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Final-state-interaction effects on one- and two-photon detachment of H^- in the presence of a static electric field

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We present a detailed theoretical formulation of the problem of an electron moving in a static electric field, a laser field, and an atomic potential. Our formulation treats the electron-atom interaction in the zero-range potential approximation and employs both the quasienergy approach and an analytic expression for the Green's function describing electron propagation in a combination of static and laser electric fields. Our formulation is applied to one- and two-photon detachment of H^- in a strong static electric field and takes into account all final-state interactions of the detached electron with the static and laser fields and with the atomic core. Our results show that rescattering effects are small in the case of one-photon detachment, where our results are close to those obtained previously by Gao and Starace [Phys. Rev. A **42**, 5580 (1990)], who ignored rescattering effects but who found a strong-field treatment of the laser field to be important, even in the limit of weak laser fields, owing to interference terms involving both the laser and static electric fields. Our results for two-photon detachment of H^-, on the other hand, show that rescattering effects are very significant. In the presence of a strong static electric field, moreover, the two-photon detachment cross section is found to be very sensitive to the magnitude of the static field. [S1050-2947(98)09406-2]

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I. INTRODUCTION

Placing negative ions and atoms in various combinations of laser fields and static electric and magnetic fields allows one to control rates of atomic processes. A typical example is negative-ion photodetachment in a static electric field leading to quantum-mechanical interference effects, which are exhibited as oscillations in the single-photon detachment cross section as a function of frequency [1]. These oscillations can be manipulated by adding a static magnetic field [2–4]. Alternatively, at fixed excitation frequency these quantum interference effects may be controlled by changing the duration of the laser pulse or by using two or more short laser pulses [4,5]. Another alternative is to consider multiphoton processes in static fields. Multiphoton detachment creates a different angular distribution for outgoing electrons as compared to that for single-photon detachment. This can change the interference pattern and in some cases enhance the oscillations in the detachment cross sections. Relevant to all of these coherent control approaches for photodetachment is the role of final-state rescattering of the photodetached electron by the residual atom and the interplay between that effect and other final-state interactions. In this paper we examine these questions (and provide answers) for the cases of one- and two-photon detachment of H^- in a strong, static electric field.

The subject of single-photon or multiphoton detachment of negative ions in the presence of a static, uniform electric field has a long history (see, e.g., the brief review given in the Introduction of the paper by Gao and Starace [6]). Nevertheless, most theoretical treatments of final-state interactions relevant to these processes have been selective and approximate, generally treating the electron–static-field interaction exactly but ignoring both the electron-laser interaction and the electron-atom rescattering interaction [2,3,7–15]. Only recently have more complete treatments of these final-state interactions appeared [6,16–20].

Nicolaides and Mercouris [16] treated all final-state interactions in principle exactly (but completely numerically) for the case of the photodetachment of Li^- and H^-. However, for weak fields their results only confirmed results of simpler theoretical calculations as well as the experimental measurements for H^- photodetachment [8]. No new effects were predicted for stronger fields. Fabrikant [17] used a frame transformation technique to treat the final-state rescattering of the detached electron by the residual atom while ignoring the final-state electron-laser interaction. He found that the rescattering effect significantly lowers[1] the photodetachment cross section of H^- for strong static, uniform electric fields near the zero-static-field threshold. Gao and Starace [6] treated the final-state electron-laser interaction exactly and showed that it leads to an additional term in the transition matrix element even in the limit of weak laser fields; this extra term was shown to result in a measurably lower photodetachment cross section near the zero-static-field threshold when the static electric field is strong. Ostrovsky and Telnov [18] have carried out an analytic study of the photodetachment of negative ions that in principle includes all final-state interaction effects. Their focus is on the particular

[1]The lowering of the cross section is relative to what the cross section would be in a perturbative treatment that ignores all final-state effects other than that between the electron and the static electric field.

case of a strong laser field and a weak static electric field. However, no numerical results are provided.

A number of theoretical studies are related tangentially to the subject of this paper. Two papers have treated photodetachment plus excitation of H^- and have included final-state electron correlation effects. Slonim and Greene [19] used a frame transformation technique and multichannel quantum defect theory to treat final-state rescattering effects on the photodetachment of H^- while ignoring the final-state interaction of the electron with the laser field. The focus of their study is on the effect of a static field on the well-known shape and Feshbach resonances near the H $(n=2)$ threshold rather than on the photodetachment cross section near the H $(n=1)$ threshold. Du, Fabrikant, and Starace [20] also used a frame transformation approach but one based on *ab initio* numerical adiabatic hyperspherical transition amplitudes (which include final-state electron-atom interaction effects) to study static electric field effects on the shape and Feshbach resonances near the H $(n=2)$ threshold in H^- photodetachment. They also ignored the final-state interaction of the electron with the laser field and did not provide predictions for the H $(n=1)$ photodetachment cross section. Finally, in the absence of a static electric field, a number of theoretical works have treated intense field multiphoton detachment [21] and ionization [22–24] for simple systems including final-state interaction effects.

In this paper we investigate the photodetachment of H^- including all final-state interactions in a regime in which our predictions differ measurably from results of calculations that ignore these effects. Specifically, our formulation includes the final-state interaction of the detached electron with both laser and static fields nonperturbatively (although the results presented are for the limit of a weak laser field). The final-state short-range interaction between the electron and the atom is represented by a three-dimensional δ function potential [25,26] whose use permits much of the theoretical work to be done analytically. The final-state electron-atom interaction is treated by a combination of the quasienergy approach [27,28] and the Green's function method. Our Green's function method uses the analytic propagator [29] describing the motion of the detached electron in both the laser and the static electric fields. In what follows, we elaborate a bit on each of these key components of our theoretical approach.

The zero-range potential model [25,26] has been used in many theoretical works to treat multiphoton detachment by a strong laser field. Physically this method is justified if the polarizability of the atomic residue is not too high and if the de Broglie wavelength of the detached electron is large compared to the effective radius of the electron-atom interaction. The advantage of this approach is that it allows one to eliminate spatial coordinates: the problem of solving the time-dependent Schrödinger equation is reduced to a one-dimensional integral equation in time. Further simplification can be achieved if the laser field is circularly polarized. In this case Berson [25] and Manakov and Rapoport [30] showed that using the quasienergy approach [27] allows one to reduce the problem to solving a transcendental equation for the quasienergy. The same approach can be applied in the case of linear and, more generally, elliptical polarization [28], but then one has to solve an infinite set of coupled transcendental equations. An alternative procedure is to solve the integral equation for the time-dependent part of the wave function by direct numerical integration [31].

While Manakov and Fainshtein [28] indicated that the quasienergy approach [27] combined with the zero-range potential model [25,26] can be applied to the calculation of negative-ion decay in the presence of a laser field and a static electric field, no numerical results were given. The same ideas were used by Slonim and Dalidchik [32] to find two-photon and three-photon photodetachment cross sections for circularly polarized light in the presence of a static electric field. The decay width was expressed in terms of a combination of Airy functions (thus allowing one, in principle, in the one-photon case to regain the well-known perturbative results of the theory of one-photon detachment in a static field [9,12]). However, Slonim and Dalidchik also present almost no numerical results.

A third ingredient of our theoretical approach (in addition to our use of a zero-range potential [25,26] and the quasienergy method [27,28]) is our use of the analytic Green's function obtained by Bao and Starace [29] for describing the propagation of an electron in the fields of both a laser and a static electric field. This Green's function is expressed by the classical action integral for an interaction of the form $\mathbf{F}(t)\cdot\mathbf{r}$, where $\mathbf{F}(t)$ includes both the laser and the static electric fields. This representation is a particular case of a general result of Feynman [33,34] holding for all Hamiltonians having only linear and quadratic dependences on spatial coordinates. This classical path approach to the evaluation of the path integral has been noted for being particularly simple and effective [35]. Having an analytical result for the path integral (which is a representation of the system's Green's function) permits one to evaluate the physical significance of each term as well as to carry out many of the relevant integrals analytically.

In Sec. II we present our Green's function approach for linearly polarized laser detachment in the presence of a static, uniform electric field directed along the axis of the laser polarization. Equations are presented for obtaining the final-state wave function for the electron including its final-state interactions with the laser and static electric fields as well as with the residual atom. In Sec. III we employ our final-state wave function to evaluate the transition matrix elements for linearly polarized laser photodetachment of H^- in the presence of a static electric field in the limit of weak laser fields and for the cases of one- and two-photon detachment. In Sec. IV we present our numerical results for the corresponding photodetachment cross section and compare our results with those of others [6,12]. In Sec. V we present our numerical results for the two-photon detachment cross section and, in the absence of the static electric field, compare our results with those of others [36]. Finally, in Sec. VI we summarize our results and present our conclusions. The three appendixes provide details of our formulation: Appendixes A and C transform some of the analytic expressions in the main text to forms suitable for numerical evaluation; Appendix B gives the relation between final-state wave functions satisfying ingoing-wave and outgoing-wave boundary conditions.

