposal of a state lying in a space of dimension $A+1>2$. In a cavity QED proposal this can be achieved by injecting a collection of A two-level atoms simultaneously into the cavity, which interacts with one mode of the electromagnetic field. Atoms going through three cavities have been proposed to generate maximally entangled field states in the context of cavity QED [14,15].

The collection of A atoms is sent through the cavities C_1 and C_2 (in the schematic diagram depicted in Fig. 2 cavity C_2 is placed between C and the Ramsey zone R_2). The field inside the cavities C_1 and C_2 is prepared in CS with zero phase $|\alpha\rangle_1$ and $|\beta\rangle_2$, respectively. The atom-field interaction is dispersive in each cavity, so the evolution is directly given by operator (3.4). After atoms leave the second cavity they go through Ramsey zone R_2, which creates entanglement between the CS lying in both cavities. The final step of the entanglement process is the measurement of the number of excited atoms in the detection zone D. The final entangled state of both cavities is

$$|\Psi\rangle = \sum_{k=0}^{A} \eta_k |\alpha_k\rangle_1 |\beta_k\rangle_2, \tag{3.11}$$

where the parameter of CS is $\lambda_k = \lambda e^{i\phi(2k-A)}$, with $\lambda = \alpha, \beta$. We have assumed the same interaction parameter in both cavities. The coefficients η_k depend on the specific initial atomic state; the interaction with classical field in the Ramsey zone and the result of the population measurement. This is explicitly given by

$$\eta_k = c_k e^{-i2\phi\nu_k} \sum_{q=0}^{A} \overline{C}_k^q C_q^l \exp(i\Omega\tau\lambda_q),$$

where l denotes the number of excited atoms detected. This simple scheme allows us a full entanglement of the macroscopic basis elements in both cavities.

IV. MACROSCOPIC SUPERPOSITIONS IN THE STRONG FIELD LIMIT

In the above discussion we have shown that a variety of superpositions can be generated in the dispersive regime of the Dicke model. An alternative scheme for generating coherent superpositions arises in the resonant case of Eq. (3.1) for the strong field limit, defined by $\bar{n}=\alpha^2 \gg A$, where α is the amplitude of the initial field as before.

In recent works it has been shown that for an initial strong quantum field the evolution of a collection of atoms can be studied perturbatively. In fact, let us consider evolution of an initial atomic coherent superposition in a strong coherent field (as before, we take zero initial field phase). It was shown in Ref. [16] that in this case the total wave function of the system approximately factorizes into field and atomic parts. However, we shall include a brief derivation of the wave-function factorization, along the lines of Ref. [16], in order to make this work self-consistent. The starting point is Hamiltonian (3.1) in the resonant case, $\Delta=0$, which can be approximately diagonalized in the field space with the following transformation:

$$\underline{H}=e^{-i\hat{\phi}(\hat{S}_z+A/2)}He^{i\hat{\phi}(\hat{S}_z+A/2)}$$
$$=2g\sqrt{\hat{n}-A/2+1/2}\hat{S}_x+O(A/\sqrt{\bar{n}}),$$

where $e^{\pm i\hat{\phi}}$ are field phase operators [17]. When we are considering the strong field limit the field does not reach the vacuum state during the interaction, so the applied transformation is unitary. It has been shown that even the zeroth-order approximation (just the first term in the above expansion) well describes all of the essential quantum phenomena. Thus the system wave function takes on the form

$$|\Psi(t)\rangle \simeq e^{i\hat{\phi}(\hat{S}_z+A/2)}e^{-i2gt\sqrt{\hat{n}+1/2}-A/2\hat{S}_x}$$
$$e^{-i\hat{\phi}(\hat{S}_z+A/2)}|\alpha\rangle_f\otimes|\underline{p}\rangle, \tag{4.1}$$

where we have assumed that the field was prepared in a coherent state $|\alpha\rangle_f$ and the atoms in a state $|\underline{p}\rangle$. After a few lines, applying the operators on Eq. (4.1) to initial field and atomic states and neglecting contributions of order $O(A/\sqrt{\bar{n}})$ we obtain

$$|\Psi(t)\rangle \simeq |A_p(t)\rangle\otimes|\Phi_p(t)\rangle,$$
$$|A_p(t)\rangle=\exp\left[-i\,\frac{gt\lambda_p(S_z+A/2)}{2\sqrt{\bar{n}-A/2+1/2}}\right]|\underline{p}\rangle, \tag{4.2}$$
$$|\Phi_p(t)\rangle=\exp[-igt\lambda_p\sqrt{\hat{n}-A/2+1/2}]|\alpha\rangle.$$

Thus, we note that the field and atomic dynamics are approximately factorized for the assumed initial conditions. We remark that even when they factorize, the effective dynamics of the collection of atoms is depending on the initial number of photons, and the field dynamics is determined by the initial atomic preparation through the parameter $\lambda_p=p-A/2$. Thus, in this limit the dynamics of one system (field) is determined by the initial condition of the other one (atoms). This together with the fact that $|\underline{p}\rangle$ states form an alternative basis of the symmetric atomic subspace allow us to create coherent field superpositions.

