Theoretical study of quantum dissipation and laser-noise effects on the atomic response

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The nonlinear dynamics of dissipative quantum systems in incoherent laser fields is studied in the framework of a master equation with the random telegraph model describing the laser noise and the Markovian approximation dealing with the system-bath couplings. Floquet theory and time-dependent perturbation methods are used to facilitate both analytical and numerical solutions. We develop a theoretical formalism that provides a powerful tool for the detailed analysis of the dissipative quantum dynamics of multilevel systems driven by intense stochastic laser fields. It is found that the system relaxes to a steady state from the effect of the laser phase and frequency noise and the kinetics of this relaxation increases with the addition of dissipative terms, introduced by the coupling to the reservoir. Amplitude fluctuations show a different behavior. Other results concerning the destruction of quantum coherence and the dynamical localization will be established and further relaxation mechanisms such as spontaneous emission and the ionization process will also be considered. [S1050-2947(98)02312-9]

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I. INTRODUCTION

The time evolution of quantum systems, which are driven by an external field and in contact with a heat bath (reservoir) has received a great deal of attention in recent years [1–4]. In quantum optics, such systems are investigated in the dressed-atom picture of resonance fluorescence [5], where a beam of atoms interacts with a coherent laser field and all the electromagnetic modes of the vacuum [6]. Moreover, it is now recognized that nearly all types of laser-atom interactions can be strongly affected by laser noise. Indeed, one practical reason for this fact is the use, in experiments, of high powers obtained in pulsed operation, at the expense of poorly stabilized laser beams. Furthermore, real atoms experience a fluctuating environment of many perturbing interactions and ideal lasers exist only in theoretical models, while the used laser sources are subjected to many types of fluctuations notably in phase, amplitude, and frequency [7–10]. Other kinds of fluctuations due to collisional effects can affect the atomic transition frequencies [9,11]. Therefore, we cannot establish, without taking into account the dissipative action of the environment and the statistical properties of the laser light, a rigorous comparison between theoretical predictions and experimental results.

Different approaches to the dissipative dynamics of open quantum systems in strong external fields have been proposed and applied to the description of atoms under the influence of thermal noise [1–4]. While for the incoherence of laser field a series of models, all based on so-called pre-Gaussian Markovian processes [7–10], have been used in order to describe the stochastic behavior of the laser field. It is important to mention a few technical features of these models. They are based on the two-state random telegraph. They are not Gaussian models but rather pre-Gaussian, with a Gaussian limit [8]. Our choice of the random telegraph is based on the simplicity of this model, which permits a unified treatment of different noisy lasers in phase, amplitude, and frequency. Several works have reported on the action of random process on a two-level system [7–10,12–15], particularly the evolution populations σ_{nn} and the ionization probability.

In the present paper we elucidate the role of quantum dissipation and laser noise on the atomic response. For this purpose we derive a master equation, which provides a general framework for the dynamics of atoms interacting with strong laser noise and a thermal reservoir.

The basic idea underlying the theoretical formalism of that paper is to take into account the exact dynamics of the interaction between an atomic system and an external field by employing the Floquet basis for the reduced system rather than the stationary unperturbed states [1,2]. The interaction of the laser-atom system with the reservoir will be treated by the time-dependent perturbation theory, which leads to a generalized quantum master equation for the reduced density matrix. Such a statistical fundamental equation, introduced in quantum optic by Burshtein [16–18], contains information concerning the atomic transition dynamics, the stochastic evolution of laser field fluctuations, and the dissipative mechanisms. We are concerned here with an important theme of contemporary research, namely, the interplay between quantum coherence and external noise. In fact, the destruction of quantum coherence by noise is central to many fields and is reflected in the many papers recently published on this subject [19–23].

Our computations are made at an exact resonance, where the effects of spontaneous emission are important [24], and for a strong laser field, where the probability to realize an ionization of atoms is highest. Therefore, we shall extend our theory here by the inclusion of the relaxation rates corresponding to the spontaneous emission and the ionization processes and present the corresponding numerical results.

The theory is developed in Sec. II by considering the case of the strong laser-atom interaction in the presence of laser noise and dissipative effects, which are introduced by the coupling to the reservoir. Within the framework of the Floquet representation and the Markovian approximation, detailed theoretical calculations are feasible to obtain the reduced matrix density elements. The account of Floquet theory given here is rather brief since the theory has been discussed at length in the recent literature (see, e.g., Refs. [25–29]). Moreover, the influence of the ionization process on the response atomic function is presented. Numerical results concerning a model of the two-level system are presented in Sec. III. A summary of our results is given in Sec. IV.

II. THEORY

We consider an atomic system that interacts with an external classical laser field. Moreover, the laser-atom system is coupled to a quantified radiation field in thermal equilibrium. In the following we will consider the behavior of the atom coupled to a reservoir with many degrees of freedom.

The aim of this paper is to provide a description of the dynamics in terms of the degrees of freedom of the atomic system alone by elimination of the reservoir variables. Since the atoms are driven strongly by an external laser field, our master equation is based on the atomic Floquet states rather than the unperturbed atomic states.

The total Hamiltonian governing the dynamics of the coupled system of matter and radiation degrees of freedom takes the form

$$H(t)=H_{A\text{-}L}(t)+H_1+H_R, \tag{1}$$

where $H_{A\text{-}L}(t)$ is the total Hamiltonian of the atomic system and the external laser field, without an interaction with the reservoir, given by

$$H_{A\text{-}L}(t)=H_A+V(t)+H_{\mathrm{SE}}, \tag{2}$$

with H_A the stationary atomic Hamiltonian, $V(t)$ the dipole interaction between the atomic system and the laser field, and H_{SE} the Hamiltonian of the simultaneous emission, which reads

$$H_{\mathrm{SE}}=-\frac{\hbar}{2}\,\Gamma, \tag{3}$$

where Γ is a diagonal matrix composed of the Einstein coefficients of the spontaneous emission process and is defined by [29]

$$\Gamma_{nn}=\sum_{n'<n}\gamma^{\mathrm{SE}}_{nn'}. \tag{4}$$

Here $\gamma^{\mathrm{SE}}_{nn'}$ is the radiative decay rate.

The Hamiltonian that describes the coupling between the matter degrees of freedom and the quantified radiation field may be written in the dipole approximation as

$$H_I=\hbar\sum_j z(\gamma_j a_j+a_j^\dagger\gamma_j^*), \tag{5}$$

where γ_j are the coupling constants, $a_j^\dagger$ and a_j are the quantum creation and annihilation operators, and z denotes the component of the dipole operator on the **OZ** axis. The free Hamiltonian of the reservoir is represented by

$$H_R=\hbar\sum_j\omega_j(a_j^\dagger a_j+\tfrac{1}{2}), \tag{6}$$

with ω_j the frequency corresponding to the jth mode of the free quantified radiation.