II. GREEN'S FUNCTION APPROACH FOR LINEARLY POLARIZED LASER DETACHMENT IN THE PRESENCE OF A STATIC ELECTRIC FIELD

Consider the H^- ion in the following combination of a parallel static electric field and a laser field, both defined along the positive z axis[2]

$$\mathbf{E}(t)=\mathbf{E}_s+\mathbf{E}_0\sin\omega t=\hat{\mathbf{z}}(E_s+E_0\sin\omega t). \tag{1}$$

The short-range interaction between the final-state electron and the atom residue will be modeled by the zero-range potential (atomic units are used throughout)

$$V(r)=\frac{2\pi}{\kappa}\delta(\mathbf{r})\frac{\partial}{\partial r}r, \tag{2}$$

where $\kappa=\sqrt{2\epsilon_i}$ and ϵ_i is the energy of the initial bound state. In the limit of $E_s\rightarrow 0$, the wave function for this bound state has the well-known expression

$$\Phi_i^{(0)}(r)=B\frac{e^{-\kappa r}}{r}, \tag{3}$$

where B is a normalization constant whose value is 0.315 52 [12].

For interaction times sufficiently short that depletion effects may be neglected, the multiphoton transition from the initial bound state of H^-, described in the short-range potential approximation by the one-electron wave function $\psi_i(\mathbf{r},t)$, and a final state of the detached electron, described by $\psi_f^{(-)}(\mathbf{r},t)$, may be calculated using the S-matrix element [37]

$$S_{fi}=-i\int_{-\infty}^{\infty}\langle\psi_f^{(-)}(\mathbf{r},t)|zE_0\sin\omega t|\psi_i(\mathbf{r},t)\rangle dt, \tag{4}$$

where the minus superscript on $\psi_f^{(-)}$ indicates that incoming-wave boundary conditions apply. This expression for the S-matrix element is exact within the short-range potential model approximation if the final-state wave function describes exactly all final-state interactions of the detached electron with the static and laser electric fields as well as the short-range atomic potential and if the initial-state wave function includes static electric field effects on the short-range potential's bound-state wave function. We present and discuss here our evaluations of $\psi_f^{(-)}$ and ψ_i in turn. In the next section we present our calculations for the S-matrix elements for one- and two-photon detachment.

[2]Note that in Ref. [17], whose results we shall comment upon later, the electric fields are defined along the negative z axis so that the force on the electron is along the positive z axis. Our formulas in this paper are written so that comparison with those in Ref. [17] requires simply changing the signs of the electric fields. (Certain normalization factors involving only the magnitude of E_s therefore employ the absolute value $|E_s|$.)

A. Final-state wave function

The final-state wave function satisfies the time-dependent Schrödinger equation

$$i\frac{\partial\psi_f^{(-)}(\mathbf{r},t)}{\partial t}=\left(-\frac{1}{2}\nabla^2+zE(t)+V(r)\right)\psi_f^{(-)}(\mathbf{r},t), \tag{5}$$

where the electric field is defined in Eq. (1). To solve this equation, we first introduce the wave function $\psi_0(\mathbf{r},t)$ which solves the time-dependent Schrödinger equation in the absence of $V(r)$:

$$i\frac{\partial\psi_0(\mathbf{r},t)}{\partial t}=\left(-\frac{1}{2}\nabla^2+zE(t)\right)\psi_0(\mathbf{r},t). \tag{6}$$

The corresponding retarded Green's function describing propagation of the detached electron in both the static and laser electric fields is

$$\left(i\frac{\partial}{\partial t}+\frac{1}{2}\nabla^2-zE(t)\right)G_0(\mathbf{r},t;\mathbf{r}',t')=\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \tag{7}$$

Using ψ_0 and G_0 as defined in Eqs. (6) and (7), the solution of Eq. (5) which satisfies the outgoing-wave boundary condition is[3]

$$\psi_f^{(+)}(\mathbf{r},t)=\psi_0(\mathbf{r},t)+\frac{2\pi}{\kappa}\int d\mathbf{r}'\int_{-\infty}^{t}dt'\,G_0(\mathbf{r},t;\mathbf{r}',t')\delta(\mathbf{r})\times\left(\frac{\partial}{\partial r'}r'\psi_f^{(+)}(\mathbf{r}',t')\right), \tag{8}$$

where we have employed Eq. (2) for the short-range potential. The main goal of this section is to solve Eq. (8) for the final-state wave function $\psi_f^{(+)}(\mathbf{r},t)$. In order to do this we begin by introducing analytic expressions for both $\psi_0(\mathbf{r},t)$ and $G_0(\mathbf{r},t;\mathbf{r}',t')$.

1. Analytic expression for $\psi_0(\mathbf{r},t)$

A momentum space representation for ψ_0 has been given in Ref. [6]; the coordinate space representation, which we require here, is given by [38]

$$\psi_0(\mathbf{r},t)=\psi_\alpha^x(x,t)\psi_\alpha^y(y,t)\psi_\alpha^z(z,t)\exp\{-i(E_0^2/8\omega^3)\sin 2\omega t-i(E_0^2/4\omega^2)t\}, \tag{9}$$

where

$$\psi_\alpha^x(x,t)=\frac{1}{\sqrt{2\pi}}\exp[i(p_x^\alpha x-\epsilon_x^\alpha t)], \tag{10}$$

$$\psi_\alpha^y(y,t)=\frac{1}{\sqrt{2\pi}}\exp[i(p_y^\alpha x-\epsilon_y^\alpha t)], \tag{11}$$

[3]We shall later obtain $\psi_f^{(-)}(\mathbf{r},t)$ using our result for $\psi_f^{(+)}(\mathbf{r},t)$.

$$\psi^z_\alpha(z,t)=\frac{2^{1/3}}{E_s^{1/6}}\text{Ai}\left[(2E_s)^{1/3}\left(z-\frac{\epsilon_z^\alpha}{E_s}-\frac{E_0}{\omega^2}\sin\,\omega t\right)\right]\times\exp\left[+i\frac{E_0E_s}{\omega^3}\cos\,\omega t+i\frac{zE_0}{\omega}\cos\,\omega t-i\epsilon_z^\alpha t\right].\tag{12}$$

Here α denotes a particular set of conserved quantum numbers $(p_x^\alpha,p_y^\alpha,\epsilon_z^\alpha)$. The energy of the final-state electron is

$$\epsilon_f=\epsilon_x^\alpha+\epsilon_y^\alpha+\epsilon_z^\alpha,\tag{13}$$

where $\epsilon_x^\alpha\equiv\frac{1}{2}(p_x^\alpha)^2$ and $\epsilon_y^\alpha\equiv\frac{1}{2}(p_y^\alpha)^2$.

2. Analytic expression for $G_0(\mathbf{r},t;\mathbf{r}',t')$

According to a general result of Feynman [33] (holding for all Hamiltonians with only linear and quadratic dependences on spatial coordinates), the Green's function we require can be represented in closed analytical form. For the Hamiltonian

$$H(\dot{\mathbf{r}},\mathbf{r},t)=\frac{1}{2}\dot{\mathbf{r}}^2+\mathbf{E}(t)\cdot\mathbf{r},\tag{14}$$

the Green's function satisfying Eq. (7) is

$$G_0(\mathbf{r}_2,t_2;\mathbf{r}_1,t_1)=\frac{-i}{[2\pi i(t_2-t_1)]^{3/2}}\exp[iI(\mathbf{r}_2,t_2;\mathbf{r}_1,t_1)]\times\theta(t_2-t_1),\tag{15}$$

where θ is a Heaviside function and where I is the classical action, defined by

$$I(\mathbf{r}_2,t_2;\mathbf{r}_1,t_1)=\int_{t_1}^{t_2}L(\dot{\mathbf{r}},\mathbf{r},t)dt,\tag{16}$$

where the classical Lagrangian is given by

$$L(\dot{\mathbf{r}},\mathbf{r},t)=\frac{1}{2}\dot{\mathbf{r}}^2-\mathbf{E}(t)\cdot\mathbf{r}.\tag{17}$$