We assume the same initial conditions as in Sec. III. However, for convenience we write the initial atomic state in $|\underline{p}\rangle$ basis. This is

$$|\psi\rangle_{\text{atoms}}=\sum_{p=0}^{A} d_p|\underline{p}\rangle,$$

where $d_p=\Sigma_k^A c_k\bar{C}_k^p$. Thus, with this initial condition and Eq. (4.2) the evolution of the system up to a time t can be written as follows:

$$|\psi(t)\rangle=\sum_{p=0}^{A} d_p|A_p(t)\rangle\otimes|\Phi_p(t)\rangle. \tag{4.3}$$

This equation is analogous to Eq. (3.9). After the atoms leave the cavity they are directly sent to the detection arrangement to measure the number of excited (unexcited) atoms. We remark that in this case an additional Ramsey zone before the atomic measurement is not necessary. In order to know in which state the field is projected after the measurement on atoms it is convenient to span the state $|A_p(t)\rangle$ in the number of atomic excitations. Thus Eq. (4.3) takes the form

$$|\psi(t)\rangle=\sum_{k=0}^{A}\left(\sum_{p=0}^{A} d_pC_p^k e^{-ik\Theta_p}|\Phi_p(t)\rangle\right)|k\rangle, \tag{4.4}$$

where $\Theta_p=gt\lambda_p/2\sqrt{\bar{n}-A/2+1/2}$. This means that the detection of l excitations in the atomic system reduces the field wave function to the superposition of $A+1$ states $|\Phi_p(t)\rangle$:

$$|\Psi_l^{(A)}(t)\rangle=\frac{1}{\sqrt{\mathcal{N}}}\sum_{p=0}^{A} d_pC_p^l e^{-il\Theta_p}|\Phi_p(t)\rangle, \tag{4.5}$$

where $|\Phi_p(t)\rangle$ is the reduced field wave function. As follows from Eq. (4.2) the evolution of the field part of the factorized wave function $|\Phi_p(t)\rangle$ is governed by the following effective Hamiltonian:

$$H_{\text{eff}}=g\lambda_p\sqrt{\hat{n}-A/2+1/2}, \tag{4.6}$$

which can be approximated (even for very long times, $gt\lesssim\bar{n}^{3/2}$) as follows:

$$H_{\text{eff}}\approx g\lambda_p\left[\sqrt{\bar{n}_A}+\frac{(\hat{n}-\bar{n})}{2\sqrt{\bar{n}_A}}-\frac{(\hat{n}-\bar{n})^2}{8\bar{n}_A^{3/2}}\right], \tag{4.7}$$

where $\bar{n}_A=\bar{n}-A/2+1/2$. The field dynamics arising from each term in the above equation has a transparent physical sense: The first term multiplies the wave function by a phase factor. The second rotates the state along the circle of radius α. The third is an intensity phase shift, which is responsible for a phase spreading. The interesting effect coming from this term is to rotate the hump around itself, producing in

addition a stretching of the hump. For interaction times close to the first revival, it gives rise to noise reduction in the field quadrature. Going back to Eq. (4.5) we note that the reduced field wave function is a superposition of rotated and stretched CS.

Let us consider explicitly the two-atom case. For initial nonexcited atoms the detection of the same state (unexcited atoms) leads to the field wave function in a superposition of three states (three-component cat), which coincides at the revival moment:

$$|\Psi_0^{(2)}(t)\rangle = \frac{1}{\sqrt{\mathcal{N}}} e^{-i\hat{\varphi}_n} |\alpha e^{igt/\sqrt{n-1/2}}\rangle + \frac{1}{\sqrt{\mathcal{N}}} (\sqrt{2}|\alpha\rangle + e^{i\hat{\varphi}_n} |\alpha e^{-igt/\sqrt{n-1/2}}\rangle), \tag{4.8}$$

where $\hat{\varphi}_n = [\sqrt{n} - (\hat{n}-\bar{n})^2/(8\bar{n}_A^{3/2})]gt$. The detection of the atoms in the excited state in the output leads to almost the same superposition for the field, but with a minus sign before the second exponent. Nevertheless, if we register one atom excited, the field wave function is represented as a superposition of just two states:

$$|\Psi_{0,1}^{(2)}(t)\rangle = \frac{1}{\sqrt{\mathcal{N}}} (|\alpha e^{-i\Theta_0 t}\rangle - e^{4igt\sqrt{n}} |\alpha e^{i\Theta_0 t}\rangle). \tag{4.9}$$

Here we have neglected the contribution of nonlinear terms.

