Quantum teleportation of a field state

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We consider the teleportation of a field state, which is in a coherent superposition of 2^n Fock states, from one cavity to another, and present an experimentally feasible scheme for this purpose.
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An interesting application of quantum nonlocality is quantum teleportation. Bennett *et al.* [1] proposed a scheme for teleporting an unknown quantum state from one observer to another through duel Einstein-Podolsky-Rosen (EPR) and classical channels. In this scheme, the sender and receiver prepare an entangled pair of states. The sender makes a joint measurement of the unknown quantum state with the EPR particle, and transmits the classical result of this measurement to the receiver. During this process the unknown quantum state is destroyed at the receiver's end, but the knowledge of the joint measurement enables the receiver to convert his EPR particle into an exact replica of the unknown quantum state.

Since this proposal by Bennett *et al.*, a number of experimentally feasible proposals have been made for the teleportation of two-level atomic states [2–6]. All these schemes rely on methods based on cavity quantum electrodynamics in which two identical high-Q cavities are initially prepared in a quantum-entangled state. Quantum teleportation of states of dynamical variables with continuous spectra has also been studied [7]. Experimental verifications of quantum teleportation have been reported recently by producing pairs of entangled photons by the process of parametric down conversion [8].

In this paper, we consider the more general question of teleporting an arbitrary field state from a high-Q cavity to another high-Q cavity. We first discuss the general considerations for the teleportation of an arbitrary superposition of states between two observers. We then propose a viable scheme for the teleportation of a field state, which is a superposition of Fock states, from one cavity to another using methods based on cavity quantum electrodynamics.

We consider the teleportation of the state of the radiation field which is a coherent superposition of N Fock states, i.e.,

$$|\psi(A)\rangle=\sum_{l=0}^{N-1} w_l|l\rangle_A \tag{1}$$

from a cavity A to cavity C. The method we describe here is valid only if $N=2^n$ (with n being an integer). The teleportation of state (1) can be done in three steps.

In the first step, we prepare another cavity B and the cavity C in the quantum-entangled state,

$$|\psi(B,C)\rangle=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}|N-1-k\rangle_B|k\rangle_C. \tag{2}$$

The combined state of the fields in cavities A, B, and C is therefore given by

$$|\Psi(A,B,C)\rangle=\frac{1}{\sqrt{N}}\sum_{j,k=0}^{N-1} w_j|j\rangle_A|N-1-k\rangle_B|k\rangle_C. \tag{3}$$

The N^2 basis states for the A-B system can be written as

$$|\psi_{n,m}(A,B)\rangle=\frac{1}{\sqrt{N}}\sum_{j=0}^{N-1} e^{2\pi inj/N}|j\rangle_A \times|(N-1-j-m)\mathrm{mod}\ N\rangle_B, \tag{4}$$

where $n,m=0,1,\ldots,(N-1)$. The combined state $|\Psi(A,B,C)\rangle$ can then be rewritten as a linear superposition of the basis states $|\psi_{n,m}(A,B)\rangle$ of the A-B system as follows:

$$|\Psi(A,B,C)\rangle=\sum_{j,k,l=0}^{N-1} e^{-2\pi ilj/N}w_l|\psi_{j,k}(A,B)\rangle| \times(k+l)\mathrm{mod}\ N\rangle_C. \tag{5}$$

In the second step we make a measurement of the A-B system. A detection of the A-B system in the state $|\psi_{j,k}(A,B)\rangle$ projects the field state in the cavity C into

$$|\psi(C)\rangle=\sum_{l=0}^{N-1} e^{-2\pi ilj/N}w_l|(l+k)\mathrm{mod}\ N\rangle_C. \tag{6}$$

The field state in the cavity C has thus been projected to a state which has all the information about the amplitudes w_n. In the third and final step of the quantum teleportation, a manipulation of the cavity C needs to be done to bring state (6) to form (1).

We now show how these steps of quantum teleportation can be accomplished using the known and experimentally accessible methods based on atom-field interaction. The first step is to produce the entangled state for the cavities B and C. Several schemes have been proposed in the literature for quantum-state preparation inside the cavity. These include methods based on atom-field state entanglement [9,10] and quantum-state mapping between multilevel atoms and cavity field [11]. The method by Vogel, Akulin, and Schleich [9], though flexible in implementation, is statistical in the sense that only a particular sequence of measurements on atomic states after they have interacted with the field leads to the desired state. On the other hand, the method by Law and Eberly [10], by using a more complicated scheme, has the advantage that an arbitrary quantum state of the radiation field can be generated inside a cavity at a prechosen time. In the method of Parkins *et al.* [11] the atomic ground-state Zeeman coherence is mapped onto the field state inside the cavity using adiabatic passage. The number of Zeeman levels available limits the number of photons in the target state.