The von Neumann equation for the statistical operator ρ of the total system reads

$$i\hbar\,\frac{\partial\rho}{\partial t}=[H(t),\rho]. \tag{7}$$

We introduce the interaction representation to treat Eq. (7) and we set

$$H_0(t)=H_{A\text{-}L}(t)+H_R, \tag{8}$$

which is the time-dependent unperturbed Hamiltonian. The evolution operator corresponding to this Hamiltonian is given by

$$U(t)=U_0(t)\otimes U_R(t) \tag{9}$$

and

$$U(t)=\left[\exp\left\{-\frac{i}{\hbar}\int_0^t H_{A\text{-}L}(t')dt'\right\}\right]_+ \times\exp\left(-\frac{i}{\hbar}H_R t\right), \tag{10}$$

where $[\ ]_+$ is a time-ordering operator. In the interaction representation, the total density operator $\rho(t)$ and the interaction Hamiltonian H_1 take the forms

$$\tilde{\rho}(t)=U^\dagger(t)\rho(t)U(t) \tag{11}$$

and

$$\tilde{H}_I(t)=U^\dagger(t)H_I U(t) \tag{12}$$

and the dynamic equation (7) becomes

$$i\hbar\,\frac{\partial\tilde{\rho}(t)}{\partial t}=[\tilde{H}_I(t),\tilde{\rho}(t)]. \tag{13}$$

We assume that the interaction between the atomic system and the reservoir is weak so that the coupling constants $\gamma_j\to 0$ and $\gamma_j^2 t=\mathrm{const}$ for $t\to\infty$. Under these conditions, Eq. (13) will be treated by the time-dependent perturbation theory. At second order in $\tilde{H}_I$, this reads

$$\frac{\partial\tilde{\rho}(t)}{\partial t}=\frac{1}{i\hbar}[\tilde{H}_I(t),\tilde{\rho}(t_0)] -\frac{1}{\hbar^2}\int_{t_0}^t dt'[\tilde{H}_I(t),[\tilde{H}_I(t'),\tilde{\rho}(t')]]. \tag{14}$$

In writing Eq. (14) it has been assumed that the interaction is adiabatically switched on at time $t_0 \to -\infty$. Prior to this, the atomic system and the reservoir are uncorrelated and the total density matrix is given by the direct product

$$\tilde{\rho}(t_0) = \tilde{\sigma}(t_0) \otimes \rho_R, \tag{15}$$

where $\tilde{\sigma}(t_0)$ is the reduced system density operator at the initial time in the interaction representation and is defined by the trace over the reservoir states. ρ_R is the reservoir distribution function at equilibrium given by

$$\rho_R = \frac{\exp(-H_R/k_B T_R)}{Z_R}. \tag{16}$$

Here Z_R and T_R are, respectively, the partition function and the temperature of the reservoir and k_B is the Boltzmann constant.

We need to define the Hamiltonian of the interaction between the atomic system and the laser field without its coupling with the bath. In the dipole approximation it reads

$$V(t) = eF_0 z \cos[\omega t + \varphi(t) + x(t)] \tag{17}$$

in the case of phase fluctuations and

$$V(t) = eF_0[1 + x(t)] z \cos[\omega t + \varphi(t)] \tag{18}$$

in the case of amplitude fluctuations, where ω is the laser frequency and e is the electron charge. F_0 is the electric-field amplitude (possibly fluctuating in magnitude) and $\varphi(t)$ is the instantaneous phase of the laser (fluctuating around the mean value). We use an intense laser field affected by a temporal stochastic process of jumps. These fluctuation mechanisms are described by the pre-Gaussian Markovian models [7–10]. In particular, we adopt the simplest example of a two-state random telegraph, which is defined by $x(t) = \pm a$, where a is the amount of the jump assigned to the stochastic signal.

Since the telegraph process that we are considering here is Markovian, the conditioned probability density function associated with it, namely, $p(s,t|s_0,t_0)$, is shown to satisfy the Chapman-Kolmogorov equation [7–10,30]

$$\frac{\partial}{\partial t} p(s,t|s_0,t_0) = -\frac{1}{T} p(s,t|s_0,t_0) + \frac{1}{T} p(-s,t|s_0,t_0). \tag{19}$$

Here s_0 is the initial state of random telegraph at the time t_0. In compact form [31] Eq. (19) is written as

$$\frac{d\mathbf{P}_s}{dt} = \sum_{s'} \mathbf{W}_s^{s'} \mathbf{P}_{s'}, \tag{20}$$

where $\mathbf{W}_s^{s'} = (1/T)\left[\begin{smallmatrix} -1 & 1 \\ 1 & -1 \end{smallmatrix}\right]$ is the relaxation matrix composed of the frequencies of the telegraph jumps process and s and s' are two different states of the random telegraph ($s = 1,2$), corresponding to the telegraph signal amplitude $\{-a, +a\}$. T denotes the dwell time (i.e., the mean time between interruptions) of the telegraph. In the following, in the presence of the noise, all physical operators, such as the reduced density operator, interaction Hamiltonian, and dipole operator, will be denoted by the index s to indicate the stochasticity influence.

By elimination of the reservoir variables in Eq. (14) we have

$$\dot{\tilde{\sigma}}_s(t) = -\frac{1}{\hbar^2} \int_{t_0}^{t} dt' \, \mathrm{Tr}_R[\tilde{H}_{Is}(t), [\tilde{H}_{Is}(t'), \tilde{\sigma}_s(t')]]. \tag{21}$$

After tracing over the reservoir variables, we combine the Chapman-Kolmogorov equation (20), which represents the stochastic evolution of the random telegraph, with Eq. (21), which represents the atomic dynamics. A master equation for the reduced density operator is derived in the interaction picture

$$\dot{\tilde{\sigma}}_s(t) = \sum_{s'} \mathbf{W}_s^{s'} \tilde{\sigma}_{s'}(t) + \tilde{D}_s(t), \tag{22}$$

where

$$\tilde{D}_s(t) = -\frac{1}{\hbar^2} \int_{t_0}^{t} dt' \, \mathrm{Tr}_R[\tilde{H}_{Is}(t), \ [\tilde{H}_{Is}(t'), \tilde{\sigma}_s(t')]] \tag{23}$$

is the time-dependent operator describing the dissipative effects induced by the coupling to the reservoir. In writing the expression of $\tilde{D}_s(t)$ we have replaced $\tilde{\rho}_s(t) \approx \tilde{\sigma}_s(t) \otimes \rho_R$ up to second order in the coupling constants by its zeroth-order approximate. Since $\tilde{H}_{Is}(t)$ is a periodic function in time, we explicitly construct the operator $U_0(t)$. To this end, it is necessary to treat the interaction with the strong incoherent laser field exactly and solve the corresponding Schrödinger equation generated by the Hamiltonian $H_{A\text{-}L}(t)$ by using the Floquet theory. According to Floquet's theorem [25–29], there exists a complete set of solutions labeled by quantum numbers α of the form

$$|\psi_s^{\alpha}(t)\rangle = \exp(-i\epsilon_\alpha t/\hbar) |\phi_{\alpha,s}(t)\rangle, \tag{24}$$

where $\hbar\epsilon_\alpha$ and $|\phi_{\alpha,s}(t)\rangle$ are, respectively, the quasienergies and the eigenstates of Floquet theory. The time-evolution operator $U_0(t)$ for the matter degrees of freedom in the Floquet representation [1,2] is given by

$$U_0(t) = \exp(i\epsilon_\alpha t/\hbar) |\phi_{\alpha,s}(t)\rangle\langle\phi_{\alpha,s}(0)|. \tag{25}$$