The coherent superposition of field states is only a consequence that the initial condition contain more than one element on the expansion on the basis $\{|p\rangle\}$, so the result is directly generalized to any other initial condition different from $|l\rangle$.

An interesting application of the proposed scheme is the generation of a quasi-Gaussian superposition of field CS along the circular arc of radius α. To achieve this goal, we consider a strong initial coherent field satisfying the condition $1 \ll A \ll \alpha$. If the collection is initially prepared in the state $|l=0\rangle$, after the interaction we must measure the same atomic state. This projects the field onto the state

$$|\Psi_G(t)\rangle = \frac{1}{\sqrt{\mathcal{N}}} \sum_{q=0}^{A} |C_0^q|^2 e^{-igt\lambda_q\sqrt{n}} |\alpha e^{-i\Theta q}\rangle. \tag{4.10}$$

Here, the interaction time is very short, so the contribution of nonlinear terms has been neglected. The C_0^q coefficients tend to a Gaussian distribution for few numbers of atoms, typically $A \sim 16$. The condition for obtaining a quasicontinuous superposition is $|\Theta_p - \Theta_{p-1}|\alpha \ll 1$, i.e., $gt \ll 1$. This class of states was originally discussed in Ref. [1].

V. SUMMARY

To conclude, we have considered the high detuning limit of the atom-field interaction in a microwave cavity. A splitting of an initial coherent state arises, generating an incoherent binomial superposition of the coherent states on the circle. In the case of a weak splitting, the initial state spreads quasicontinuously in phase, giving rise to the effective phase diffusion when the final state of the atom remains unknown.

We also have proposed two different schemes for generating coherent superpositions of CS inside a microwave cavity. The schemes are based on effective dispersive interactions that arise both in the large detuning limit and in the strong field limit of a quantum field interacting with an atomic collection of two-level atoms. Experimental realizations of both proposals are closely related to the microwave regime of cavity QED. The present experimental setups are mainly focused in the field of small amplitude, $|\alpha|^2 \sim 10$ [18]. Thus, at the present state of the art of experiments only the dispersive regime arising from the large detuning limit appears to be well suited for an experimental realization. In this case some difficulties appear on the initial atomic preparation, which in principle can be overcome for few atoms [19]. The others parameters are in good agreement with reported experimental setups [6]. The current interest in small field amplitude has as its main goal to emphasize some quantum features of atom-field interactions. However, in principle there are not inseparable experimental difficulties in achieving the dispersive regime in the strong field limit. Also, interaction of many two-level atoms with a quantum field in a driven microwave cavity has been recently reported [20].

The proposed method for generating a coherent superposition of quantum states can be directly mapped onto experiments in laser-cooled trapped ions, in the corresponding dispersive interaction limit. In this case the superposition corresponds to the vibrational mode of a single ion and the measurement must be performed at the ionic level [21]. The interaction of a collection of two-level atoms with a quantum field in the weak field limit ($\sqrt{n} \ll A$) provides an interesting mechanism for measuring the field density matrix elements. This corresponds to the inverse problem of this paper. It is presently being studied and will be published elsewhere.

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APPENDIX

In this Appendix we include a brief derivation of the effective evolution operator given in Eq. (3.4). The starting point is the Dicke Hamiltonian [9], in the standard dipole and rotating wave approximations, which reads as follows (disregarding a constant term):

$$H = \Delta S_z + g(aS_+ + a^\dagger S_-),$$

where the collective atomic operators are defined in Eq. (3.2), and a and $a^\dagger$ are usual field operators. This Hamiltonian can be rewritten as

$$H = \Delta S_z + g_0(qS_x - pS_y), \tag{A1}$$

with $g_0 = \sqrt{2}g$ and

$$S_x=\frac{1}{2}(S_+ + S_-), \quad S_y=-\frac{i}{2}(S_+ - S_-),$$
$$q=\frac{1}{\sqrt{2}}(a+a^\dagger), \quad p=\frac{i}{\sqrt{2}}(a^\dagger - a). \tag{A2}$$

Now, we apply the following unitary transformation:

$$U_1=\exp(i\phi S_x).$$

Under this transformation and assuming that ϕ is a small parameter Hamiltonian (3.1) goes to

$$\begin{aligned} H_1 &= U_1 H U_1^\dagger \\ &= (\Delta S_z + g_0 U_1 q U_1^\dagger S_x - g_0 U_1 p U_1^\dagger S_y) \\ &\quad + (\Delta S_y + g_0 U_1 p U_1^\dagger S_z)\phi, \end{aligned} \tag{A3}$$

where we have performed a series expansion up to first order in ϕ. We choose the parameter as $\phi=(g_0/\Delta)p$, with this choice H_1 explicitly reads as

$$H_1=\Delta S_z + g_0 q S_x + \frac{g_0^2}{\Delta}\left(S_x^2+\frac{1}{4}[p^2 S_z + S_z p^2]\right).$$