It appears that no systematic method of generating arbitrary entangled states in two or more cavities has been considered so far. Here we propose a generalization of the method by Vogel, Akulin, and Schleich to generate the entangled state (2). We send $N-1$ two-level atoms in the excited state through the cavities B and C which interact with resonant modes of the electromagnetic field inside the two cavities via the Jaynes-Cummings Hamiltonian. The field inside the two cavities is initially in the vacuum state. The interaction times for the atoms with the field can be controlled by controlling the velocity of the atoms. We thus have a choice of $2N-2$ interaction parameters, two for each atom in the cavities B and C. These parameters can be chosen such that if all the atoms are found in the ground state after the passage through the cavities, the entangled state (2) will be generated. If, however, any one of the $N-1$ atoms is found to be in the excited state after the passage through the cavities, we will be required to *empty* the cavities and start all over again.

If the first $j-1$ atoms are found to be in the ground state $|b\rangle$ of the two-level atoms after the passage through the pair of cavities, and the resulting entangled field state in the two cavities is found to be

$$|\psi^{(j-1)}(B,C)\rangle=\sum_{n,m=0}^{j-1} c_{nm}^{(j-1)}|n\rangle_B|m\rangle_C, \tag{7}$$

then the field state projected after the passage of the jth atom initially in the excited state $|a\rangle$ through the two cavities and detected subsequently in its ground state is given by

$$\begin{aligned}|\psi^{(j)}(B,C)\rangle=-i\sum_{n,m=0}^{j-1} c_{nm}^{(j-1)}[&\cos(g\tau_{j_B}\sqrt{n+1})\sin(g\tau_{j_C}\sqrt{m+1})|n\rangle_B|m+1\rangle_C+\sin(g\tau_{j_B}\sqrt{n+1})\cos(g\tau_{j_C}\sqrt{m})\\&\times|n+1\rangle_B|m\rangle_C],\end{aligned} \tag{8}$$

where g is the vacuum Rabi frequency and τ_{j_B} and τ_{j_C} are the passage times for the jth atom inside the cavities B and C, respectively.

As an example, we consider a superposition of only four states, i.e.,

$$|\psi(A)\rangle=w_0|0\rangle_A+w_1|1\rangle_A+w_2|2\rangle_A+w_3|3\rangle_A. \tag{9}$$

In this case we want to produce the entangled state

$$|\psi(B,C)\rangle=\tfrac{1}{2}[|3\rangle_B|0\rangle_C+|2\rangle_B|1\rangle_C+|1\rangle_B|2\rangle_C+|0\rangle_B|3\rangle_C] \tag{10}$$

for the cavities B and C. The interaction times that would generate the entangled state (10) are given by $g\tau_{1_B}=5.0915$ $g\tau_{1_C}=1.5708$, $g\tau_{2_B}=7.2596$, $g\tau_{2_C}=1.1107$, $g\tau_{3_B}=7.1704$, and $g\tau_{3_C}=6.7436$. We have chosen these parameters such that the probability of detecting the first two atoms in ground state is unity. The probability of detecting the third atom in the ground state is, however, about 4%. Thus an average of 25 tries are required before the desired entangled state is produced in the cavities B and C. Note that the field inside the cavity A remains unaffected during the process of generating the entangled state between B and C.

In the next step, we find the state $|\psi_{n,m}(A,B)\rangle$ of the A-B system. A careful look at expression (4) of $|\psi_{n,m}(A,B)\rangle$ reveals that the subscript m can be inferred from the total number of photons inside the two cavities, whereas the subscript n is inferred from the relative phase of the states in the sum which is independent of m. Thus the state of the A-B system can be determined in two sets of measurements, the first determining m via the total number of photons inside the two cavities, and the second determining n via the relative phase.

