

# Analysis of dynamical suppression of spontaneous emission

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It has been shown recently [see, for example, S.-Y. Zhu and M. O. Scully, Phys. Rev. Lett. **76**, 388 (1996)] that a dynamical suppression of spontaneous emission can occur in a three-level system when an external field drives transitions between a metastable state and *two* decaying states. What is unusual in the decay scheme is that the decaying states are coupled directly by the vacuum radiation field. It is shown that the decay dynamics required for total suppression of spontaneous emission necessarily implies that the level scheme is isomorphic to a three-level  $\Lambda$  system, in which the lower two levels are *both* metastable and each is coupled to the decaying state. As such, the total suppression of spontaneous emission can be explained in terms of conventional dark states and coherent population trapping. [S1050-2947(98)00812-9]

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## I. INTRODUCTION

Following the work of Fontana and Srivastava [1], Agarwal [2], Cardimona, Raymer, and Stroud [3], and Zhu and Scully [4], a number of articles have appeared containing proposals for suppressing spontaneous emission [5–12]. In contrast to the suppression of spontaneous emission that one can achieve by placing an atom in a cavity whose radiation modes differ from those of free space, it is suggested in these articles that spontaneous emission in free space can be suppressed by applying an external radiation field to an atom having a specified level scheme. This is a rather remarkable result since one might imagine that, owing to the very short correlation time of the vacuum field, such modification of spontaneous emission rates would be strictly forbidden. A prototypical level scheme that leads to suppression of spontaneous emission is that of Zhu and Scully [4] (see Fig. 1). Two excited states  $|2\rangle$  and  $|3\rangle$  are separated in frequency by  $\omega_{32}$ . These states decay to the ground state  $|0\rangle$  with rates  $\Gamma_2$  and  $\Gamma_3$ , respectively. What makes the decay scheme somewhat unusual is that states  $|2\rangle$  and  $|3\rangle$  are *coupled directly by the vacuum field*. An external radiation field couples an auxiliary, *metastable* state  $|1\rangle$  to both states  $|2\rangle$  and  $|3\rangle$ . For certain values of the field strength and atom-field detunings, it is found that one can have a nonvanishing, significant, steady-state probability for the atom to be in states  $|2\rangle$  or  $|3\rangle$ . As such, spontaneous emission from these levels is suppressed by the presence of the driving field. Xia *et al.* [13] claim to have observed this effect in an experiment on sodium dimers.

The suppression of spontaneous emission has been explained in terms of a dressed state of the atom-field system that is decoupled from the vacuum radiation field [3,4,8,10,11,13]. How is this decoupling accomplished? Is there any underlying structure in the proposed level schemes that can help one to understand this most surprising result? It is the purpose of this article to address these questions. By considering a model problem I will show that states  $|2\rangle$  and  $|3\rangle$  can be viewed as superpositions of two states, *one of which is metastable*. It is this metastable state that is necessary for the total suppression of spontaneous emission. Moreover, the decay dynamics required for total suppression of spontaneous emission implies that the level scheme of

Fig. 1 is isomorphic to a three-level  $\Lambda$  system. The lower two levels of the  $\Lambda$  system are *both* metastable and each is coupled to the decaying state. As such, the total suppression of spontaneous emission can be explained in terms of conventional dark states and coherent population trapping [14]. The experiment of Xia *et al.* [13] will also be discussed. While the level scheme they study is relevant to this class of problems, the results they obtained cannot be classified as a suppression of spontaneous emission.

## II. EQUATIONS OF MOTION

In the absence of the driving field, the equations for the evolution of the state amplitudes  $a_2$  and  $a_3$  given by Zhu and Scully [4] are