In the interaction picture, the annihilation, creation, and dipole operators a_j, $a_j^{\dagger}$, and z, respectively take the forms

$$\tilde{a}_j = U^{\dagger}(t) a_j U(t) = \exp(-i\omega_j t) a_j, \tag{26}$$

$$\tilde{a}_j^\dagger = \exp(i\omega_j t) a_j^\dagger \tag{27}$$

and

$$\tilde{z}_s(t) = U^\dagger(t) z U(t) = \sum_{\alpha,\beta,k}{}' \exp[i\epsilon_{\alpha\beta}(k)t] Z_{\alpha\beta,s}(k) |\phi_{\alpha,s}(t)\rangle\langle\phi_{\alpha,s}(0)|, \tag{28}$$

where

$$Z_{\alpha\beta,s}(k) = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt\, \exp[-ik\omega t]\langle\phi_{\alpha,s}(t)|z|\phi_{\beta,s}(t)\rangle \tag{29}$$

are the dipole matrix elements between Floquet states. The prime in the sum of Eq. (28) indicates that only the triplets (α,β,k) that verify the condition $\epsilon_\alpha - \epsilon_\beta + k\hbar\omega > 0$ will be considered, with the purpose of eliminating the degenerate Floquet eigenstates. In order to calculate the dissipation operator $\tilde{D}_s(t)$, we follow the methodology formulated in Ref. [1]. We have

$$\tilde{H}_{Is}(t) = \sum_{\alpha,\beta,k}{}' \sum_j \gamma_j [e^{i[\epsilon_{\alpha\beta}(k)-\omega_j]t} F_{\alpha\beta,s} a_j + \text{H.c.}], \tag{30}$$

with H.c. the Hermitian conjugate,

$$F_{\alpha\beta,s} = |\phi_{\alpha,s}(0)\rangle\langle\phi_{\beta,s}(0)|, \tag{31}$$

and

$$\epsilon_{\alpha\beta}(k) = \epsilon_\alpha - \epsilon_\beta + k\hbar\omega, \tag{32}$$

where $-\infty < k < +\infty$. By taking into account Eq. (30), $\tilde{D}_s(t)$ reads

$$\begin{aligned}\tilde{D}_s(t) = \sum_{\alpha,\beta,k}{}' \sum_{\alpha',\beta',k'}{}' \sum_j \gamma_j^2 &\times \int_{t_0}^{t} dt' \{\langle a_j^\dagger a_j\rangle [A(t,t')F_{\alpha\beta,s}\tilde{\sigma}_s(t')F_{\beta\alpha,s}\\ &- A^+(t,t')F_{\beta\alpha,s}F_{\alpha'\beta',s}\tilde{\sigma}_s(t')]\\ &+ \langle a_j a_j^\dagger\rangle [A^+(t,t')F_{\beta\alpha,s}\tilde{\sigma}_s(t')F_{\alpha'\beta',s}\\ &- A(t,t')F_{\alpha\beta,s}F_{\beta'\alpha,s}\tilde{\sigma}_s(t')] + \text{H.c.}\},\end{aligned} \tag{33}$$

with

$$\begin{aligned}A(t,t') = Z_{\alpha\beta,s}(k) Z^*_{\beta'\alpha',s}(k') \exp\{i[\epsilon_{\alpha\beta}(k) - \omega_i]t\}\\ \times \exp\{i[\epsilon_{\alpha'\beta'}(k') - \omega_j]t'\}\end{aligned} \tag{34}$$

and

$$\langle a_j^\dagger a_j\rangle = N(\omega_j) = \left[\exp\left(\frac{\hbar\omega_j}{k_B T_R}\right) - 1\right]^{-1}. \tag{35}$$

$N(\omega_j)$ is the photon number operator. Since the degrees of freedom of the bath are infinite, we can make the substitution $\Sigma_j \gamma_j^2 \cdots \rightarrow \int d\omega\, J(\omega)\cdots$, where $J(\omega)$ is a function that is proportional to the bath spectral density. To perform integration, which is present in the expression of $\tilde{D}_s(t)$ [Eq. (33)], further conditions must be imposed on the reservoir to prevent the energy, initially in the atomic system, from returning from the heat bath to the system in any finite time, i.e., the coupling of the reduced system to the reservoir must be treated as an irreversible process. At this stage, we make two approximations.

(i) Equation (33) contains $\tilde{\sigma}_s(t')$ in the integral and hence the behavior of the atomic system depends on its history from $t' = t_0$ to t. The motion of the atomic system is, however, damped by the coupling to the reservoir and damping destroys the knowledge of the past behavior of the system. Therefore, the first assumption is that $\dot{\tilde{\sigma}}(t)$ depends only on its present value $\tilde{\sigma}_s(t)$ (Markovian approximation) [1,2].

(ii) Let us consider an operator **B** of the bath and its time correlation function $\langle \mathbf{B}(t-t')\mathbf{B}^+\rangle$. The reservoir is assumed to be large and Markovian. Thus it is expected that $\langle \mathbf{B}(t-t')\mathbf{B}^+\rangle$ will be nonzero for some time interval $t - t' < \tau_R$, where τ_R is the correlation time of the reservoir. Interactions at times t and t' become progressively less correlated for $t - t' \gg \tau_R$. The correlation function $\langle \mathbf{B}(t-t')\mathbf{B}^+\rangle$ is maximum at $t = t'$. Therefore the upper bound of integration in Eq. (33) tends to infinity $(t\rightarrow\infty)$.

With these two approximations and using the expression for the initial time $t_0 \rightarrow -\infty$,

$$\int_{-\infty}^{+\infty} dt' \exp\{i[\omega - \epsilon_{\alpha'\beta'}(k')]t'\} = 2\pi\delta(\omega - \epsilon_{\alpha'\beta'}(k')), \tag{36}$$

the integro-differential equation reads

$$\begin{aligned}\tilde{D}_s(t) = 2\pi \sum_{\alpha,\beta,k}{}' \sum_{\alpha',\beta',k'}{}' J[\epsilon_{\alpha'\beta'}(k')]\\ \times (N[\epsilon_{\alpha'\beta'}(k')][A'(t)F_{\alpha\beta,s}\tilde{\sigma}_s(t')F_{\beta\alpha,s}\\ - A'^+(t)F_{\beta\alpha,s}F_{\alpha'\beta',s}\tilde{\sigma}_s(t)] + \{1 + N[\epsilon_{\alpha'\beta'}(k')]\}\\ \times [A'^+(t)F_{\beta\alpha,s}\tilde{\sigma}_s(t)F_{\alpha'\beta',s}\\ - A'(t)F_{\alpha\beta,s}F_{\beta'\alpha',s}\tilde{\sigma}_s(t)] + \text{H.c.}),\end{aligned} \tag{37}$$

where

$$A'(t) = \exp\{i[\epsilon_{\alpha\beta}(k) - \epsilon_{\alpha'\beta'}(k)]t\} Z_{\alpha\beta,s}(k) Z^*_{\alpha'\beta',s}(k'). \tag{38}$$

This last quantity is maximal for

$$\epsilon_{\alpha\beta}(k) - \epsilon_{\alpha'\beta'}(k') = 2n\pi, \tag{39}$$

where n is a positive or negative integer. For the case of $n=0$ only terms such as $(\alpha,\beta,k)=(\alpha',\beta',k')$ must be kept in Eq. (37). Equation (33) takes the final form

$$\begin{aligned}\tilde{D}_s(t)=\sum_{\alpha,\beta,k}{}' \ \Omega_{\alpha\beta}(k)(N[\epsilon_{\alpha\beta}(k)]\{[F_{\alpha\beta,s}\tilde{\sigma}_s(t),F_{\beta\alpha,s}]\\ +[F_{\alpha\beta,s},\tilde{\sigma}_s(t)F_{\beta\alpha,s}]\}+\{1+N[\epsilon_{\alpha\beta}(k)]\}\\ \times\{[F_{\beta\alpha,s}\tilde{\sigma}_s(t),F_{\alpha\beta,s}]+[F_{\beta\alpha,s},\tilde{\sigma}_s(t)F_{\alpha\beta,s}]\}),\end{aligned} \tag{40}$$

with

$$\Omega_{\alpha\beta}(k)=2\pi J[\epsilon_{\alpha\beta}(k)]|Z_{\alpha\beta,s}(k)|^2. \tag{41}$$