Apart from the state with $m=0$ which contains $N-1$ photons in all its constituent states, an arbitrary state $|\psi_{n,m}(A,B)\rangle$ has two possible number of photons, $N-1-m$ and $2N-1-m$. For example, when $N=4$, the states $|\psi_{0,m}(A,B)\rangle$ are given by

$$\begin{aligned}|\psi_{0,0}(A,B)\rangle&=\tfrac{1}{2}(|0\rangle_A|3\rangle_B+|1\rangle_A|2\rangle_B+|2\rangle_A|1\rangle_B+|3\rangle_A|0\rangle_B),\\|\psi_{0,1}(A,B)\rangle&=\tfrac{1}{2}(|0\rangle_A|2\rangle_B+|1\rangle_A|1\rangle_B+|2\rangle_A|0\rangle_B+|3\rangle_A|3\rangle_B),\\|\psi_{0,2}(A,B)\rangle&=\tfrac{1}{2}(|0\rangle_A|1\rangle_B+|1\rangle_A|0\rangle_B+|2\rangle_A|3\rangle_B+|3\rangle_A|2\rangle_B),\\|\psi_{0,3}(A,B)\rangle&=\tfrac{1}{2}(|0\rangle_A|0\rangle_B+|1\rangle_A|3\rangle_B+|2\rangle_A|2\rangle_B+|3\rangle_A|1\rangle_B).\end{aligned} \tag{11}$$

It is clear that the states $|\psi_{n,0}(A,B)\rangle$, $|\psi_{n,1}(A,B)\rangle$, $|\psi_{n,2}(A,B)\rangle$, and $|\psi_{n,3}(A,B)\rangle$ have three (two or six), (one or five), and (zero or four) photons, respectively.

The number of photons can be determined via Ramsey interferometry. In such a scheme two-level atoms that are nonresonant with the fields in the two cavities and which are initially prepared in coherent superposition of states ($|a\rangle$

$+|b\rangle)/\sqrt{2}$ are passed through the two cavities. The interaction times in the two cavities are taken to be equal. Due to the dispersive nature of atomic interaction with the cavity fields, the level $|b\rangle$ picks up a relative phase shift which is proportional to the total number of photons in the two cavities and the interaction time with no change in the number of photons, i.e., the resulting state of the atom is

$$\frac{1}{\sqrt{2}}(|a\rangle+e^{ip\theta}|b\rangle), \tag{12}$$

where p is the total number of photons in the two cavities and θ is a parameter that depends on atom-field coupling, detuning, and interaction time [12]. By changing the interaction time, we can control θ. The atom is now passed through a resonant classical field. The interaction time and the coupling parameters are chosen such that $|a\rangle\rightarrow(|a\rangle+|b\rangle)/\sqrt{2}$ and $|b\rangle\rightarrow(|a\rangle-|b\rangle)/\sqrt{2}$. The final atomic state is thus

$$e^{ip\theta/2}[\cos(p\theta/2)|a\rangle-i\ \sin(p\theta/2)|b\rangle]. \tag{13}$$

The complete atom-field state is entangled and is rather complicated. We have, therefore, not reproduced it here. It is however clear that a measurement of the atom in state $|a\rangle$ or $|b\rangle$ would reduce the fields inside the cavities to states with only appropriate number of total photons in the two cavities.

The first atom is sent such that $\theta=\pi$. It follows from Eq. (13) that if the atom is found to be in the excited state $|a\rangle$, the number of photons in the two cavities is even, i.e., m is odd, and if the atom is detected in the lower state $|b\rangle$, the number of photons in the two cavities is odd, i.e., m is even. If m is odd, we send another atom, such that $\theta=\pi/2$. The detection of the atom in state $|a\rangle$ implies field states corresponding to $m=3,7,\ldots,N-1$, and the detection of atoms in state $|b\rangle$ implies field states corresponding to $m=1,5,\ldots,N-3$. If, however, the first atom is detected in state $|b\rangle$, we first add one photon in the cavity A and send a second atom such that $\theta=\pi/2$. The second atom found in the state $|a\rangle$ implies $m=0,4,\ldots,N-4$, whereas the atom found in the state $|b\rangle$ implies $m=2,6,\ldots,N-2$. If the second atom is found in state $|a\rangle$, the third atom is sent with $\theta=\pi/4$. If, however, the second atom is found to be in the state $|b\rangle$, we add two photons in cavity A before the third atom is sent with $\theta=\pi/4$, and the process is repeated. If the atom is found in the state $|a\rangle$, we send the fourth atom with $\theta=\pi/8$, whereas if atom is found in the state $|b\rangle$, three photons are added and then the fourth atom is sent with $\theta=\pi/8$. This process is repeated until l (such that $z^l=N$) atoms are sent with appropriate interaction times. The N possible outcomes of the atomic states of these l atoms uniquely determine the value of m.