$$\dot{a}_2 = i(\omega_{32}/2)a_2 - \gamma_2 a_2 - \gamma_{3,2} a_3, \tag{1a}$$

$$\dot{a}_3 = -i(\omega_{32}/2)a_3 - \gamma_3 a_3 - \gamma_{3,2} a_2, \tag{1b}$$

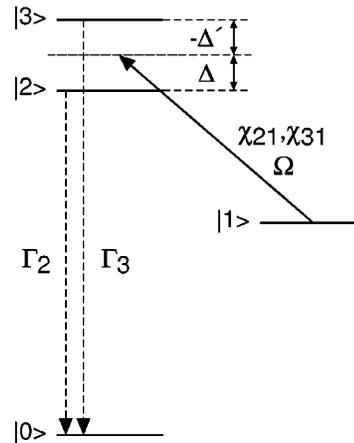


FIG. 1. Level scheme proposed by Zhu and Scully (see Ref. [4]) to observe total suppression of spontaneous emission. The driving field having frequency  $\Omega$  couples state  $|1\rangle$  to both states  $|2\rangle$  and  $|3\rangle$ . Spontaneous emission is totally suppressed if  $\Delta\chi_{31}^2 + \Delta'\chi_{21}^2 = 0$ , where  $\chi_{21}$  and  $\chi_{31}$  are the Rabi frequencies associated with the 1-2 and 1-3 transitions, respectively, and  $\Delta \equiv \Omega - \omega_{21}$  and  $\Delta' \equiv \Omega - \omega_{31}$ .

where

$$\gamma_2 = \Gamma_2/2, \quad \gamma_3 = \Gamma_3/2, \quad \gamma_{3,2} = \sqrt{\gamma_2 \gamma_3}. \quad (2)$$

The first question we must ask is whether or not these equations correctly describe the interaction of an atom with the vacuum radiation field. The answer to this question is not obvious. If we consider the energy levels shown in Fig. 1 to be those of an isolated atom in free space, we immediately run into some problems. From the dipole selection rules, it is easy to show that the vacuum coupling from state  $|2\rangle$  to state  $|3\rangle$  must conserve orbital, spin-orbit, and total angular momenta  $\mathbf{L}$ ,  $\mathbf{J}$ , and  $\mathbf{F}$ , as well as the  $z$  component of total angular momentum. As a consequence, states  $|2\rangle$  and  $|3\rangle$  must belong to different electronic state manifolds. This in turn implies that  $\omega_{32}$  corresponds to a frequency that is orders of magnitude larger than the decay rates  $\Gamma_2$  and  $\Gamma_3$ , respectively. The rapid oscillation of state amplitudes  $a_2$  and  $a_3$  with frequency  $\omega_{32}$  brings into question the validity of the Weisskopf-Wigner approximation used for the derivation of Eqs. (1). There is perhaps a more subtle point involved. Starting from state  $|2\rangle$ , one can emit a photon taking the atom to state  $|0\rangle$ , reabsorb this photon taking the atom to virtual state  $|3\rangle$ , reemit a photon taking the atom to state  $|0\rangle$ , and reabsorb this photon returning the atom to state  $|2\rangle$ . This overall process constitutes an  $\alpha^5$  (Rydberg) contribution to the Lamb shift of state  $|2\rangle$ . Consequently, if the atomic states are renormalized to include the Lamb shift, it is questionable as to whether the vacuum coupling between states  $|2\rangle$  and  $|3\rangle$  should be included in Eqs. (1) [15].

It thus appears unlikely that one can achieve the vacuum coupling indicated in Eqs. (1) if these states correspond to eigenstates of a free, isolated atom, dressed by the vacuum field. On the other hand, it is possible to achieve this vacuum coupling if states  $|2\rangle$  and  $|3\rangle$  correspond to eigenstates of an atom plus some external field or, in some cases, to the states of a molecule [15]. The most obvious atom candidate is a hydrogen atom in a static electric field [16]. States  $|2\rangle$  and  $|3\rangle$  could then be chosen as linear combinations of the  $2S$  and  $3P$  states of hydrogen. The idea of using a hydrogen atom in a static electric field to modify the spontaneous emission spectrum is not new. Zhu and Scully mention it in their 1996 article [4] and Fontana and Srivastava gave a detailed analysis of the decay in their 1973 article [1]. Alternatively, one could use a level scheme similar to that used by Xia *et al.* [13], in which states  $|2\rangle$  and  $|3\rangle$  are superposition of singlet and triplet states in a molecule. I will return to the experiment of Xia *et al.* in Sec. III.