Note that in Eqs. (14)–(17), $\mathbf{r}_2\equiv\mathbf{r}(t_2)$ and $\mathbf{r}_1\equiv\mathbf{r}(t_1)$, where classically $\mathbf{r}(t)$ is defined by Newton's equations for the particle. The classical action may be obtained analytically by doing the integration in Eq. (16) along the classical path [29]:

$$I(\mathbf{r}_2,t_2;\mathbf{r}_1,t_1)=\frac{1}{2}\frac{(\mathbf{r}_2-\mathbf{r}_1)^2}{t_2-t_1}-\int_{t_1}^{t_2}dt\mathbf{E}(t)\cdot\left[\mathbf{r}_2\frac{t-t_1}{t_2-t_1}+\mathbf{r}_1\frac{t_2-t}{t_2-t_1}\right]+\frac{1}{2}\int_{t_1}^{t_2}dt\int_{t_1}^{t_2}dt'G_f(t,t')\mathbf{E}(t)\cdot\mathbf{E}(t'),\tag{18}$$

where

$$G_f(t,t')=-\frac{1}{t_2-t_1}[(t_2-t)(t'-t_1)\theta(t-t')+(t_2-t')(t-t_1)\theta(t'-t)].\tag{19}$$

For later convenience, we change the variables t_2 and t_1 to t and $t-t'$, $\mathbf{r}_2$ to $\mathbf{r}$, let $\mathbf{r}_1$ go to zero, and use the explicit form of $\mathbf{E}$ in Eq. (1), in order to obtain from Eq. (18) the following analytic result for the classical action:

$$\begin{aligned}I(\mathbf{r},t;0,t-t')=&\frac{x^2+y^2+z^2}{2t'}+\frac{E_0z}{\omega}\cos\omega t\\&-\frac{E_0z}{\omega^2t'}[\sin\omega t-\sin\omega(t-t')]\\&-\frac{E_szt'}{2}-\frac{E_s^2t'^3}{24}-\frac{E_0^2t'}{4\omega^2}\\&+\frac{E_0E_s}{\omega^3}[\cos\omega t-\cos\omega(t-t')]\\&+\frac{E_0E_st'}{2\omega^2}[\sin\omega t+\sin\omega(t-t')]\\&+\frac{E_0^2}{2\omega^4t'}[\sin\omega t-\sin\omega(t-t')]^2\\&-\frac{E_0^2}{8\omega^3}[\sin2\omega t-\sin2\omega(t-t')].\end{aligned}\tag{20}$$

3. Solution of the integral equation for $\psi_f^{(+)}(\mathbf{r},t)$

Before solving Eq. (8) for the final-state wave function $\psi_f^{(+)}(\mathbf{r},t)$, we make two observations. First, owing to the δ function in the integral equation for $\psi_f^{(+)}(\mathbf{r},t)$ in Eq. (8), the integral over the spatial coordinates is determined by the behavior of the integrand near the origin. Second, owing to the periodicity of the laser field, we can introduce the quasienergy representation [27,28]

$$\psi_f^{(+)}(\mathbf{r},t)=\exp(-i\epsilon t)\Phi_\epsilon^{(+)}(\mathbf{r},t),\tag{21}$$

$$\psi_0(\mathbf{r},t)=\exp(-i\epsilon t)\Phi_\epsilon^{(0)}(\mathbf{r},t),\tag{22}$$

where ϵ is the quasienergy, and $\Phi_\epsilon^{(+)}(\mathbf{r},t)$ and $\Phi_\epsilon^{(0)}(\mathbf{r},t)$ are periodic functions of t. Furthermore, $\Phi_\epsilon^{(+)}(\mathbf{r},t)$ must satisfy the following boundary condition appropriate for the short-range potential in Eq. (8) [25,26]:

$$\Phi_\epsilon^{(+)}(\mathbf{r},t)|_{r\to0}=\left(\frac{1}{r}-\kappa\right)u(t).\tag{23}$$

Substituting Eqs. (21)–(23) into Eq. (8), multiplying both sides by $e^{i\epsilon t}$, and changing the integration variable t' to $\tau\equiv t-t'$, we obtain the following result:

$$\Phi_\epsilon^{(+)}(\mathbf{r},t)=\Phi_\epsilon^{(0)}(\mathbf{r},t)-2\pi\int_0^\infty d\tau e^{i\epsilon\tau}G_0(\mathbf{r},t;0,t-\tau)\times u(t-\tau). \quad (24)$$

From Eq. (15), we have, then,

$$G_0(\mathbf{r},t;0,t-\tau)=\frac{-i}{(2\pi i\tau)^{3/2}}\exp[iI(\mathbf{r},t;0,t-\tau)], \quad (25)$$

where the action I is given by Eq. (18) for the electric field defined by Eq. (1). Substituting Eq. (25) into Eq. (24) gives

$$\Phi_\epsilon^{(+)}(\mathbf{r},t)=\Phi_\epsilon^{(0)}(\mathbf{r},t)+\frac{1}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}\times\exp[iI(\mathbf{r},t;0,t-\tau)+i\epsilon\tau]u(t-\tau). \quad (26)$$

Now for $r\to 0$, the singularity in the integrand (stemming from the $\tau^{-3/2}$ factor) for $\tau\to 0$ is canceled by the rapid oscillations of exp(iI) [cf. Eq. (18) and note that $\tau\to 0$ implies $t_2-t_1\to 0$ in that equation]. However, for $r=0$, $I(0,t;0,t-\tau)$ does not lead to rapid oscillations in exp(iI) as $\tau\to 0$ and thus one must deal with the singularity at $\tau\to 0$.

In order to treat the singularity in Eq. (26) for $r=0$ and $\tau\to 0$, we follow the procedure of Manakov and Fainshtein [28] and introduce the free-particle (FP) Green's function

$$G_{\rm FP}(r,t;0,t-\tau)\equiv\frac{-i}{(2\pi i\tau)^{3/2}}\exp[iI_0(r,\tau)], \quad (27)$$

where

$$I_0(r,\tau)\equiv\frac{r^2}{2\tau}. \quad (28)$$

This result follows from Eqs. (15) and (18) upon setting the electric fields equal to zero and making the appropriate change of variables; one may also easily verify from these equations that

$$G_0(r=0,t;0,t-\tau)\underset{\tau\to 0}{\to}G_{\rm FP}(r=0,t;0,t-\tau)=\frac{-i}{(2\pi i\tau)^{3/2}}. \quad (29)$$

Because the free-particle Green's function equals G_0 as $\tau\to 0$ [and thus permits us to eliminate the singularity as $\tau\to 0$ in Eq. (26) for $r=0$], we add and subtract the free-particle Green's function to or from the Green's function in Eq. (25) and substitute the result in the integrand in Eq. (26) to obtain

$$\Phi_\epsilon^{(+)}(r,t)=\Phi_\epsilon^{(+)}(r,t)+\frac{1}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}e^{i\epsilon\tau}\times[e^{iI(r,t;0,t-\tau)}u(t-\tau)-e^{iI_0(r,\tau)}u(t)]+\frac{u(t)}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}e^{iI_0(r,\tau)+i\epsilon\tau}. \quad (30)$$

One observes in Eq. (30) that the first integral on the right-hand side is now well behaved for $r=0$ when $\tau\to 0$ because the quantities in brackets exactly cancel. On the other hand, the second integral in Eq. (30) involving the free-particle Green's function can be carried out analytically,

$$\frac{1}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}e^{ir^2/2\tau+i\epsilon\tau}=\frac{e^{ikr}}{r}, \quad (31)$$

where $k\equiv\sqrt{2\epsilon}$. Substituting this result in Eq. (30) gives, then,

$$\Phi_\epsilon^{(+)}(\mathbf{r},t)=\Phi_\epsilon^{(0)}(\mathbf{r},t)+\frac{1}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}\times e^{i\epsilon\tau}[e^{iI(r,t;0,t-\tau)}u(t-\tau)-e^{iI_0(r,\tau)}u(t)]+\frac{u(t)e^{ikr}}{r}. \quad (32)$$

Two observations regarding Eq. (32) must now be made. First, although the singularity in the integral has now been removed by adding and subtracting the free-particle Green's function to the Green's function for the photoelectron in the presence of the laser and static electric fields, we still have a singularity at $r\to 0$ from the integral of the free-particle Green's function. Second, everything on the right-hand side of Eq. (32) is known analytically except for the time-dependent function $u(t)$. Hence, if we can determine this function, we shall have determined the exact final-state wave function $\Phi_\epsilon^{(+)}(\mathbf{r},t)$, which can then be obtained directly from Eq. (32).