In the above discussion, a crucial step in determining the value of m involved adding one, two, three, or more photons in cavity A. Thus, for example, adding one photon in the cavity A would transform the state

$$|\psi_{0,2}(A,B)\rangle=\tfrac{1}{2}(|0\rangle_A|1\rangle_B+|1\rangle_A|0\rangle_B+|2\rangle_A|3\rangle_B+|3\rangle_A|2\rangle_B) \tag{14}$$

into

$$\tfrac{1}{2}(|1\rangle_A|1\rangle_B+|2\rangle_A|0\rangle_B+|3\rangle_A|3\rangle_B+|4\rangle_A|2\rangle_B). \tag{15}$$

This step can be carried out using the recently proposed method based on quantum-state mapping between multilevel atoms and cavity light field via adiabatic passage [11]. Here we briefly describe this method, as it shall be crucial to our proposed method for determining the value of n in $|\psi_{n,m}(A,B)\rangle$.

We consider a single three-level atom in the Λ configuration. The lower levels $|b_1\rangle$ and $|b_2\rangle$ are coupled to the upper level $|a\rangle$ via a classical field of Rabi frequency $\Omega(t)$ and a cavity mode field with coupling strength $g(t)$, respectively. The interaction Hamiltonian for this system is given by

$$H(t)=\hbar g(t)(|a\rangle\langle b_2|a+a^\dagger|b_2\rangle\langle a|)-\frac{\hbar\Omega(t)}{2}(|a\rangle\langle b_1|+|b_1\rangle\langle a|), \tag{16}$$

where a and $a^\dagger$ are the destruction and creation operators of the cavity field. An eigenstate of this Hamiltonian is given by

$$|E_n\rangle=\frac{g(t)\sqrt{n+1}|b_1,n\rangle+\Omega(t)/2|b_2,n+1\rangle}{\sqrt{g(t)^2(n+1)+\Omega(t)^2/4}}. \tag{17}$$

This eigenstate does not contain the upper level $|a\rangle$. The asymptotic behavior of the state E_n as a function of time is given by

$$|E_n\rangle\rightarrow\begin{cases}|b_{1,}n\rangle & \text{for } \Omega(t)/g(t)\rightarrow 0\\ |b_{2,}n+1\rangle & \text{for } g(t)/\Omega(t)\rightarrow 0.\end{cases} \tag{18}$$

Now, according to the adiabatic theorem as applied to the time-varying Hamiltonian $H(t)$, if the Hamiltonian at time t_0 is in an eigenstate of $H(t_0)$, and the evolution from time t_0 to time t_1 is sufficiently slow, then the system will evolve into the eigenstate $H(t_1)$. Thus it follows that if the atom-cavity system is initially in state $|b_1\rangle$ then, for the pulse sequence in which $\Omega(t)$ is time delayed with respect to $g(t)$, the final state as the atom leaves the interaction region will be $|b_2,n+1\rangle$. This results in a single-photon shift in the cavity field state. A reverse sequence can be used to remove the single photon from the field state.

In a more general situation where the cavity field induces multiphoton transition between levels $|b_2,n+m\rangle$ and $|a,n\rangle$, described by the Hamiltonian

$$H(t)=\hbar g(t)(|a\rangle\langle b_2|a^m+(a^\dagger)^m|b_2\rangle\langle a|)-\frac{\hbar\Omega(t)}{2}(|a\rangle\langle b_1|+|b_1\rangle\langle a|), \tag{19}$$

the energy eigenstate is given by

$$|E_n\rangle = \frac{g(t)\sqrt{(n+1)(n+2)\cdots(n+m)}|b_1,n\rangle + \Omega(t)/2|b_2,n+m\rangle}{\sqrt{g(t)^2(n+1)(n+2)\cdots(n+m)+\Omega(t)^2/4}}. \quad (20)$$

Following the above argument, an m-photon transfer becomes possible in this case. In Eq. (19) we did not include the terms associated with the dynamic Stark shifts. These terms can be ignored if the integrated dynamic Stark shift is an integral multiple of 2π [13].

Now we return to the question of determining the value of n in $|\psi_{n,m}(A,B)\rangle$.