In order to gain additional insight into this problem, I consider the level scheme shown in Fig. 2. States  $|0\rangle$ ,  $|0'\rangle$ ,  $|b\rangle$ ,  $|d\rangle$ , and  $|1\rangle$  are eigenstates of an unperturbed Hamiltonian. States  $|b\rangle$  and  $|d\rangle$  have opposite parity and are coupled by a constant potential  $\hbar V$  [16]. An external radiation field, having frequency  $\Omega$ , couples state  $|1\rangle$ , which is assumed to be metastable, to state  $|b\rangle$  only. States  $|b\rangle$  and  $|d\rangle$  decay to states  $|0\rangle$  and  $|0'\rangle$  with rates  $\Gamma_b$  and  $\Gamma_d$ , respectively. (Note that this level scheme could correspond to hydrogen with state  $|b\rangle$  corresponding to  $|n=2, L=1, m_\ell=0\rangle$ , state  $|d\rangle$  to  $|n=2, L=0, m_\ell=0\rangle$ , and states  $|0\rangle$ ,  $|0'\rangle$ , and  $|1\rangle$  to  $|n=1, L=0, m_\ell=0\rangle$  [17]. In this case,  $\Gamma_d \approx 0$  for the  $2S$  state.) The goal of this calculation is to show that the

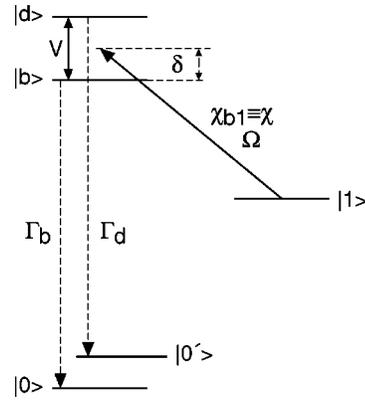


FIG. 2. Level scheme equivalent to that in Fig. 1 under conditions of total suppression of spontaneous emission. A static field  $V$  couples states  $|b\rangle$  and  $|d\rangle$  and a driving field having frequency  $\Omega$  couples state  $|1\rangle$  to state  $|b\rangle$  only, with associated Rabi frequency  $\chi$ . Spontaneous emission is totally suppressed if the detuning  $\delta \equiv \Omega - \omega_{b1} = \omega_{db}$  and  $\Gamma_d = 0$ .

level scheme of Fig. 2, a level scheme exhibiting conventional decay dynamics, can be mapped onto that of Fig. 1, a level scheme exhibiting somewhat unconventional decay dynamics. Thus the suppression of spontaneous emission can equally well be analyzed using the level schemes of Fig. 1 or 2. It will be seen that the suppression of spontaneous emission can be explained in terms of conventional dark states when the level scheme of Fig. 2 is used. The mapping between the two level schemes is achieved by identifying states  $|2\rangle$  and  $|3\rangle$ , appearing in Fig. 1, as eigenstates of the unperturbed Hamiltonian associated with Fig. 2, plus the potential  $\hbar V$ .

In the absence of decay, the effective Hamiltonian for the level scheme of Fig. 2, in the rotating wave approximation, in an interaction representation, and with the energy of level  $b$  taken equal to zero, can be written as [18]

$$\mathbf{H}_0 = \hbar \begin{pmatrix} \delta & \chi & 0 \\ \chi & 0 & V \\ 0 & V & \delta' \end{pmatrix}, \quad (3)$$

where the order of the states is  $|1\rangle, |b\rangle, |d\rangle$ ,  $\chi$  is a Rabi frequency (taken to be real), and [16]

$$\delta = \Omega - \omega_{b1}, \quad \delta' \equiv \omega_{db}. \quad (4)$$

States  $|0\rangle$  and  $|0'\rangle$  have not been included in Eq. (3) since they are not needed for the present discussion of the decay dynamics of the excited states.

The Hamiltonian (3) can be diagonalized without much difficulty; however, the desired comparison between the level schemes of Figs. 1 and 2 is achieved by diagonalizing the  $(b, d)$  subspace only. The new eigenstates are given by

$$|2\rangle = c|b\rangle - s|d\rangle, \quad (5a)$$

$$|3\rangle = c|d\rangle + s|b\rangle, \quad (5b)$$

where

$$c \equiv \cos \theta = \sqrt{\frac{1}{2} \left( 1 + \frac{\delta'}{R_A} \right)}, \quad s \equiv \sin \theta,$$

$$\tan 2\theta = 2V/\delta', \quad R_A = \sqrt{\delta'^2 + 4V^2}. \quad (5c)$$

In terms of these eigenstates, the transformed Hamiltonian takes the form

$$\mathbf{H}'_0 = \hbar \begin{pmatrix} \delta - \frac{\delta'}{2} & c\chi & s\chi \\ c\chi & \frac{-R_A}{2} & 0 \\ s\chi & 0 & \frac{R_A}{2} \end{pmatrix}, \quad (6)$$