The effect of U_1 transformation is a partial elimination of atomic population transfer due to the interaction with the field, i.e., the contribution of the term proportional to pS_y is canceled out. By following the same procedure we perform a next transformation to H_1, which is given by

$$U_2=\exp\left(i\frac{g_0}{\Delta}qS_y\right).$$

Applying this new transformation the Hamiltonian reads as

$$\begin{aligned} H_2 &= U_2 H_1 U_2^\dagger \\ &= \Delta S_z + 2\frac{g^2}{\Delta}nS_z + \frac{g^2}{\Delta}(S^2 - S_z^2 + S_z) \\ &\quad + \frac{g^2}{2\Delta}(S_+^2 + S_-^2) + O\left(\frac{1}{\Delta^2}\right). \end{aligned} \tag{A4}$$

The last term in the above equation does not affect essentially the system dynamics and can be removed using a rotation-wave-like approximation. If we consider this term as a perturbation to the first two, one can note that the first-order correction to the eigenvalues vanishes and the second-order correction is of order $(g/\Delta)^3$. Finally, the system Hamiltonian takes the form (3.4). One can easily see that dynamics of any observable will not be affected by the transformations U_1 and U_2, because these transformations are time independent and would only introduce small corrections to coefficients. The time range where the Hamiltonian H_2 well describes the system dynamics is defined by neglected terms and one can show that it is of order $At(\sqrt{\bar{n}}g_0)^3/\Delta^2 \ll 1$.

[1] J. Jansky and V. Vinogradov, Phys. Rev. Lett. **64**, 2771 (1990); P. Adam, J. Jansky, and V. Vinogradov, *ibid.* **68**, 3816 (1992); M. Orszag, R. Ramirez, J. C. Retamal, and C. Saavedra, *ibid.* **68**, 3815 (1992); G. S. Agarwal and R. Simon, Opt. Commun. **92**, 105 (1992); I. Földesi, P. Adam, and J. Jansky, Phys. Lett. A **173**, 97 (1993); V. Buzek, A. Vidiella Barranco, and P. L. Knight, Phys. Rev. A **45**, 6570 (1992); P. Adam, I. Földesi, and J. Jansky, *ibid.* **49**, 1281 (1994); P. Domokos, P. Adam, and J. Jansky, *ibid.* **50**, 4293 (1994).

[2] K. Vogel, V. M. Akulin, and W. P. Schleich, Phys. Rev. Lett. **71**, 1816 (1993).

[3] E. Schrödinger, Naturwissenschaften **23**, 807 (1935); **23**, 823 (1935); **23**, 8844 (1935). English translation by J. D. Trimmer, Bull. Am. Phys. Soc. **124**, 3225 (1980).

[4] C. Monroe, D. M. Meekhof, and D. T. Wineland, Science **272**, 1131 (1996).

[5] L. Davidovich, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **53**, 1295 (1996).

[6] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996).

[7] M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1977).

[8] L. Davidovich, N. Zagury, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **50**, R895 (1994).

[9] R. Dicke, Phys. Rev. **93**, 99 (1954).

[10] G. S. Agarwal, R. R. Puri, and R. P. Singh, Phys. Rev. A **56**, 2249 (1997).

[11] Christopher C. Gerry and Rainer Grove, Phys. Rev. A **56**, 2390 (1997).

[12] N. Vilenkin and A. Klimyk, *Representation of Lie Groups and Special Functions* (Kluwer Academic, Dordrecht, 1991), Vols. 1–3.

[13] C. H. Bennet, G. Brassard, C. Creépeau, R. Jozsa, A. Peres, and W. K. Wooters, Phys. Rev. Lett. **70**, 1895 (1993).

[14] Christopher C. Gerry, Phys. Rev. A **54**, R2529 (1996).

[15] János A. Bergou and Mark Hillery, Phys. Rev. A **55**, 4585 (1997).

[16] A. B. Klimov and S. M. Chumakov, Phys. Lett. A **202**, 145 (1995).

[17] R. Loudon, *The Quantum Theory of Light* (Clarendon Press, Oxford, UK, 1973), p. 141.

[18] For instance, in Ref. [6] an average photon number varying from 0 to 10 is reported.

[19] J. M. Raimond (private communication).

[20] W. Lange, Ph.D. thesis, Max-Planck-Institut für Quantenoptik, München, 1994 (unpublished).

[21] J. C. Retamal and N. Zagury, Phys. Rev. A **55**, 2387 (1997).