4. Determination of $u(t)$

One may obtain an equation that permits one to determine $u(t)$ by considering the $r\to 0$ limit of Eq. (32). Specifically, substitute Eq. (23) on the left of Eq. (32) and replace e^{ikr} by $1+ikr$ on the right to obtain

$$-u(t)(\kappa+i\sqrt{2\epsilon})=\Phi_\epsilon^{(0)}(0,t)+\frac{1}{(2\pi i)^{1/2}}\times\int_0^\infty\frac{d\tau}{(\tau)^{3/2}}e^{i\epsilon\tau}[\exp(iI(0,t,0,t-\tau)u(t-\tau)-u(t)]. \quad (33)$$

Although this equation is similar to one obtained by Manakov and Fainshtein [cf. Eq. (4) of Ref. [28]], there are two important differences. First, we use the length rather than the velocity gauge. Second, we look for the final-state wave function for a fixed real quasienergy ϵ rather than for the complex quasienergy considered in Ref. [28] since we are solving the final-state scattering problem with outgoing-wave boundary conditions. Therefore we have the additional inhomogenious term on the right-hand side of our equation for $u(t)$. Expanding $u(t)$ and $\Phi_\epsilon^{(0)}(0,t)$ in Floquet series,

$$u(t)=\sum_{n=-\infty}^{+\infty} U_n\exp(-in\omega t), \tag{34}$$

$$\Phi_\epsilon^{(0)}(0,t)=\sum_{n=-\infty}^{+\infty}\Phi_n^{(0)}\exp(-in\omega t), \tag{35}$$

and taking the Fourier transform of Eq. (33), we obtain a system of coupled algebraic equations for the coefficients U_n which determine $u(t)$:

$$-(\kappa+i\sqrt{2\epsilon})U_n=\Phi_n^{(0)}+\sum_m M_{nm}U_m, \tag{36}$$

where

$$M_{nm}=\frac{\omega}{2\pi(2\pi i)^{1/2}}\int_{-\pi/\omega}^{\pi/\omega}dt\exp[i\omega t(n-m)]\times\int_0^\infty\frac{d\tau}{(\tau)^{3/2}}e^{i\epsilon\tau}\{\exp[iI(0,t,0,t-\tau)+in\omega\tau]-1\}. \tag{37}$$

To summarize, Eqs. (36) and (37) permit one to determine the coefficients U_m which define $u(t)$ [according to Eq. (34)]. With $u(t)$ determined as well as $\Phi_\epsilon^{(0)}$, I, and I_0, Eq. (32) may be used to obtain the desired final-state wave function.

5. Evaluation of U_0 and U_{-1} in the limit $E_0\to 0$

As will be shown in Sec. III, in which we evaluate the transition matrix element [cf. Eq. (4)] for the cases of one- and two-photon detachment in the limit of weak laser fields, the only coefficients U_n [cf. Eq. (36)] that we require are U_0 and U_{-1}. We evaluate these here.

We show first that in the limit $E_0\to 0$, the summation in Eq. (36) is severely truncated. Substitute the $\mathbf{r}=0$ value of the action in Eq. (20) into Eq. (37), expand the exponential up to terms linear in E_0, and carry out the integral over t to obtain

$$M_{nm}=\delta_{nm}\frac{1}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}e^{i\epsilon\tau}\left\{\exp\left[i\left(n\omega\tau-\frac{E_s^2\tau^3}{24}\right)\right]-1\right\}+\delta_{n+1,m}\frac{iE_0E_s}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}\exp\left[i\left((\epsilon+n\omega)\tau-\frac{E_s^2\tau^3}{24}\right)\right]Q_+(\tau)+\delta_{n-1,m}\frac{iE_0E_s}{(2\pi i)^{1/2}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}\times\exp\left[i\left((\epsilon+n\omega)\tau-\frac{E_s^2\tau^3}{24}\right)\right]Q_-(\tau), \tag{38}$$

where

$$Q_+=Q_-^*\equiv\frac{(1-e^{-i\omega\tau})}{2\omega^3}+\frac{(1+e^{-i\omega\tau})}{4i\omega^2}\tau. \tag{39}$$

Second, expand the wave function in Eq. (12) up to first order in E_0 and compare the result with Eq. (35) to obtain

$$\Phi_0^{(0)}\equiv\frac{2^{1/3}}{2\pi|E_s|^{1/6}}\mathrm{Ai}(-\xi), \tag{40}$$

$$\Phi_{-1}^{(0)}\equiv\frac{iE_0}{2^{4/3}\pi\omega^2}\left(\frac{E_s^{1/3}}{|E_s|^{1/6}}\right)\left\{\mathrm{Ai}'(-\xi)+\frac{E_s^{2/3}}{2^{1/3}\omega}\mathrm{Ai}(-\xi)\right\}, \tag{41}$$

where

$$\xi\equiv(2/E_s^2)^{1/3}\epsilon_z^\alpha, \tag{42}$$

and where the prime on the Airy function in Eq. (41) is the total derivative (and not the derivative with respect to ξ).

Finally, substituting Eq. (38) into Eq. (36) we may solve for the coefficients U_0 and U_{-1} [keeping only terms of order $(E_0)^0$ in U_0 and of order $(E_0)^1$ in U_{-1}]:

$$U_0=-\Phi_0^{(0)}(-y_\epsilon+\kappa)^{-1}, \tag{43}$$

$$U_{-1}=-(\Phi_{-1}^{(0)}+M_{-1,0}U_0)(-y_{\epsilon-\omega}+\kappa)^{-1}, \tag{44}$$

where we have defined, for later convenience,

$$-y_{\epsilon+n\omega}\equiv M_{nn}+i\sqrt{2\epsilon}. \tag{45}$$

In Eqs. (43) and (44), $\Phi_0^{(0)}$ and $\Phi_{-1}^{(0)}$ are given in Eqs. (40) and (41); the matrix elements M_{00} (equivalently y_ϵ), M_{-1-1} (equivalently $y_{\epsilon-\omega}$), and $M_{-1,0}$ are defined by Eq. (38) and our method for their evaluation is presented in Appendix A. Equations (43) and (44) are necessary for determining the final-state wave function in the limit $E_0\to 0$. We shall employ these results when we evaluate the S-matrix elements for one- and two-photon detachment in this limit in Sec. III.

B. Initial-state wave function

In our approach [cf. Eq. (4)] the initial state should include all interactions other than the atom-laser interaction. Then it can be represented by the function

$$\psi_i(\mathbf{r},t)=e^{-i\epsilon_i t}\Phi_i(r)=2\pi BG_s(\mathbf{r},0), \tag{46}$$

where B is a normalization constant (whose numerical value equals 0.315 52 when $E_s=0$ [12]), and $G_s(\mathbf{r},\mathbf{r}')$ is the stationary Green's function for the electron in the static field E_s. Owing to the possibility of decay in the static field, the wave function in Eq. (46) is not, strictly speaking, stationary, and should be calculated for a complex energy whose imaginary part gives the decay width. However, in this paper, as in previous similar treatments of the problem [6–12], we consider the limit of pure real ϵ_i. Physically it means that for the fields under consideration the rate for detachment in the laser field is much greater than the rate for detachment in the static field. Although this condition puts certain limitations on the strength of the static field, the latter might still be relatively strong. Typically it is possible to have a field E_s up to 3 MV/cm without inclusion of static-field-decay effects. It should be emphasized that in spite of this approximation, the radial wave function in Eq. (46) does include the static field effect. [Note, however, that we have found the static field effects on the initial-state wave function to be small for the field strengths we consider; thus, in practice, our calculations have employed the zero-static-field result for the initial-state wave function given in Eq. (3).]