By projecting on the Floquet basis $\{|\phi_{\alpha,s}(0)\rangle\}$, the master equation for the diagonal and the off-diagonal elements $\tilde{\sigma}_{\alpha\alpha,s}(t)$ and $\tilde{\sigma}_{\alpha\beta,s}(t)$, respectively, read

$$\begin{aligned}\dot{\tilde{\sigma}}_{\alpha\alpha,s}(t)=\sum_{s'} W_s^{s'}\tilde{\sigma}_{\alpha\alpha,s'}(t)\\ +\sum_{\gamma}{}' \ [M_{\gamma\alpha}\tilde{\sigma}_{\gamma\gamma,s}(t)-M_{\alpha\gamma}\tilde{\sigma}_{\alpha\alpha,s}(t)]\end{aligned} \tag{42}$$

and

$$\begin{aligned}\dot{\tilde{\sigma}}_{\alpha\beta,s}(t)=\sum_{s'} W_s^{s'}\tilde{\sigma}_{\alpha\beta,s'}(t)\\ -\tfrac{1}{2}\left[\sum_{\gamma}{}' \ (M_{\alpha\gamma}+M_{\beta\gamma})\right]\tilde{\sigma}_{\alpha\beta,s}(t),\end{aligned} \tag{43}$$

where the coefficients $M_{\alpha\beta}$ are defined by

$$\begin{aligned}M_{\alpha\beta}=2\sum_k (\{1+N[\epsilon_{\alpha\beta}(k)]\}\Omega_{\alpha\beta}(k)\\ +N[\epsilon_{\alpha\beta}(k)]\Omega_{\beta\alpha}(k))\end{aligned} \tag{44}$$

and their solutions are given by

$$\tilde{\sigma}_{\alpha\alpha,s}(t)=\sum_{\beta,s'} [\exp(-\Lambda 1 t)]_{\alpha\beta,ss'}\tilde{\sigma}_{\beta\beta,s'}(0) \tag{45}$$

and

$$\tilde{\sigma}_{\alpha\beta,s}(t)=\sum_{\gamma,s'} [\exp(-\Lambda 2 t)]_{\alpha\gamma,ss'}\tilde{\sigma}_{\gamma\beta,s'}(0), \tag{46}$$

where

$$\Lambda 1_{\alpha\beta,ss'}=-W_s^{s'}\delta_{\alpha\beta}-\left(M_{\beta\alpha}-\delta_{\alpha\beta}\sum_{\eta} M_{\alpha\eta}\right)\delta_{ss'} \tag{47}$$

and

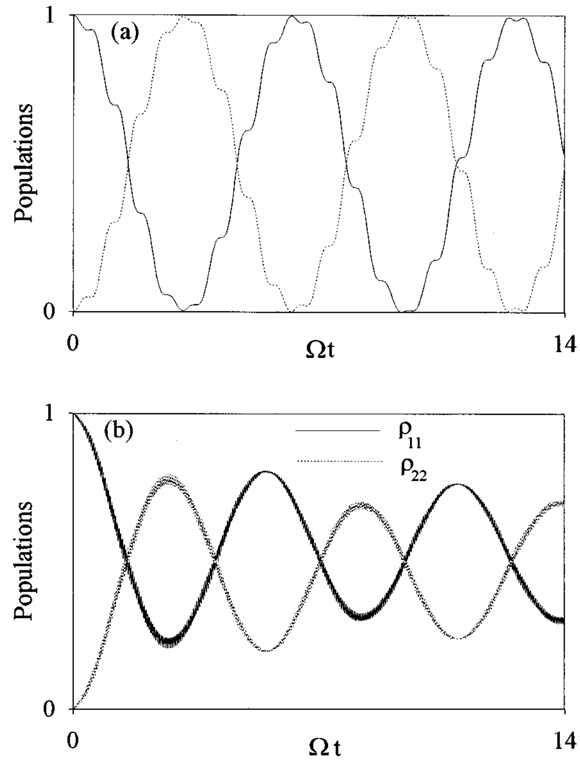

FIG. 1. Populations σ_{nn} versus time (in units of the inverse Rabi frequency Ω) for two-level atoms resonantly excited by an intense electric laser field such that the Rabi frequency $\Omega=1$ a.u. The emission spontaneous coefficient $\gamma_{21}=\Omega/10^5$ a.u. (a) The effects of both noise and dissipation are neglected (coherent laser and no coupling to the reservoir). (b) Noise is absent and dissipation is considered.

$$\Lambda 2_{\alpha\gamma,ss'}=-W_s^{s'}\delta_{\alpha\gamma}+\tfrac{1}{2}\delta_{\alpha\gamma}\delta_{ss'}\left(\sum_{\eta}{}' \ (M_{\alpha\eta}+M_{\beta\eta})\right). \tag{48}$$

The theoretical expressions for populations and coherence of quasienergie states, Eqs. (45) and (46), respectively, have to be transformed back into the atomic basis, which yields σ_{nn} and $\sigma_{nn'}$. In the Schrödinger picture we then obtain

$$\dot{\sigma}_s(t)=-\frac{i}{\hbar}[H_0(t),\sigma_s(t)]+\sum_{s'} W_s^{s'}\sigma_{s'}(t)+D_s(t), \tag{49}$$

with

$$D_s(t)=U_0\tilde{D}_sU_0^+, \tag{50}$$

$$\begin{aligned}D_s(t)=\sum_{\alpha,\beta,k}{}' \ \Omega_{\alpha\beta}(k)(\{1+N[\epsilon_{\alpha\beta}(k)]\}\\ \times\{[R^+_{\alpha\beta,s}(t),\sigma_s(t)R_{\alpha\beta,s}(t)]\\ +[R^+_{\alpha\beta,s}(t)\sigma_s(t),R_{\alpha\beta,s}(t)]\}\\ +N[\epsilon_{\alpha\beta}(k)]\{[R_{\alpha\beta,s}(t),\sigma_s(t)R^+_{\alpha\beta,s}(t)]\\ +[R_{\alpha\beta,s}(t)\sigma_s(t),R^+_{\alpha\beta,s}(t)]\}),\end{aligned} \tag{51}$$