where the order of the states is  $|1\rangle, |2\rangle, |3\rangle$  and a constant energy  $\hbar \delta'/2$  has been subtracted from the energy of each state. The equations of motion for the state amplitudes are

$$\dot{a}_1 = -i \left( \delta - \frac{\delta'}{2} \right) a_1 - ic\chi a_2 - is\chi a_3, \quad (7a)$$

$$\dot{a}_2 = i(R_A/2)a_2 - ic\chi a_1, \quad (7b)$$

$$\dot{a}_3 = -i(R_A/2)a_3 - is\chi a_1. \quad (7c)$$

It is now a simple matter to include decay into these equations. Since spontaneous decay is governed by  $\dot{a}_b = -\gamma_b a_b$  and  $\dot{a}_d = -\gamma_d a_d$ , where

$$\gamma_b = \Gamma_b/2, \quad \gamma_d = \Gamma_d/2, \quad (8)$$

and since  $a_2 = ca_b - sa_d$  and  $a_3 = ca_d + sa_b$ , it follows that Eqs. (7), including decay, can be written as

$$\dot{a}_1 = -i \left( \delta - \frac{\delta'}{2} \right) a_1 - ic\chi a_2 - is\chi a_3, \quad (9a)$$

$$\dot{a}_2 = -\gamma_2 a_2 - \gamma_{3,2} a_3 + i(\omega_{32}/2)a_2 - ic\chi a_1, \quad (9b)$$

$$\dot{a}_3 = -\gamma_3 a_3 - \gamma_{2,3} a_2 - i(\omega_{32}/2)a_3 - is\chi a_1, \quad (9c)$$

where

$$\gamma_2 = c^2 \gamma_b + s^2 \gamma_d, \quad (10a)$$

$$\gamma_3 = c^2 \gamma_d + s^2 \gamma_b, \quad (10b)$$

$$\gamma_{3,2} = \gamma_{2,3} = sc(\gamma_b - \gamma_d), \quad (10c)$$

$$\omega_{32} = R_A. \quad (10d)$$

This form of the equations is *almost* identical to that used in theories of suppression of spontaneous decay [compare with Eq. (1)] based on the level scheme of Fig. 1. For the equations to be *identical* and for the level schemes of Figs. 1 and 2 to be isomorphic, one must require that

$$\gamma_{3,2} = \sqrt{\gamma_2 \gamma_3}. \quad (11)$$

It follows from Eqs. (10) that the only way this equation can be satisfied is to have  $\gamma_d = 0$ . In other words, the form of the vacuum coupling given in Eqs. (1) for the level scheme of Fig. 1 in theories of total suppression of spontaneous emission *necessarily implies that state  $|d\rangle$  of the equivalent level scheme of Fig. 2 must be metastable.*

Since both states  $|d\rangle$  and  $|1\rangle$  are metastable and do not undergo spontaneous emission in the isolated atom, it is reasonable to ask whether or not the level scheme of Fig. 1 legitimately qualifies to be labeled as one in which spontaneous emission has been suppressed. In order to determine if the driving field suppresses spontaneous emission, one must first establish that spontaneous emission of states  $|2\rangle$  and  $|3\rangle$  always occurs in the *absence* of the driving field. Setting  $\chi = 0$  in Eqs. (9), one finds that the only steady-state solution is  $a_2 = a_3 = 0$ . Any initial-state population in states  $|2\rangle$  and  $|3\rangle$  decays if the driving field is absent. This is easily understood in terms of the original  $|b\rangle, |d\rangle$  basis; although state  $|d\rangle$  is metastable, it is coupled to the decaying state  $|b\rangle$  by the potential  $\hbar V$ . No matter how weak the coupling strength  $V$ , any initial population in state  $|d\rangle$  eventually leaks out via state  $|b\rangle$ .

Does the presence of the driving field suppress this spontaneous emission? The answer to this question is affirmative if the initial state is an arbitrary superposition of states  $|2\rangle$  and  $|3\rangle$  and their remains population trapped in states  $|2\rangle$  and  $|3\rangle$  as the time approaches infinity. An initial condition in which the atom is in state  $|1\rangle$ , corresponding to the initial condition in the experiment of Xia *et al.* [13], cannot be used directly to establish total suppression of spontaneous emission, but can provide indirect evidence for this effect, as discussed in Sec. III below.