At the end of the last step, after we have determined the value of m in $|\psi_{n,m}(A,B)\rangle$, the entangled state of the two cavities A and B is

$$|\psi_{n,m}(A,B)\rangle = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{2\pi inj/N}|j\rangle_A |(N-1-j-m)\mathrm{mod}N + x\rangle_B, \quad (21)$$

where x is the number of photons added in the cavity in the process of measuring m. We now *empty* the cavity B by removing the photons one by one using the method based on adiabatic passage described above. We send a stream of $N-1+x$ three-level atoms in Λ configuration in level $|b_2\rangle$ interacting with the field inside cavity B only and a classical field via Hamiltonian (16). For the first x atoms the state of the atom, after the passage through the cavity, would be found in state $|b_1\rangle$ corresponding to the removal of x photons. The subsequent $N-1$ atoms are sent in such a way that the levels $|b_1\rangle$ and $|b_2\rangle$ are mixed by a strong field after the passage through the cavity such that $|b_1\rangle \to (|b_1\rangle + |b_2\rangle)/\sqrt{2}$ and $|b_2\rangle \to (|b_1\rangle - |b_2\rangle)/\sqrt{2}$. It can be verified that a detection of the atom in state $|b_1\rangle$ does not add any phase. However, a detection in state $|b_2\rangle$ would add a π phase (negative sign) for those constituent states that have initially no photons in cavity B. The resulting state of the cavity field in A would be decoupled from that of cavity B. The price, however, would be a random but known distribution of π phases depending on the outcome of the atomic states. Certain sequences may not make it possible to find the value of n and we may have to abandon the effort and start all over again. However, in certain cases it should be possible to proceed as follows.

We consider only the case when all the $n-1$ atoms are found in $|b_1\rangle$. In this case, the cavity B having no photons is decoupled from A. The state of cavity A is

$$|\psi_n(A)\rangle = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{2\pi inj/N}|j\rangle. \quad (22)$$

The step we follow now is to send $N/2$ three-level atoms of the type described above in state $|b_2\rangle$ through the cavity B in the following way. The first atom removes $N/2$ photons from atomic states with $N/2$ or more photons via an interaction of the type (19), and ends up in the atomic state $|b_1\rangle$. This is followed by the passage of the atom through a strong classical field which transforms the states $|b_1\rangle$ and $|b_2\rangle$ into $(|b_1\rangle - |b_2\rangle)/\sqrt{2}$ and $(|b_1\rangle + |b_2\rangle)/\sqrt{2}$, respectively. The atom is then detected in state $|b_1\rangle$ or $|b_2\rangle$. If the atom is detected in level $|b_1\rangle$, then $n=0,2,\ldots,(N-2)$ whereas if the atom is detected in level $|b_2\rangle$, then $n=1,3,\ldots,(N-1)$. In both cases the field state inside the cavity reduces to

$$\frac{1}{\sqrt{N/2}} \sum_{j=0}^{N/2-1} e^{2\pi inj/N}|j\rangle. \quad (23)$$

The second atom removes $N/4$ photons from atomic states having an adequate number of photons, followed by a strong field that transforms the states $|b_1\rangle$ and $|b_2\rangle$ into $[|b_1\rangle + i\exp(i\varphi)|b_2\rangle]/\sqrt{2}$ and $[i\exp(-i\varphi)|b_1\rangle + |b_2\rangle]/\sqrt{2}$, respectively. We chose $\varphi=\pi/2$ or π depending upon whether the first atom was detected in state $|b_1\rangle$ or $|b_2\rangle$. The $b_1b_1, b_1b_2, b_2b_1, b_2b_2$ sequences imply $n=0,4,\ldots,N$, $n=2,6,\ldots,(N-2)$, $n=1,5,\ldots,(N-3)$, $n=3,7,\ldots,(N-1)$, respectively. In all these cases the field state inside the cavity reduces to

$$\frac{1}{\sqrt{N/4}} \sum_{j=0}^{N/4-1} e^{2\pi inj/N}|j\rangle. \quad (24)$$

The third atom removes $N/8$ photons followed by the strong classical field and the detection process. We chose $\varphi=\pi/2$, π, $3\pi/4$, or $5\pi/4$ if the sequence of measurements are b_1b_1, b_1b_2, b_2b_1, or b_2b_2, respectively. The sequence of measurements $b_1b_1b_1$, $b_1b_1b_2$, $b_1b_2b_1$, $b_1b_2b_2$, $b_2b_1b_1$, $b_2b_1b_2$, $b_2b_2b_1$, and $b_2b_2b_2$ imply $n=0,8,\ldots$, $n=4,12,\ldots$, $n=2,10,\ldots$, $n=6,14,\ldots$, $n=1,9,\ldots$, $n=5,13,\ldots$, $n=3,11,\ldots$, and $n=7,15,\ldots$, respectively. The subsequent atoms remove $N/16$, $N/32,\ldots,1$ photons from atomic states having an adequate number of photons, followed by a strong classical field with appropriate choice of the phase φ and the detection process. The N outcomes of the final states uniquely determine the value of n in $|\psi_{n,m}(A,B)\rangle$. For example, when $N=4$, the final outcomes b_1b_1, b_2b_1, b_1b_2, and b_2b_2 yield $n=0$, 1, 2, and 3, respectively.