where

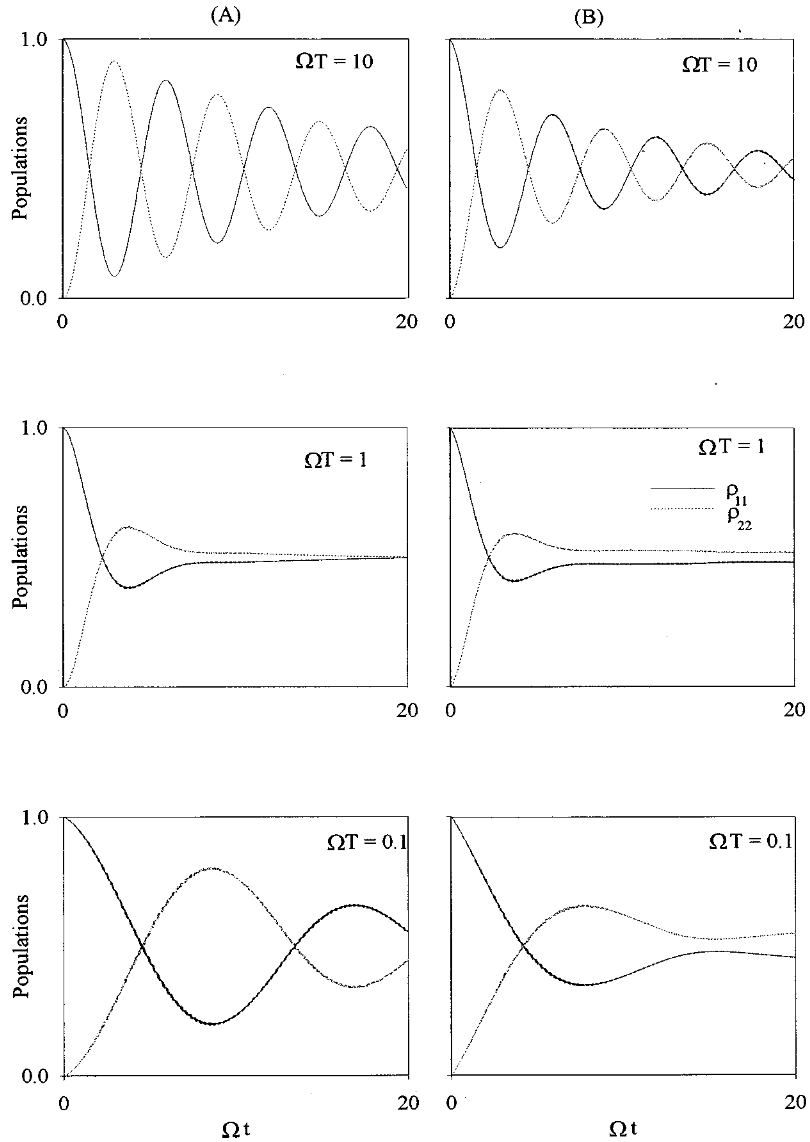

FIG. 2. Populations σ_{nn} versus time (in units of the inverse Rabi frequency Ω) for two-level atoms resonantly excited by random telegraph phase noise. Successive frames are for different values of the phase switching rates $\Omega T = 0.1$, 1, and 10. We use a strong laser field such that the Rabi frequency $\Omega = 1$ a.u. The phase jump parameter $a = 0.4\pi$ and the emission spontaneous coefficient $\gamma_{21} = \Omega/10^5$ a.u. Column (A) represents the effects of phase noise. Column (B) represents the same situation as column (C), but with the dissipative effects.

$$
\begin{aligned}
R_{\alpha\beta,s}(t) &= U_0(t) F_{\alpha\beta,s} U_0^{+}(t) \\
&= \exp[-i(\epsilon_\alpha - \epsilon_\beta)] | \phi_{\alpha,s}(t) \rangle \langle \phi_{\beta,s}(t) | .
\end{aligned} \tag{52}
$$

The main difficulty of typical problems lies in the correct averaging of the matrix density over all realizations of noise. In fact, what is physically wanted is $\langle \sigma_{nn'} \rangle$, that is, the solution to the master equation in the atomic states and averaged over the ensemble of jumps of the implicit telegraph $x(t)$. To obtain $\langle \sigma_{nn'} \rangle$ one proceeds indirectly, by defining a marginal average $\sigma_{nn',s}(t)$, given by the equation

$$
\langle \sigma_{nn'} \rangle = \sum_s g(s) \sigma_{nn',s} , \tag{53}
$$

where $g(s)$ is the initial probability distribution of the random process and $\sigma_{nn',s}(t)$ the average value of $\sigma_{nn'}(t)$ under the condition that $x(t)$ is fixed at the value s at time t. By projecting on the atomic basis $\{|n\rangle\}$, the master equation in the Schrödinger picture finally reads

$$
\begin{aligned}
\dot{\sigma}_{mn,s}(t) = \sum_{s'} W_s^{s'} \sigma_{mn,s'}(t) \\
&+ \sum_{m'} \left(D1_{m'n,s}(t) + \frac{i}{\hbar} H_0(t)_{m'n,s}(t) \right) \sigma_{mm',s}(t) \\
&+ \sum_{m'} \left(D1_{mm',s}(t) - \frac{i}{\hbar} H_0(t)_{mm',s} \right) \sigma_{mm',s}(t) \\
&+ \sum_{m',n'} D2_{mm'n'n,s}(t) \sigma_{m'n',s}(t),
\end{aligned} \tag{54}
$$

with the two terms responsible for the dissipation defined by

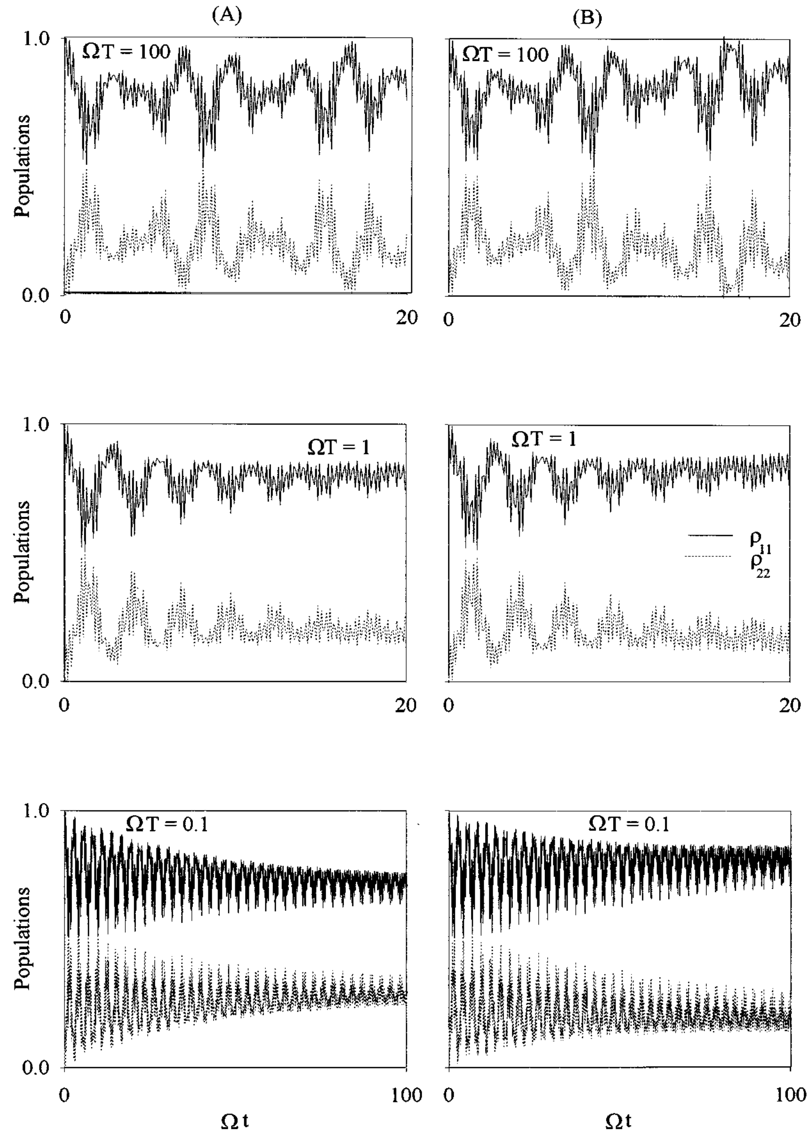

FIG. 3. Same as Fig. 2, but for an amplitude noise with the jump parameter $a=0.1$ a.u. and three switching rates $\Omega T=0.1$, 1, and 100.

$$D1_{mn,s}(t)=-\sum_{\alpha,\beta,k}{}' \; \Omega_{\alpha\beta}(k) \times(\{1+N[\epsilon_{\alpha\beta}(k)]\}\phi_{m\alpha,s}(t)\phi^{\dagger}_{n\alpha,s}(t) +N[\epsilon_{\alpha\beta}(k)]\phi_{m\beta,s}(t)\phi^{\dagger}_{n\beta,s}) \tag{55}$$

and

$$D2_{mm'n'n,s}(t)=2\sum_{\alpha,\beta,k}{}' \; \Omega_{\alpha\beta}(k)(\{1+N[\epsilon_{\alpha\beta}(k)]\} \times\phi_{m\beta,s}(t)\phi^{\dagger}_{m'\alpha,s}(t)\phi_{n'\alpha,s}(t)\phi^{\dagger}_{n,\beta,s}(t) +N[\epsilon_{\alpha\beta}(k)]\phi_{m\alpha,s}(t)\phi^{\dagger}_{m'\beta,s}(t) \times\phi_{n'\beta,s}(t)\phi^{\dagger}_{n\alpha,s}(t)), \tag{56}$$

where $\phi_{n\alpha,s}(t)$ are the Floquet states, which are projected on the atomic basis $\{|n\rangle\}$.