Having established that state  $|d\rangle$  must be metastable to satisfy the requirements for spontaneous emission suppression, it is now an easy matter to understand the total suppression of spontaneous emission by returning to the original Hamiltonian (3). An inspection of this Hamiltonian reveals that it is identical to a Hamiltonian, written in a field interaction representation [18], that characterizes a three-level atom in a  $\Lambda$  scheme driven by two fields. The field having Rabi frequency  $\chi$  and detuning  $\delta = \Omega - \omega_{b1}$  drives the  $1-b$  transition and the field having Rabi frequency  $V$  and detuning  $\delta' \equiv \omega_{bd}$  drives the  $b-d$  transition [16]. Total suppression of spontaneous emission occurs if one can find an eigenstate consisting of a superposition of state amplitudes  $a_1$  and  $a_d$  that is decoupled from state amplitude  $a_b$ . This *dark state* [14] does not decay since it is a superposition of nondecaying states. In other words, we seek values of  $\alpha$  and  $\beta$  for which the superposition of state amplitudes of the form

$$a_I = \alpha a_1 + \beta a_d \quad (12)$$

satisfies the equation of motion

$$\dot{a}_I = -i\omega_I a_I. \quad (13)$$

From Schrödinger's equation with the Hamiltonian (3) it follows that

$$\dot{a}_I = \alpha \dot{a}_1 + \beta \dot{a}_d = -i(\alpha\chi + \beta V)a_b - i(\alpha\delta a_1 + \beta\delta' a_d). \quad (14)$$

Equation (13) can be satisfied only if

$$\alpha\chi + \beta V = 0, \quad (15)$$

$$\delta = \delta' \equiv \omega_{db}, \quad (16)$$

which implies that  $\omega_I = \delta$ . The driving field must be tuned to the frequency that would correspond to a ‘‘hole’’ in the emission spectrum from the 2-3 state manifold [1]. Thus, if  $\delta = \omega_{bd}$  there always exists a dark state amplitude of the system

$$a_I = \frac{Va_1 - \chi a_d}{R_B}, \quad (17)$$

where

$$R_B = \sqrt{V^2 + \chi^2}, \quad (18)$$

which does not decay. The other eigenstate amplitudes

$$a_{II} = \frac{\chi a_1 - (R_D + \delta/2)a_b + Va_d}{\sqrt{(R_D + \delta/2)^2 + R_B^2}}, \quad (19a)$$

$$a_{III} = \frac{\chi a_1 + (R_D - \delta/2)a_b + Va_d}{\sqrt{(R_D - \delta/2)^2 + R_B^2}}, \quad (19b)$$

where

$$R_D = \sqrt{R_B^2 + (\delta/2)^2}, \quad (20)$$

contain an admixture of state amplitude  $a_b$  and decay as  $t \sim \infty$ . As a consequence, any initial condition for which  $a_I(0) \neq 0$  has a metastable component that does not decay as the time approaches infinity.

It remains only to establish that an initial condition of the form  $|\psi(0)\rangle = a_2(0)|2\rangle + a_3(0)|3\rangle$  leads to a final state that has some population trapped in states  $|2\rangle$  and  $|3\rangle$  (or, equivalently, in state  $|d\rangle$ ). As  $t \sim \infty$ , the solution for the total dressed-state amplitudes  $a_I, a_{II}, a_{III}$  is  $a_I(t) \sim a_I(0)e^{-i\delta t}$ ,  $a_{II}(t) \sim 0$ , and  $a_{III}(t) \sim 0$ , which when reexpressed in terms of the bare-state initial conditions [with  $a_1(0) = 0$ ] is  $a_I(t) \sim -(\chi/R_B)a_d(0)e^{-i\delta t}$ . The final-state populations are

$$|a_1(\infty)|^2 = \left( \frac{\chi V}{V^2 + \chi^2} \right)^2 |a_d(0)|^2, \quad (21a)$$

$$|a_b(\infty)|^2 = 0, \quad (21b)$$

$$|a_d(\infty)|^2 = \left( \frac{\chi^2}{V^2 + \chi^2} \right)^2 |a_d(0)|^2, \quad (21c)$$