III. FORMULAS FOR ONE- AND TWO-PHOTON DETACHMENT CROSS SECTIONS IN THE WEAK-LASER-FIELD LIMIT

The S-matrix element in Eq. (4) requires $\psi_f^{(-)}(r,t)$, which is the appropriate wave function for photodetachment processes, whereas in the previous section we have obtained $\psi_f^{(+)}(r,t)$, which is the appropriate wave function for electron-atom scattering processes. As shown in Appendix B, however, these two final-state solutions are related as follows:

$$\psi_f^{(-)}(\mathbf{r},t,E_0,\mathbf{p}^\alpha)=\psi_f^{(+)*}(\mathbf{r},-t,-E_0,-\mathbf{p}^\alpha). \tag{47}$$

The matrix element in Eq. (4) actually requires $\psi_f^{(-)*}$, which equals

$$\psi_f^{(-)*}(\mathbf{r},t,E_0,\mathbf{p}^\alpha)=e^{+i\epsilon_f t}\Phi_E^{(0)*}(\mathbf{r},t,E_0,\mathbf{p}^\alpha)+e^{+i\epsilon_f t}\Phi_{\epsilon_f}^a(\mathbf{r},-t,-E_0), \tag{48}$$

where the first term on the right-hand side is obtained from Eqs. (9)–(13) and (22), and the second term on the right-hand side is defined by Eqs. (26) and (34) [4]:

$$\begin{aligned}\Phi_{\epsilon_f}^a(\mathbf{r},-t,-E_0)&\equiv\Phi_{\epsilon_f}^{(+)}(\mathbf{r},-t,-E_0)-\Phi_{\epsilon_f}^{(0)}(\mathbf{r},-t,-E_0,-\mathbf{p}^\alpha)\\&=\frac{1}{(-2\pi i)^{1/2}}\int_{-\infty}^{0}\frac{d\tau}{\tau^{3/2}}\exp\{-i[I(r,t;0,t-\tau;E_0)+\epsilon_f\tau]\}\\&\quad\times\sum_{n=-\infty}^{\infty}U_n(-E_0)e^{+in\omega(t-\tau)}.\end{aligned} \tag{49}$$

In obtaining Eq. (49) we have used the fact that $I(\mathbf{r},-t;-t+t',-E_0)=-I(\mathbf{r},t;t-t',E_0)$ [cf. Eq. (20)] as well as Eq. (34) for $u(-t+\tau)$. In Eq. (48), $U_n(-E_0)$ is obtained from Eq. (36) [using the analytic expressions for $\Phi_n^{(0)}(-E_0)$ and $M_{nm}(-E_0)$]. Substituting Eq. (48) into Eq. (4), we obtain

$$S_{fi}\equiv S_{fi}^0+S_{fi}^a, \tag{50}$$

where

$$S_{fi}^0\equiv-i\int_{-\infty}^{\infty}e^{+i\epsilon_f t}\langle\Phi_{\epsilon_f}^{(0)}|zE_0\sin\,\omega t|\psi_i(\mathbf{r},t)\rangle dt \tag{51}$$

is the S-matrix element ignoring rescattering by the atom and

$$S_{fi}^a\equiv-i\int_{-\infty}^{\infty}e^{+i\epsilon_f t}\langle\Phi_{\epsilon_f}^{a*}(\mathbf{r},-t,-E_0)|zE_0\sin\,\omega t|\psi_i(\mathbf{r},t)\rangle dt \tag{52}$$

is the rescattering correction. In what follows, we obtain the S-matrix elements in Eqs. (51) and (52) appropriate for one- and two-photon detachment in the limit of weak laser fields E_0 but strong static fields E_s (although not so strong that field ionization of the ground state is significant).

A. One- and two-photon S-matrix elements ignoring rescattering

The part of the total S-matrix element S_{fi} in Eq. (50) which ignores rescattering effects, S_{fi}^0, has been treated by Gao and Starace, whose results we employ here.[5] As shown in Ref. [6], upon carrying out the time integration in Eq. (51) one may write

$$S_{fi}^0=\sum_N S_{fi}^{0(N)}\delta\left(\epsilon_f+\frac{E_0^2}{4\omega^2}-\epsilon_i-N\omega\right), \tag{53}$$

where N is the number of photons absorbed by the H^- ion in the photodetachment process, $E_0^2/4\omega^2$ is the ponderomotive shift, and $S_{fi}^{0(N)}$ is given in the limit $E_0\rightarrow 0$ by Eq. (62) of Ref. [6]. For $N=1$ and 2 one obtains [using Eq. (3)]

$$S_{fi}^{0(N=1)}=\frac{2^{2/3}\pi BE_0}{\omega^2}\left(\frac{E_s^{1/3}}{|E_s|^{1/6}}\right)\left[\mathrm{Ai}'(-\xi)+\frac{(E_s^2/2)^{1/3}}{\omega}\mathrm{Ai}(-\xi)\right] \tag{54}$$

and

$$S_{fi}^{0(N=2)}=\frac{i\pi BE_0^2}{(2^{2/3}\omega^3|E_s|^{1/6})}\left\{\left(\frac{E_s^{2/3}\xi}{2^{1/3}\omega}-\frac{1}{4}-\frac{E_s^2}{2\omega^3}\right)\mathrm{Ai}(-\xi)-\frac{2^{1/3}E_s^{4/3}}{\omega^2}\mathrm{Ai}'(-\xi)\right\}. \tag{55}$$

B. One- and two-photon S-matrix elements including rescattering

In the limit $E_0\rightarrow 0$, because of the factor E_0 coming from the electric dipole interaction between the laser and the initial state of the ion, only terms independent of E_0 (respectively, of order E_0) in $\Phi_{\epsilon_f}^a$ contribute to the rescattering part of the S matrix in Eq. (52) for a one-photon (respectively, two-photon) transition. We thus expand $\Phi_{\epsilon_f}^a$ in powers of E_0 up to first order and, in order to carry out the time integrations, convert all factors of time to exponential form. The result is

[4]Note that the singularity in the integral in Eq. (49) at $\tau=0$ for $r\rightarrow 0$ is treated by adding and subtracting the free-particle Green's function, as discussed in Sec. II A 3 above. For simplicity of notation, we have not indicated this procedure explicitly in Eq. (49).

[5]Note that although Ref. [6] employs a different gauge for the laser field than that employed here, Ref. [6] has shown that for an initial state of the form in Eq. (3) the S-matrix element in Eq. (51) is gauge invariant.

$$\Phi^{a}_{\epsilon_f}(\mathbf{r},-t,-E_0)=-2\pi\sum_{n=-\infty}^{+\infty}e^{+in\omega t}U_n(-E_0)\times\sum_{q=-1}^{+1}e^{-iq\omega t}Q_q^n i^{|q|}(-E_0)^{|q|}, \quad (56)$$

where

$$Q_q^n\equiv\int_0^\infty d\tau e^{i(\epsilon_f+n\omega)\tau}G_s(r,\tau)\chi_q(r,\tau). \quad (57)$$

G_s is the Green's function for an electron moving in the static electric field,

$$G_s(r,\tau)\equiv\frac{-i}{(2\pi\tau)^{3/2}}\exp\left\{i\left(\frac{r^2}{2\tau}-\frac{E_s z\tau}{2}-\frac{E_s^2\tau^3}{24}\right)\right\}, \quad (58)$$

and

$$\chi_{q=0}=1, \quad (59)$$

$$\chi_{q=\pm1}=\frac{z}{2\omega}-\frac{qz}{2i\omega^2\tau}[1-e^{-iq\omega\tau}]+\frac{E_s}{2\omega^3}[1-qe^{-iq\omega\tau}]+\frac{qE_s\tau}{4i\omega^2}[1+e^{-iq\omega\tau}]. \quad (60)$$

The rescattering contribution to the S-matrix elements for one- and two-photon detachment may now be calculated by substituting Eqs. (56)–(60) into Eq. (52) and carrying out the time integrations. For one-photon detachment, only terms of order E_0 are included, while for two-photon detachment, only terms of order E_0^2 are included and, of course, the time integral must result in an energy-conserving delta function $\delta(\epsilon_f+E_0^2/4\omega^2-N\omega-\epsilon_i)$, where $N=1$ or 2. These considerations give the following result for $N=1$:

$$S_{fi}^{a(N=1)}=-2\pi^2U_0E_0\int d\mathbf{r}G_s(r,\epsilon_f)z\Phi_i^{(0)}(r), \quad (61)$$

where[6]

$$G_s(r,\epsilon_f)\equiv\int_0^\infty d\tau e^{i\epsilon_f\tau}G_s(r,\tau). \quad (62)$$

For $N=2$ we obtain

$$S_{fi}^{a(N=2)}=+2\pi^2iE_0^2U_0\int d\mathbf{r}\Pi_{\epsilon_f}(r)z\Phi_i^{(0)}(r,t)-2\pi^2E_0U_{-1}(-E_0)\times\int d\mathbf{r}G_s(r,\epsilon_f-\omega)z\Phi_i^{(0)}(r,t), \quad (63)$$

where

$$\Pi_{\epsilon_f}(r)\equiv\int_0^\infty d\tau e^{i\epsilon_f\tau}G_s(r,\tau)\chi_{+1}(r,\tau). \quad (64)$$

A convenient way to evaluate Π_{ϵ_f} is given in Appendix C.