This completes the step of the detection of A-B system. A determination of the A-B system in the state $|\psi_{j,k}(A,B)\rangle$ reduces the state of the field inside the cavity C to the form (6), i.e.,

$$|\psi(C)\rangle = \sum_{l=0}^{N-1} e^{-2\pi ilj/N} w_l |(l+k)\mathrm{mod}\ N\rangle_C \quad (25)$$

This state depends on the amplitudes w_l, but is different from state (1) due to the presence of the phase factors and the displacement of photon numbers. If $k=0$, state (25) is different from Eq. (1) only in terms of the phase factors. The Ramsey interferometry can be employed to remove the phase factors as follows. A nonresonant atom in its superposition of states $(|a\rangle + |b\rangle)/\sqrt{2}$ is passed through the cavity C; the interaction time is chosen such that the ground state $|b\rangle$ picks up a relative phase $+2\pi ilj/N$ which is proportional to the photon number in the state. The atom-field state after the passage of the atom through the cavity is given by

$$\frac{1}{\sqrt{2}}\sum_{l=0}^{N-1}(e^{-2\pi ilj/N}w_l|l\rangle_A|a\rangle+w_l|l\rangle_A|b\rangle). \qquad (26)$$

The detection of atom in state $|a\rangle$ leaves the cavity field in state (27), whereas the detection of the atom in state $|b\rangle$ brings the cavity field into state (1). If the atom is detected in state $|a\rangle$, we keep repeating the procedure until an atom is detected in state $|b\rangle$.

The above is true when $k=0$ which would happen only $1/N$ times on the average. When $k\neq 0$, the process to bring state (25) to form (1) is somewhat complicated. In that case we first add $N-k$ photons, using the method based on adiabatic passage discussed above. This will transform state (25) into

$$|\psi(C)\rangle=\sum_{l=0}^{N-1}e^{-2\pi ilj/N}w_l|[(l+k)\mathrm{mod}\ \ N]+N-k\rangle_C. \qquad (27)$$

This makes the Fock states in the expansion with coefficients w_l with $l\geqslant k$ as $|l\rangle$ and with $l<k$ as $|l+N\rangle$. We thus remove N photons from state $|\psi(C)\rangle$ via the N-photon absorption process in the adiabatic passage scheme. This removes N photons from states $|l+N\rangle$ (with $l<k$) only, leaving states $|l\rangle$ (with $l\geqslant k$) in expansion (27) untouched. However, the full atom-field state is entangled, i.e., the resulting atom-field state is

$$|\psi(C)\rangle=\sum_{l=0}^{k-1}e^{-2\pi ilj/N}w_l|l\rangle_A|b_2\rangle+\sum_{l=k}^{N-1}e^{-2\pi ilj/N}w_l|l\rangle_A|b_1\rangle, \qquad (28)$$

where $|b_1\rangle$ and $|b_2\rangle$ are the atomic levels. We now prepare states $|b_1\rangle$ and $|b_2\rangle$ in a coherent superposition so that $|b_1\rangle\rightarrow(|b_1\rangle-|b_2\rangle)/\sqrt{2}$ and $|b_2\rangle\rightarrow(|b_1\rangle+|b_2\rangle)/\sqrt{2}$. The detection of the atom in state $|b_1\rangle$, followed by a correction of phase factors as discussed above, reduces the state of the cavity field to form (1). However, the detection of the atom in state $|b_2\rangle$ adds negative signs in the coefficients of $|l\rangle$ for $l<k$, which survive after the correction of the phase factor $\exp(-2\pi ilj/N)$ in Eq. (28).

We have thus presented an experimentally viable scheme for the quantum teleportation of a field state of form (1) from a cavity at the sender's end to another cavity at the receiver's end (apart from known phase factors). The proposed scheme relies on the preparation of quantum entangled states of type (2) between two cavities, the optical Ramsay interferometry, and single and many photons transfer via adiabatic following in three-level atoms. All these lie within the realm of the presently accessible experimental methods. However, cavity damping and controlling the interaction times of the atom and the cavity to a high precision may pose difficulties. The proposed method of removing n photons via an adiabatic following may limit us to small values of N.

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