$$\begin{aligned} |a_2(\infty)|^2 &= s^2 |a_d(\infty)|^2 \\ &= \frac{1}{2} \left( 1 - \frac{\omega_{db}}{\sqrt{\omega_{db}^2 + 4V^2}} \right) \left( \frac{\chi^2}{V^2 + \chi^2} \right)^2 |a_d(0)|^2, \end{aligned} \quad (21d)$$

$$\begin{aligned} |a_3(\infty)|^2 &= c^2 |a_d(\infty)|^2 \\ &= \frac{1}{2} \left( 1 + \frac{\omega_{db}}{\sqrt{\omega_{db}^2 + 4V^2}} \right) \left( \frac{\chi^2}{V^2 + \chi^2} \right)^2 |a_d(0)|^2, \end{aligned} \quad (21e)$$

where Eqs. (4), (5c), and (18) were used. Thus we see that population is always trapped in states  $|2\rangle$  and  $|3\rangle$ .

Equations (21) for the probabilities  $|a_1(\infty)|^2$ ,  $|a_2(\infty)|^2$ , and  $|a_3(\infty)|^2$  can be written in terms of the couplings and detunings in the  $|1\rangle, |2\rangle, |3\rangle$  basis. Referring to Eq. (6) and using Eqs. (4), (5c), (10d), (16), and (18), one finds the appropriate relationships,  $\omega_{db} = (\Delta + \Delta')$ ,  $V = \sqrt{-\Delta\Delta'}$ ,  $\chi^2 = \chi_{21}^2 + \chi_{31}^2$ ,  $a_d = ca_3 - sa_2$ ,  $c = \sqrt{\Delta/\omega_{32}}$ , and  $s = \sqrt{-\Delta'/\omega_{32}}$ , subject to the constraint  $\Delta\chi_{31}^2 + \Delta'\chi_{21}^2 = 0$ . Under conditions of total suppression of spontaneous emission, the field is tuned to the energy of the metastable level  $d$ , that is,  $\Delta = \omega_{d2} > 0$  and  $\Delta' = \omega_{d3} < 0$ .

As an aside, I note that the results can be reinterpreted as a suppression of absorption [3,19] if one starts with all population initially in state  $|1\rangle$ , for which  $a_I(t) \sim (V/R_B)a_1(0)e^{-i\delta t}$  as  $t \sim \infty$ . In the absence of the coupling potential  $\hbar V$ , the steady-state population  $|a_1(\infty)|^2$  would vanish, but it does not vanish in the presence of this coupling. In this sense, it is closely related to electromagnetically induced transparency [20].

### III. DISCUSSION

It has been shown that the origin of the suppression of spontaneous emission proposed by Zhu and Scully [4] and others [7,8,10–12] can be traced to a metastable state that is ‘‘hidden’’ in their calculations. Once this hidden state is revealed, the suppression of spontaneous emission can be understood in terms of a conventional dark state and coherent population trapping [14] that can arise when an atom having a three-level  $\Lambda$  scheme is driven by two fields. The dark state in this instance is a superposition of two metastable states and so is itself metastable. The dynamical suppression of spontaneous emission is a real effect. If the external driving field  $\chi$  were not present, the two state manifold consisting of states  $|2\rangle$  and  $|3\rangle$  would always decay. In some sense, the driving field allows one to access the metastable level  $|d\rangle$  contained in both states  $|2\rangle$  and  $|3\rangle$ . This type of dynamical suppression could be used, for example, to reduce spontaneous emission in the  $2S$ - $2P$  manifold of hydrogen resulting from stray fields that couple the  $S$  state to the  $P$  state. It would be necessary to drive the  $2S$ - $2P$  transition using a rf field and the  $1S$ - $2P$  with an uv field having frequency  $\Omega = \omega_{2S,1S} - \Omega_{rf}$  [16].