Substituting Eqs. (43) and (44) for U_0 and U_{-1} [using Eqs. (40), (41), and (45)] into the rescattering part of the S-matrix elements in Eqs. (61) and (63) and combining the results with the S-matrix elements ignoring rescattering effects [Eqs. (54) and (55)], we obtain finally the complete S-matrix elements including rescattering effects. For one-photon detachment we obtain

$$S_{fi}^{(N=1)}=S_{fi}^{0(N=1)}+S_{fi}^{a(N=1)}\equiv\frac{2^{2/3}\pi BE_0}{\omega^2}\left(\frac{E_s^{1/3}}{|E_s|^{1/6}}\right)\left\{\mathrm{Ai}'(-\xi)+\mathrm{Ai}(-\xi)\left[\frac{(E_s^2/2)^{1/3}}{\omega}+\omega^2\frac{(-y_{\epsilon_f}+\kappa)^{-1}}{(2|E_s|)^{1/3}}\int d\mathbf{r}G_s(\mathbf{r},\epsilon_f)(z/r)e^{-\kappa r}\right]\right\}. \quad (65)$$

In Eq. (65), the evaluation of y_ϵ is discussed in Appendix A. For two-photon detachment we obtain

$$S_{fi}^{(N=2)}=S_{fi}^{0(N=2)}+S_{fi}^{a(N=2)}=\frac{-i\pi BE_0^2}{2^{8/3}\omega^3|E_s|^{1/6}}\left\{\mathrm{Ai}'(-\xi)\left[\frac{2^{7/3}E_s^{4/3}}{\omega^2}+\frac{2^{7/3}E_s^{1/3}\omega}{(-y_{\epsilon_f-\omega}+\kappa)}\int d\mathbf{r}G_s(\mathbf{r},\epsilon_f-\omega)(z/r)e^{-\kappa r}\right]+\mathrm{Ai}(-\xi)\left[1+\frac{2E_s^2}{\omega^2}-\frac{2^{5/3}E_s^{2/3}}{\omega}\xi+\frac{8\omega^3}{(-y_{\epsilon_f}+\kappa)}\int d\mathbf{r}\Pi_{\epsilon_f}(r)(z/r)e^{-\kappa r}+\frac{4E_s}{(-y_{\epsilon_f-\omega}+\kappa)}\left(1+i\frac{2M_{-1,0}(-E_0)}{E_sE_0}\frac{\omega^3}{(-y_{\epsilon_f}+\kappa)}\right)\int d\mathbf{r}G_s(r,\epsilon_f-\omega)(z/r)e^{-\kappa r}\right]\right\}. \quad (66)$$

In Eq. (66) the matrix element $M_{-1,0}$ and the functions $y_{\epsilon+n\omega}$ are discussed in Appendix A, and the integral involving $\Pi_{\epsilon_f}(r)$ is discussed in Appendix C.

[6] Note that Ref. [17] employs the sign convention for Green's functions in which a minus sign appears on the right-hand side of an equation such as Eq. (62). Comparisons of formulas in this paper with those in Ref. [17] must take this into account.

C. Generalized one- and two-photon cross sections

The S-matrix elements $S_{fi}^{(N)}$ for $N=1$ and 2 in Eqs. (65) and (66) may now be used to obtain the corresponding cross sections, which are defined as follows [6]:

$$\sigma^{(N)}=\frac{1}{F}\int W_{fi}^{(N)}dp_x^f dp_y^f d\epsilon_z^f, \tag{67}$$

where the transition rate $W_{fi}^{(N)}$ is defined by

$$W_{fi}^{(N)}=(2\pi)^{-1}|S_{fi}^{(N)}|^2\delta\left(\epsilon_f+\frac{E_0^2}{4\omega^2}-\epsilon_i-N\omega\right) \tag{68}$$

and where the photon flux is given by

$$F=\frac{cE_0^2}{8\pi\omega}. \tag{69}$$

Our results in the next section are presented for the generalized cross section [39], given by

$$\hat{\sigma}^{(N)}=\frac{\sigma^{(N)}}{F^{N-1}}, \tag{70}$$

since this generalized cross section is independent of E_0 in the lowest order in E_0. In evaluating Eqs. (67) and (68), we note that we may write $dp_x^f dp_y^f\equiv p_\perp^f dp_\perp^f d\theta_\perp=d(\frac{1}{2}p_\perp^2)d\theta_\perp$, where $\epsilon_f=\frac{1}{2}p_\perp^2+\epsilon_z^f$. Combining Eqs. (67)–(70), we obtain, for the generalized cross section,

$$\hat{\sigma}^{(N)}=\left(\frac{8\pi\omega}{cE_0^2}\right)^N\int_{-\infty}^{\epsilon_i+N\omega-E_0^2/4\omega}|S_{fi}^{(N)}|_{\overline{p_\perp}}^2 d\epsilon_z^f, \tag{71}$$

where the S matrix is evaluated at $p_\perp=\overline{p_\perp}$, where $\frac{1}{2}\overline{p_\perp^2}\equiv\epsilon_i+N\omega-E_0^2/4\omega-\epsilon_z^f$. From Eqs. (42) and (65), we obtain, for the single-photon detachment cross section,

$$\sigma^{(1)}=\hat{\sigma}^{(1)}=\frac{16\pi^3B^2|E_s|}{c\omega^3}\int_{-\infty}^{\xi_1}d\xi\left|\mathrm{Ai}'(-\xi)+\mathrm{Ai}(-\xi)\left[\frac{(E_s^2/2)^{1/3}}{\omega}+\frac{\omega^2(\kappa-y_{\epsilon_f})^{-1}}{(2|E_s|)^{1/3}}\int d\mathbf{r}G_s(\mathbf{r},\epsilon_f)(z/r)e^{-\kappa r}\right]\right|^2. \tag{72}$$

Similarly, from Eqs. (42) and (66) we obtain

$$\begin{aligned}\hat{\sigma}^{(2)}=&\frac{\pi^4B^2(2|E_s|)^{1/3}}{c^2\omega^4}\int_{-\infty}^{\xi_2}d\xi\Bigg|\mathrm{Ai}'(-\xi)\left[\frac{(2^7E_s^4)^{1/3}}{\omega^2}+\frac{(2^7E_s)^{1/3}\omega}{(\kappa-y_{\epsilon_f-\omega})}\int d\mathbf{r}G_s(\mathbf{r},\epsilon_f-\omega)(z/r)e^{-\kappa r}\right]\\&+\mathrm{Ai}(-\xi)\Bigg[1+\frac{2E_s^2}{\omega^2}-\frac{(2^5E_s^2)^{1/3}}{w}\xi+\frac{8\omega^3}{(\kappa-y_{\epsilon_f})}\int d\mathbf{r}\Pi_{\epsilon_f}(r)(z/r)e^{-\kappa r}\\&+\frac{4E_s}{(\kappa-y_{\epsilon-\omega})}\left(1+i\frac{2M_{-1,0}(-E_0)\omega^3}{E_sE_0(\kappa-y_{\epsilon_f})}\right)\int d\mathbf{r}G_s(\mathbf{r},\epsilon_f-\omega)(z/r)e^{-\kappa r}\Bigg]\Bigg|^2.\end{aligned} \tag{73}$$

The upper limits of the integrations in Eqs. (72) and (73) are defined by

$$\xi_N\equiv(\epsilon_i+N\omega-E_0^2/4\omega)(2/E_s^2)^{1/3}. \tag{74}$$

IV. RESULTS FOR PHOTODETACHMENT OF H^- IN A STRONG, STATIC ELECTRIC FIELD

Our results in this section and the next take full account of all three final-state interactions affecting an electron photodetached from H^-: its interactions with the static electric field, with the laser field, and with the atomic potential (which we treat in the zero-range potential approximation). Two points should be emphasized concerning the field strengths. First, our results are presented in the weak-laser-field limit. However, our formulation treats the laser field nonperturbatively and we then take the limit of our results as $E_0\to 0$. As shown in Ref. [6], even in this limit an additional term appears that is not included in formulations that treat the laser field in the lowest order of perturbation theory. The