It is important to note that the general master equation (54) contains dissipative terms (55) and (56) that explicitly depend on time. This is the main difference from the usual optical Bloch equations. The physical interpretation of this fact is the strong distortion of the atomic dipole moment, which is induced by the external laser field. Since the atom couples to the environment via its dipole moment, the laser field also strongly influences the dissipation process [2].

III. RESULTS AND DISCUSSION

In this section we gather typical numerical results for the excitation and ionization of two-level atoms by strong laser fields in the presence of noise and dissipation mechanisms. To illustrate the effects of dissipation and laser noise on the atomic response, we present the evolution of atomic populations $\sigma_{nn}(t)$, which are obtained by numerical integration of the master equation (54). Our theoretical formalism is valid for the general case of multilevel systems, but in order to keep the discussion simple we will restrict our application to the two-level atoms for which a detailed study of the dissipative nonlinear dynamics will be presented. Particular attention will be paid to the case of a strong laser field, where the

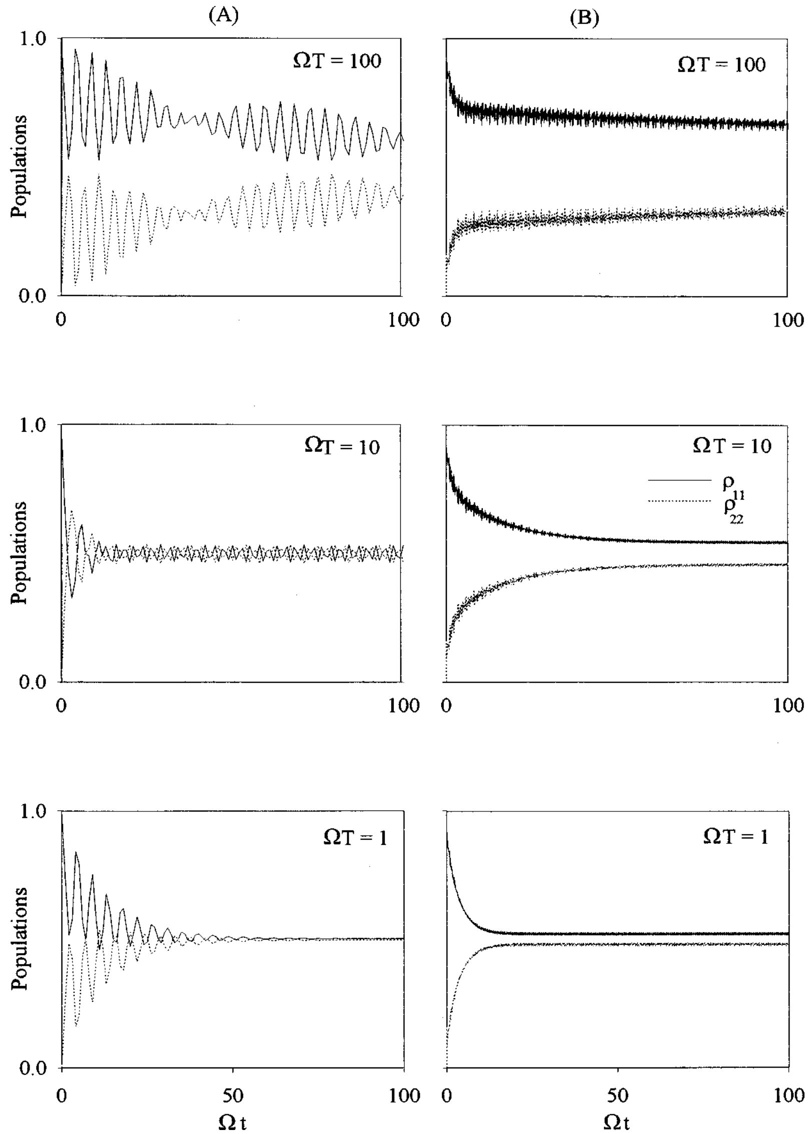

FIG. 4. Same as Fig. 3, but for frequency noise.

dipole operator is taken between the Floquet eigenstates $\{|\phi_{\alpha,s}(t)\rangle\}$ rather than between unperturbed atomic states $\{|n\rangle\}$.

We have established the effects of strong laser noise on the atomic response and explored several features of different sources of noise (phase, amplitude, and frequency). We now concentrate our attention on the examination of quantum dissipation induced by coupling to the reservoir and when the noise is added to the laser field. We choose the inverse Rabi frequency Ω as the time unit to analyze the results obtained in terms of the noise magnitude. We are interested in a large light intensity such that the Rabi frequency is set equal to the atomic unit ($\Omega = 1$ a.u.). This certainly is a very strong intensity.

A. Dissipative nonlinear quantum dynamics in the excitation of two-level systems

We begin by representing the effects of quantum dissipation on the atomic response. Figures 1(a) and 1(b) show the time evolution of populations of a two-level atomic system driven by a strong *coherent laser field* sufficiently intense to remove a significant fraction of the population from the atomic ground state. One might think that the only consequence of a field this intense would be to lower the overall response of the atom. The optical field is nearly resonant with the allowed transition between discrete states of the two-level atom. In the absence of spontaneous emission decay and ionization effects, Fig. 1(a) represents the atomic response without dissipation. The atomic system oscillates between the ground state $|1\rangle$ and some other discrete level $|2\rangle$ and we have ordinary Rabi oscillations. In Fig. 1(b) the dissipation effect introduces a damping of Rabi oscillations. If damping effects are present, we expect that the Rabi oscillations will eventually become damped out and the population's difference will approach some steady-state value for a large scale of time. Hence Rabi oscillations are not present in the steady state.

It is interesting to note the presence of an irregular behavior of the oscillations of the two populations for a strong laser field. In fact, we observe small oscillations that superpose the Rabi oscillations; their amplitude is weak and disappears when the electric field strength F_0 becomes small with respect to the atomic unit of the field strength. These little oscillations represent the fast nonrotating variable phases $\exp[\pm i(\omega+\omega_{nn'})t]$ in the laser-atom interaction, which is treated in a nonperturbative way (Floquet theory).

Figure 2 shows some typical results. We take into account both laser phase noise and dissipation influences. We have a phase jump $a=0.4\pi$ and three switching rates $\Omega T=0.1$, 1, and 10. We display the time evolution populations in two columns. In column (A) only the effect of laser noise is considered. We remark, in this case, that for a switching rate $\Omega T=1$, i.e., the noise frequency $1/T$ is of the same size order as the Rabi frequency Ω, a destruction of the atomic coherence is observed. The damping rate is strong and the relaxation to the steady state is rapid. The Rabi oscillations are restored when we consider the case of slow fluctuations ($\Omega T=10$) and fast fluctuations ($\Omega T=0.1$) and the damping rate is weak. Column (B) represents the same situation, but with the laser noise of the dissipative terms [see Eqs. (55) and (56)]. The behavior is similar to that of column (A), but the damping rate is more intense. For a switching rate $\Omega T=0.1$, a partial destruction of the atomic coherence is induced by the dissipation effects. The kinetics of the population relaxation is more rapid than in column (A). One of the effects of the quantum dissipation is the breaking of the atomic coherence especially for $\Omega T=0.1$ and the establishment of the dynamical localization regime for $\Omega T=1$. In Fig. 3 we plot the time evolution of two-level atom populations in two situations: In the first we neglect the effect of quantum dissipation [column (A)] and consider only an amplitude telegraph noise and in the second we combine the influences of noise and quantum dissipation. We have an amplitude jump $a=0.1$ a.u. and three different switching rates $\Omega T=0.1$, 1, and 100. Column (A) shows pronounced quasioscillations. We note a very weak damping at $\Omega T=1$ and 100 and rapid relaxation for $\Omega T=0.1$ with respect to $\Omega T=1$ and 100. In order to lead the system to the steady state we must use a larger number of Rabi periods than in the case of phase fluctuations. Column (B) shows behavior similar to that in column (A). The complicated structure of these curves is a consequence of the action of the amplitude laser noise on the reduced system dynamics. In fact, one observes a separation between the two occupation probabilities. Each population $\sigma_{11}(t)$ and $\sigma_{22}(t)$ performs independently irregular oscillations, which converge to a stationary state. As a constraint on the phase, the addition of dissipation in Fig. 3 [column (B)] introduces a weakness of the damping.