The use of the equivalent  $|1\rangle, |b\rangle, |d\rangle$  basis rather than the  $|1\rangle, |2\rangle, |3\rangle$  basis greatly simplifies the interpretation of the results. From the analysis of Sec. II it is clear that the final-

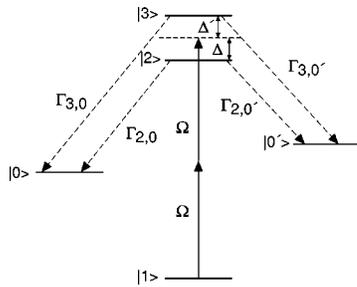


FIG. 3. Level scheme used in the experiment of Xia *et al.* (Ref. [13]).

state probabilities depend only on the initial-state amplitude  $a_I(0) = [Va_1(0) - \chi a_d(0)]/R_B$  and not on the decay rate if state  $|b\rangle$  decays to state  $|0\rangle$  only. If state  $|b\rangle$  decays to state  $|1\rangle$  as well as to state  $|0\rangle$  or if states  $|1\rangle$  and  $|0\rangle$  actually correspond to the same state, the final-state probabilities are modified, but the steady state still corresponds to a dark state for which there is total suppression of absorption. On the other hand, if state  $|1\rangle$  is not metastable, there cannot be total suppression of spontaneous emission since the dark state amplitude  $a_I(t) = [Va_1(t) - \chi a_d(t)]/R_B$  decays to zero as  $t \sim \infty$ .

Finally I should like to discuss the experiment of Xia *et al.* [13], who used the level scheme shown in Fig. 3, corresponding to molecular states in the sodium dimer. States  $|2\rangle$  and  $|3\rangle$  are superpositions of singlet and triplet states that are mixed by a spin-orbit interaction. In the spirit of this calculation, one can associate the singlet and triplet states with states  $|b\rangle$  and  $|d\rangle$ , respectively, in the Hamiltonian (3) and the spin-orbit mixing with the potential  $\hbar V$ . Of course, it is not possible to “turn off” the mixing potential in this case. The singlet component of states  $|2\rangle$  and  $|3\rangle$  decays to state  $|0\rangle$  and the triplet component of states  $|2\rangle$  and  $|3\rangle$  decays to state  $|0'\rangle$ , while the singlet component of states  $|2\rangle$  and  $|3\rangle$  is driven by a two-photon transition from the ground state. Since *both* the singlet and triplet components decay, the conditions for suppression of spontaneous emission are not met (recall that it was necessary that state  $|d\rangle$ , which corresponds to the triplet state, be metastable).

In their experiment, Xia *et al.* are not measuring spontaneous emission, as it is normally defined. Instead, they are

measuring *scattering* via the *three-photon* process in which two photons are absorbed from the driving field and a vacuum photon is emitted taking the atom to state  $|0\rangle$  (singlet channel) or  $|0'\rangle$  (triplet channel). They found that, for a tuning of the incident field midway between levels  $|2\rangle$  and  $|3\rangle$ ,  $2\Omega = (\omega_{31} + \omega_{21})/2$ , the scattering in the singlet channel was suppressed and that in the triplet channel was enhanced. This constitutes strong evidence that states  $|2\rangle$  and  $|3\rangle$  are coupled directly by the vacuum field and that the singlet and triplet states are degenerate in the absence of the spin-orbit coupling i.e.,  $\omega_{db} = 0$  [21]. Although this experiment is important insofar as it provides an example of a system in which vacuum coupling of two, distinct excited states occurs, it does not demonstrate suppression of spontaneous emission. There will be no steady-state population in states  $|2\rangle$  and  $|3\rangle$ . On the other hand, Xia *et al.* have shown that scattering in a specific channel can be suppressed.

As was noted above, total suppression of absorption occurs under the same conditions as total suppression of spontaneous emission, so that the existence of one implies the other. Consequently, if one can demonstrate total suppression of absorption, the system will also exhibit total suppression of spontaneous emission. To establish total suppression of absorption, one can either (i) prove that there is no absorption of the driving field or (ii) show that there is no scattered radiation for *all* polarizations and directions of the scattered field. The existence of scattered radiation in the triplet channel in the experiment of Xia *et al.* necessarily implies that there is *not* total suppression of absorption. On the other hand, the absorption rate from the ground state is decreased by a factor  $2\gamma_d/(\gamma_d + \gamma_b)$  relative to that which would have occurred if states  $|2\rangle$  and  $|3\rangle$  were not coupled by the vacuum field. Consequently, one can say that the spontaneous emission rate or the absorption rate is decreased in this system if  $\gamma_d(\text{triplet}) \ll \gamma_b(\text{singlet})$ . The data seem to indicate that  $\gamma_d$  and  $\gamma_b$  are comparable.

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