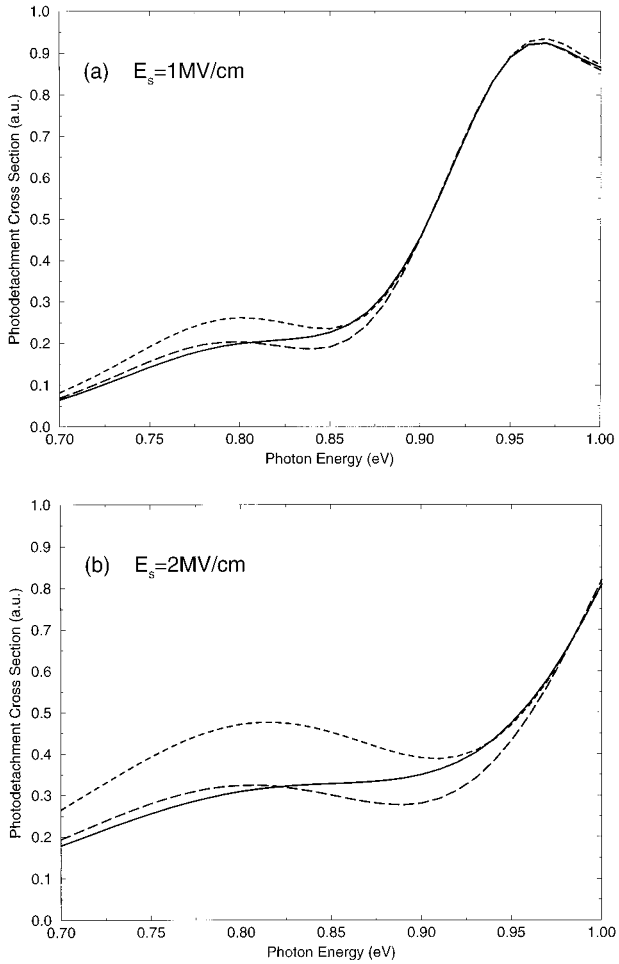

FIG. 1. Photodetachment cross section as a function of photon energy for a static electric field strength of 1 MV/cm (a) and 2 MV/cm (b). Short-dashed line: results of a weak (laser) field theory without rescattering effects [12]. Long-dashed line: results of a strong (laser) field theory without rescattering effects [6]. Solid line: present results [using Eq. (72)], which include all final-state interactions.

effect of this extra term is observable for strong static fields [6]. (In fact, it arises as a result of interference between the static and laser fields [6].) Second, we present our results for static electric fields of the order of 1 MV/cm. As shown in Ref. [6], such high static field strengths are still not so high that they significantly field ionize the H^- ion. They are, nevertheless, much higher than is typically employed in the laboratory. Nevertheless, such field strengths have been achieved in experiments using a relativistic H^- beam to convert a laboratory magnetic field into a static electric field in the H^- rest frame [8,40,41].

Our results for the photodetachment of H^- are presented in Figs. 1 and 2 and are compared with the theoretical predictions of Du and Delos [12] and of Gao and Starace [6]. The predictions of Ref. [12] are typical of weak (laser) field theories that ignore final-state interactions of the electron with the laser field as well as with the atomic potential [2,3,7–15]. The predictions of Ref. [6] represent results of a strong (laser) field theory that ignores final-state electron-atom (rescattering) interactions.

Figure 1 compares our results [obtained using Eq. (72)] for the photodetachment of H^- in a static electric field of 1 MV/cm [Fig. 1(a)] and 2 MV/cm [Fig. 1(b)] with predictions of both weak-field [12] and strong-field [6] theories which ignore rescattering effects. Near the zero-static-field threshold, we predict the plateau region to be substantially lower than the results of the weak-field theory [12]. On the other hand, our present results are close to the strong-field results of Gao and Starace [6]. This indicates that the rescattering effect is small. The results of previous calculations of one of us [17], in which rescattering effects were calculated in the weak-laser-field approximation using the frame transformation theory, overestimate the rescattering effect for strong static electric fields.

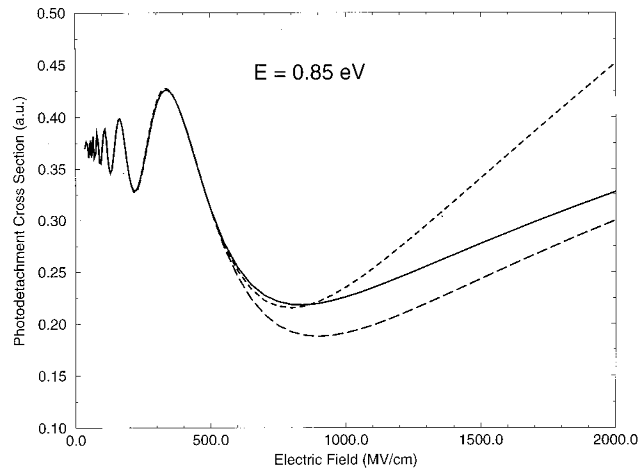

FIG. 2. Photodetachment cross section of H^- as a function of the static electric field strength at $\hbar\omega = 0.85$ eV. Curves are labeled as in Fig. 1.

Figure 2 plots the photodetachment cross sections for H^- in an external static electric field for a particular photon energy, 0.85 eV, which corresponds to the right edge of the plateau near the zero-static-field threshold, as a function of the static field strength. Note that the magnitude of the cross section and the differences between our results and those of the weak-field [12] and strong-field [6] predictions without rescattering decrease as the static electric field becomes smaller. The cross sections converge to the weak-field prediction [12] when the external static field decreases to zero, as expected. These conclusions are confirmed by recent calculations of Mese and Potvliege [42] who found the cross sections for photodetachment in a static electric field using the Sturmian-Floquet approach with a screened Coulomb potential for the e^--H interaction. Their results are quite close to ours for the cross section as a function of field strength.

V. RESULTS FOR TWO-PHOTON DETACHMENT OF H^- BOTH WITH AND WITHOUT A STRONG, STATIC ELECTRIC FIELD

It is well known that accurate prediction of the two-photon detachment cross section of H^- for the case of linearly polarized light requires that one take into account the $^1S^e$ phase shift of the detached electron [36,43]. This implies that the rescattering effects we treat should be strong for the two-photon detachment process. Hence we present our predictions for zero static electric field for the purpose of determining the reliability of our approach by comparison with

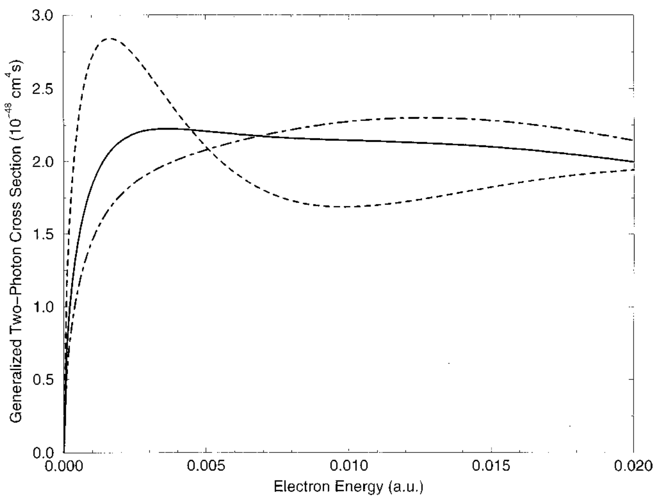

FIG. 3. Generalized two-photon detachment cross section for H^- plotted versus photoelectron kinetic energy. Solid curve: present zero-range potential model results. Dash-dotted curve: effective range model results [36]. Dashed curve: present free-electron model results.

results of others. We then present our results for two-photon detachment of H^- in the presence of a strong static electric field. (Our remarks given in the first paragraph of Sec. IV apply also to this section.)

Figure 3 presents generalized two-photon detachment cross sections for H^- without the presence of a static electric field in three levels of approximation: the free-electron approximation (which includes no final-state electron-atom interactions), the present zero-range potential model approximation (which includes final-state electron-atom interactions), and an effective range theory result from Ref. [36]. The effective range theory is more realistic than the zero-range potential model. Specifically, our zero-range potential approximation corresponds to the following expression for the phase shift:

$$k \cot\delta_s = -\kappa, \tag{75}$$

whereas effective range theory [44] provides a more precise expression for δ_s,

$$k \cot\delta_s = -\kappa + \frac{1}{2} r_{eff}(\kappa^2 + k^2), \tag{76}$$

where the variationally determined value of r_{eff} is 2.646 for H^- [45]. The two-photon detachment cross section incorporating this latter phase shift was obtained by Liu *et al.* [36]. One sees from Fig. 3 that the final-state rescattering interaction effect changes drastically the cross section near threshold (as compared to the free-electron approximation result). Furthermore, the effect is described reasonably well within the zero-range potential approximation, as evidenced by the good agreement with the effective range model results. This good agreement provides support for the reliability of our results in the presence of a static electric field, in which case there are no other results with which to compare.