Figure 4 illustrates the case of frequency fluctuations. This kind of noise is introduced by collisional effects. The transition frequency ω_{21} can also fluctuate around its fixed value. The simplest model of such interruption collisions [9,11] assumes that the atomic transition frequency ω_{21} should be replaced by $\omega_{21}(t)=\omega_{21}+x(t)$. We have a jump parameter $a=0.1$ a.u. and three different frequency switching rates $\Omega T=1$, 10, and 100. Note the damped quasiperiodic oscillations in column (A). $\Omega T=10$ corresponds to strong damping without any convergence to a steady state. The relaxation to an equilibrium state of value $\frac{1}{2}$ is clear for a switching rate $\Omega T=1$. The damping becomes weak for $\Omega T=100$ and two independent beat phenomena are observed. The complicated time evolution of populations is a

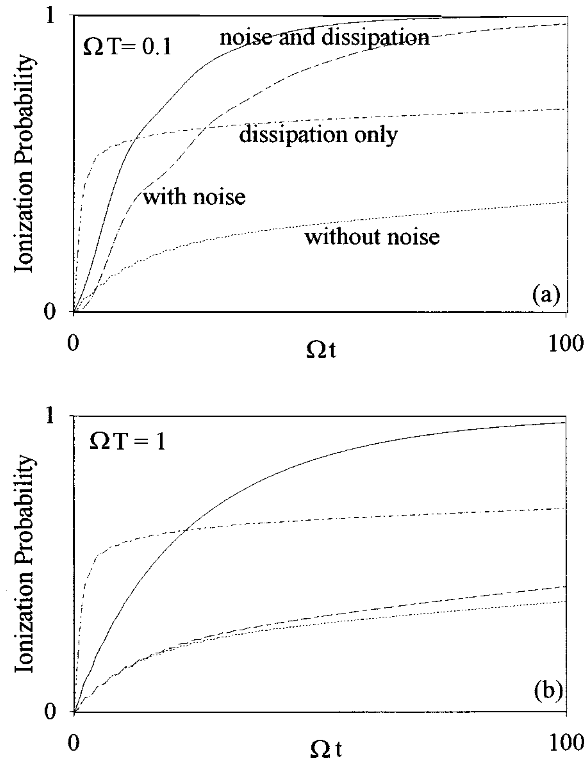

FIG. 5. Ionization probability $P_{\rm ion}(t)$ versus time (in units of the inverse Rabi frequency Ω). The parameters are the same as in the previous figures. The relaxation rate from bound states to the continuum is $R_{2c}=\Omega/100$. (a) Dotted line, $P_{\rm ion}(t)$ in the absence of noise and dissipation; dashed line, only the effect of phase noise for $\Omega T=0.1$ is considered; dash-dotted line, only the effect of dissipation is considered; solid line, the effects of both noise and dissipation are considered. (b) Same as (a), but for an amplitude noise with $\Omega T=1$.

result of Rabi oscillation interference. In column (B), where we take into account quantum dissipation, the two populations relax to an equilibrium state. The thermal noise induced by coupling to the bath introduces a complete destruction of the atomic coherence. The kinetics of relaxation and rate damping decreases from the case of fast fluctuations ($\Omega T=0.1$) to slow fluctuations ($\Omega T=100$).

In Figs. 2 and 4 the comparison between columns (A) and (B) shows that the dissipation that behaves as noise (thermal noise) leads the system to an equilibrium state with rapid kinetics of relaxation. The damping rates become large when we introduce the dissipation terms [column (B)]. We clearly see the destruction of atomic coherence, which increases when we take into account dissipation. The dynamical localization regime appears for phase and frequency noises. However, Fig. 3 shows important asymmetries. This behavior is justified by the fact that in the case of amplitude fluctuations, the jump parameter a, assigned to stochastic processes appears in terms of the laser intensity $F_0(1\pm a)$, while in the case of phase noise the dependence occurs in terms of $\exp(\pm ia)$.

B. Dissipative nonlinear quantum dynamics in the ionization of two-level systems

We have established a formal framework for the excitation of atoms by laser noise in the presence of the reservoir

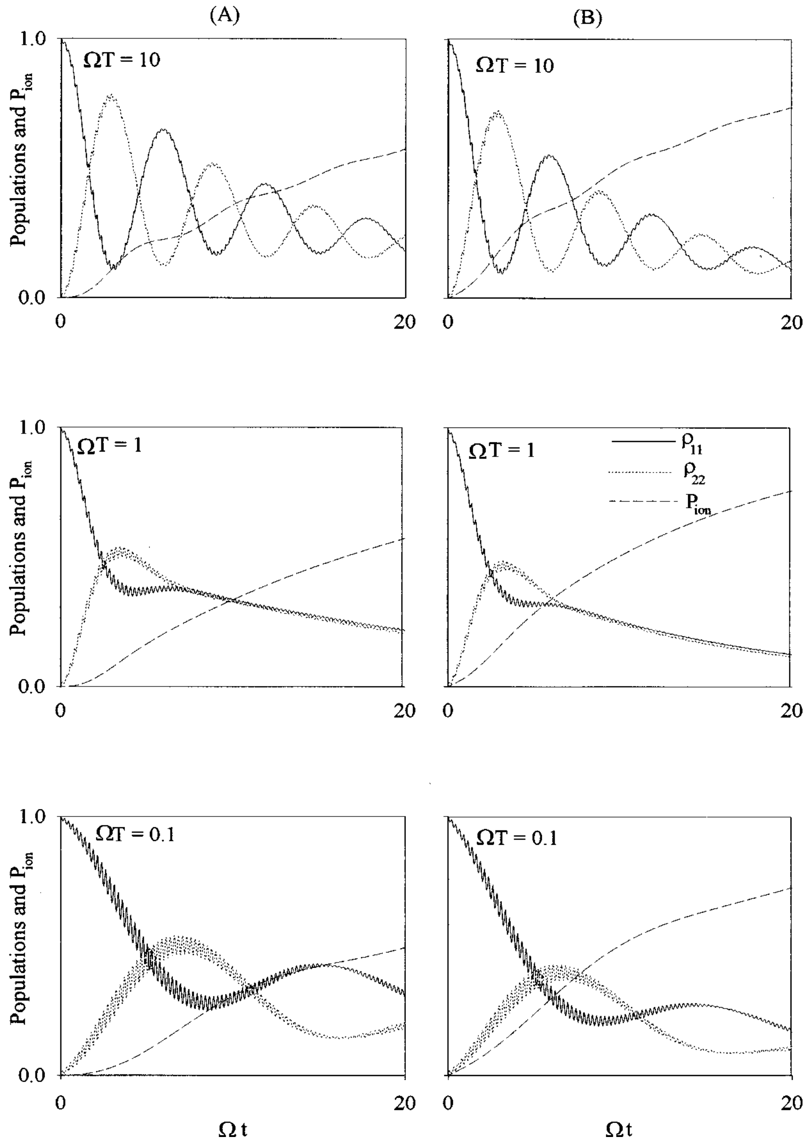

FIG. 6. Columns (A) and (B) are the same as Figs. 2(a) and 2(b), respectively, but take into account the ionization process represented by the probability $P_{\rm ion}(t)$. The relaxation rate from bound states to the continuum is $R_{2c}=\Omega/100$.

action and explored some of its general predictions. We turn now to the examination of ionization effects on the populations and the illustration of the modifications generated by the different kinds of noise and quantum dissipation on the ionization rates. In order to analyze the ionization effects we adopt the extended two-level system model proposed by Yeh and Eberly [14,32]. The computation of ionization probability is made by the incorporation of the term responsible for ionization $[-\tau_{EC}\sigma_{mn,s}$, with $\tau_{EC}=R_{mc}\delta_{mn}+1/2(R_{mc}+R_{nc})(1-\delta_{mn})$ and R_{nc} the relaxation rate from the excited state $|n\rangle$ to the continuum $|c\rangle]$ in the motion equation (54). The trace of σ over a complete set of atomic states leads to the expression $P_{\rm ion}(t)=1-\Sigma_{n=1}^{2}\sigma_{nn}$ for the total ionization probability of the system [4,13,33].

We begin by showing successively the effects of laser noise and reservoir dissipation on the ionization probability. As illustrated in Fig. 5, we have plotted the total ionization as a function of Rabi periods. We have an intense electric laser field such as $\Omega=1$ a.u. and a resonant laser frequency. The four curves of Fig. 5 correspond respectively to the situations where noise and dissipation are neglected, only the noise is considered, only the dissipation effect is retained, and both noise and dissipation exist. In Fig. 5(a) we have considered a phase noise of $a=0.4\pi$. The results depend on the fluctuations time scale $(1/T)$ compared to the other characteristic time scales of the problem such as the Rabi frequency Ω. The minimum variations of the ionization probability are obtained, where we neglect both noise and dissipation. When we take into account dissipation, the ionization probability increases. For a large Rabi period, the phase fluctuation effect is more important than the reservoir action. In fact, the ionization probability rapidly increases when we introduce phase fluctuations corresponding to $\Omega T=0.1$. In the presence of dissipation terms, the ionization probability variations remain almost constant. We conclude that noise and dissipation rapidly lead the atom to the ionization states. Figure 5(b) shows the amplitude noise, where the jump parameter $a=0.1$ a.u. and a switching rate $\Omega T=1$. The behavior is similar to that observed in Fig. 5(a), but the amplitude noise effect is very weak. The technique

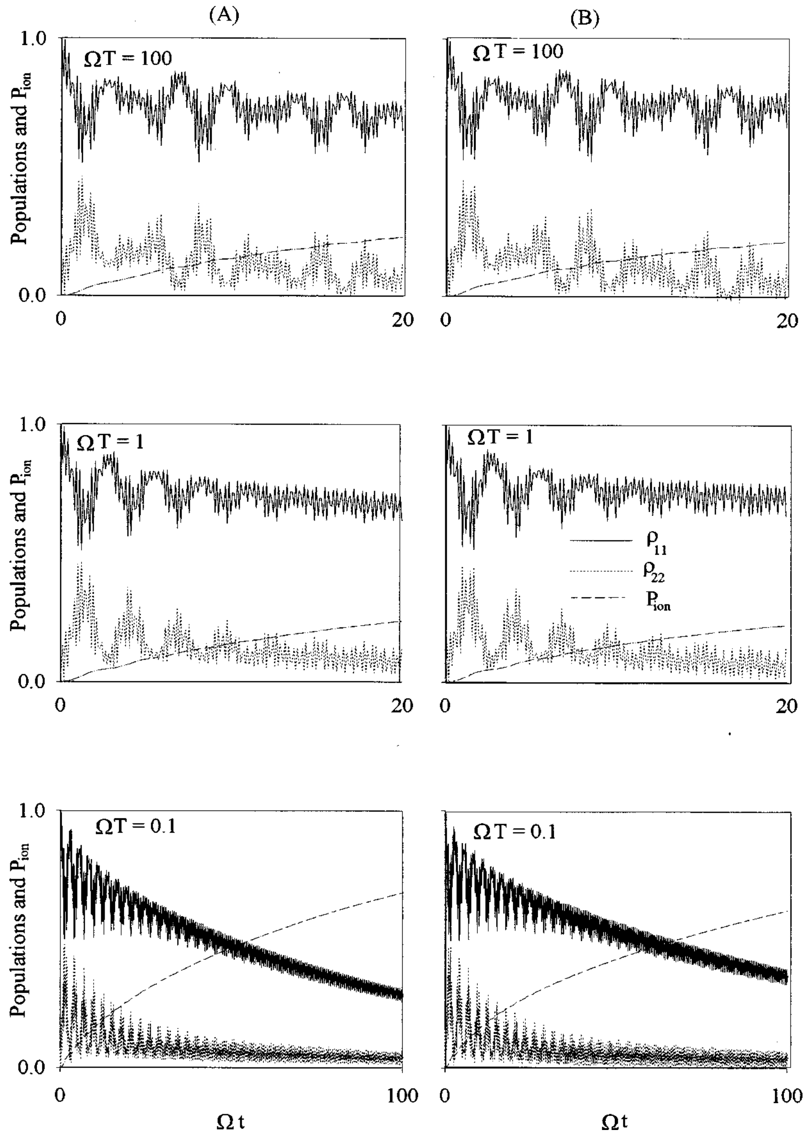

FIG. 7. Columns (A) and (B) are the same as Figs. 3(A) and 3(B), respectively, but take into account the ionization process represented by the probability $P_{\mathrm{ion}}(t)$. The relaxation rate from bound states to the continuum is $R_{2c}=\Omega/100$.

that we have used will be applied subsequently to frequency noise.

A plot that gives a pictorial sense of how ionization proceeds in time is given in Fig. 6. The total probability of ionization $P_{\mathrm{ion}}(t)$ as a function of the Rabi period is plotted together with the occupation probabilities of bound states. The oscillations in these curves reflect the Rabi oscillations of the atom between the resonantly coupled states $|1\rangle$ and $|2\rangle$. These oscillations are damped by ionization on a small number of Rabi frequencies. This behavior is well known from the study of bound states coupled by an intense field. Column (A) of Fig. 6 shows the response of a two-level system in the presence of phase noise and [column (B)] by considering the effect of dissipation. The parameters are the same as in Fig. 2. The populations, which have not been lost through direct ionization to the atomic continuum, oscillate in the same manner as in the absence of the ionization effect, but there is a progressive decay to a zero probability. The ionization probability can be viewed as dominant in a few Rabi periods and increases rapidly in time when the quantum dissipative effects are considered. Figure 7 displays the same behavior as in Fig. 6, but for an amplitude noise.

IV. CONCLUSION

In this paper we have investigated at length the nonlinear dynamics of dissipative quantum atomic systems subjected to the action of a heat bath and periodically driven by a strong laser field that is affected by classical noise. We have derived and solved a master equation for atoms in strong noisy laser fields and in the presence of reservoir dissipative effects. Such an equation, based on the Floquet states rather than the unperturbed atomic states, has given typical and interesting results concerning the atomic dynamics. In fact, we have demonstrated how the master equation formalism, the Floquet theory, the pre-Gaussian models of laser noise, and the Markovian coupling of a quantum system to an environment can be combined together to tackle a general theoretical formalism and be used as a powerful tool for the detailed analysis of the interaction of an atomic system with an intense incoherent laser field and with a large reservoir.

We have shown that the dissipation terms, which are time dependent with respect to those in the Bloch equations, force the system to settle to some ''preferred state,'' that is, the dynamical localization regime observed in the cases of phase and frequency noises, as we have explored in this paper. Moreover, under the action of these decay mechanisms, the atomic system exhibits different regimes such as the destruction of coherence and the relaxation to equilibrium state. In general, the strength of damping and the kinetics of relaxation increase with the addition of dissipation effects, but the amplitude fluctuations show a different behavior. We have also analyzed the modifications induced by ionization effects.

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