In Fig. 4 we present the frequency dependence of the generalized two-photon detachment cross section for two

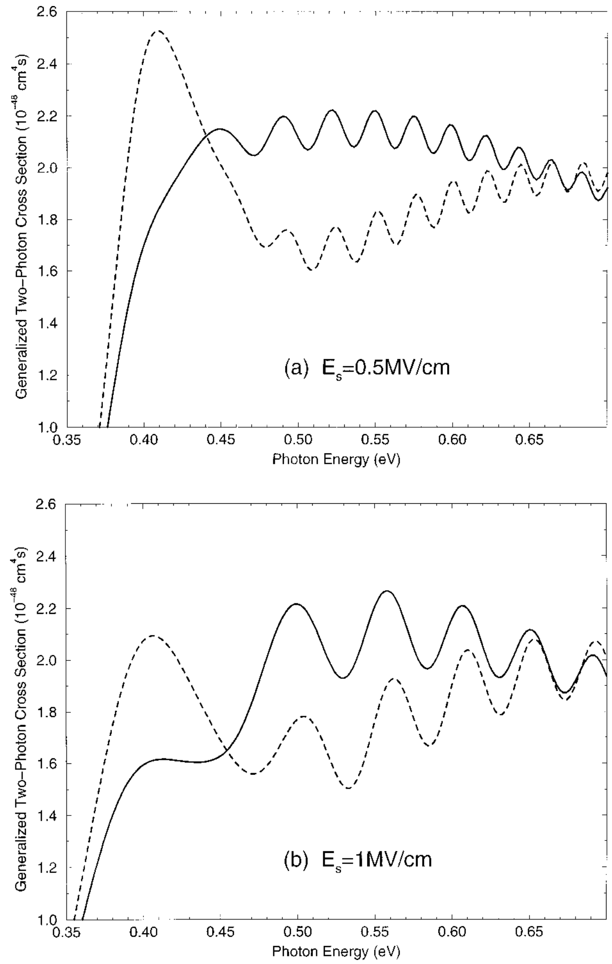

FIG. 4. Generalized two-photon detachment cross section for H^- plotted versus photon frequency for $E_s = 0.5$ MV/cm (a) and 1.0 MV/cm (b). Solid curve: present zero-range potential model results. Dashed curve: present free-electron model results.

values of the static field: $E_s = 0.5$ and 1.0 MV/cm. The amplitude of oscillations grows with increasing E_s, and the whole cross section pattern exhibits a high degree of sensitivity to final-state interactions. This sensitivity provides one an opportunity to extract information on the s-wave scattering phase shift from any experimental results for the photodetachment cross section. In principle, it can be done even for zero static field. However, by adding a nonzero static field, one can manipulate the detachment cross section and make it more sensitive to final-state interactions. This would allow a more precise determination of the phase shift.

VI. SUMMARY AND CONCLUSIONS

In this paper we have treated theoretically the effect of final-state interactions on the photodetachment cross section of H^- in a strong static electric field in the zero-range potential model approximation. As this model permits much of the theoretical work to be done analytically, we have presented the necessary development and the final results in detail, in a form suitable for numerical evaluation. Indeed, many of the details presented here are difficult to find elsewhere; we present them for the benefit of others who may wish to employ the zero-range potential model, the quasienergy approach, and/or the analytic Green's function that have been key ingredients of our formulation of the theory. These ingredients have enabled us to treat final-state interactions of the detached electron with the static electric field, the laser field, and the atomic potential. Although our results are presented for the weak-laser-field limit, we emphasize that this limit is obtained from our strong-field formulation, based on Ref. [6], which leads to terms that are not present in a formulation which treats the laser field perturbatively.

For single-photon detachment of H^- we find that final-state rescattering effects have only a very modest influence on the cross sections. Our results are similar to those of Gao and Starace [6], who treat final-state interactions of the detached electron with the static and laser electric fields, but who ignore rescattering effects. Our results also indicate that results of calculations employing a frame transformation theory [17] apparently overestimate the effect of rescattering interactions for strong static electric fields.

For two-photon detachment of H^- without a static electric field, for which rescattering effects are known to be strong [36,43], we find that our zero-range potential model results are in reasonable agreement with results of an effective range theory treatment [36]. This agreement is evidence that our results in the presence of a static electric field are also likely to be reliable. Our results show that the generalized two-photon detachment cross sections for H^- are highly sensitive to both final-state rescattering interactions and to the magnitude of the static electric field.

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APPENDIX A: EXPRESSIONS FOR M_{nm}

In the applications presented in this paper we evaluate the matrix elements M_{nm}, defined by Eq. (37), to first order in E_0. The analytic result is given in Eq. (38). This result, however, is not convenient for numerical computations. Much more readily evaluated expressions may be obtained by employing the stationary Green's function for an electron moving in a static electric field and its representation in terms of Airy functions. We obtain these expressions here in turn for a general diagonal matrix element M_{nn} and for the particular off-diagonal element M_{-10} which we require.

1. Expressions for M_{nn}

The diagonal element M_{nn} is given by the first line of Eq. (38). This expression for M_{nn} may be rewritten as the following limiting expression:

$$M_{nn}=-\lim_{r\to 0}\left[2\pi G_s(r,\epsilon+n\omega)+\frac{1}{r}+i\sqrt{2\epsilon}\right]. \quad \text{(A1)}$$

In this equation, G_s is the stationary Green's function for an electron moving in a static electric field [cf. Eqs. (58) and (62)] and the last two terms on the right-hand side stem from the equality in Eq. (31) upon expanding the exponential on the right-hand side of that equation to first order in kr ($\equiv\sqrt{2\epsilon}r$). In order to handle the apparent singularity near $r=0$ in Eq. (A1) we introduce the quantity y_ϵ, where

$$y_\epsilon\equiv 2\pi\frac{\partial}{\partial r}[rG_s(r,\epsilon)]\big|_{r=0}. \quad \text{(A2)}$$

Consider now the behavior of $G_s(r,\epsilon)$ near $r=0$. As discussed in Sec. II A 3, we use the procedure of Ref. [28] to treat the singularity in $G_s(r,\epsilon)$ for $r\to 0$ near the limit $\tau=0$ in the integral representation for this Green's function [cf. Eqs. (58) and (62)]; namely, we add and subtract the free-particle Green's function [cf. Eqs. (27) and (28)], as follows:

$$G_s(r,\epsilon)=\int_0^\infty d\tau e^{i\epsilon\tau}[G_s(r,\tau)-G_{\rm FP}(r,\tau)] + \int_0^\infty d\tau e^{i\epsilon\tau}G_{\rm FP}(r,\tau). \quad \text{(A3)}$$

Now, from Eq. (31),

$$\int_0^\infty d\tau e^{i\epsilon\tau}G_{\rm FP}(r,\tau)=-\frac{e^{ikr}}{2\pi r} \quad \text{(A4)}$$

and the integral of the difference between G_s and $G_{\rm FP}$ is well behaved near $r=0$. Comparison with Eq. (A1) shows that M_{nn} is well behaved near $r=0$ because of a cancellation of the r^{-1} singular terms. Numerically, however, it is advantageous to employ the function y_ϵ, which is a completely smooth function of r. We have only now to relate y_ϵ to M_{nn}. Formally carrying out the derivative in Eq. (A2), we obtain

$$y_\epsilon=2\pi G_s(r,\epsilon)\big|_{r=0}+2\pi r\frac{\partial G_s}{\partial r}(r,\epsilon)\big|_{r=0}. \quad \text{(A5)}$$

The only nonzero contribution to the derivative term in Eq. (A5) comes from the second integral on the right-hand side of Eq. (A3), whose value is given by Eq. (A4). Expanding the exponential in powers of r, one obtains

$$\lim_{r\to 0}2\pi r\frac{\partial G_s}{\partial r}(r,\epsilon)=\frac{1}{r}. \quad \text{(A6)}$$

Comparison of Eqs. (A5) and (A6) with Eq. (A1) shows that

$$M_{nn}=-y_{\epsilon+n\omega}-i\sqrt{2\epsilon}. \quad \text{(A7)}$$

This relationship was employed in Eqs. (43)–(45).

The function y_ϵ defined by Eq. (A2) is most conveniently calculated by using the representation for the Green's function $G_s(r,\epsilon)$ in terms of Airy functions obtained by Dalidchik and Slonim [46]:

$$G_s(r,\epsilon)=\frac{-1}{2r}[\mathrm{Ai}(\xi_1)\mathrm{Ci}'(\xi_2)-\mathrm{Ai}'(\xi_1)\mathrm{Ci}(\xi_2)], \quad \text{(A8)}$$

where

$$\mathrm{Ci}(\xi_2)=\mathrm{Bi}(\xi_2)+i\ \mathrm{Ai}(\xi_2) \tag{A9}$$

and where the arguments are defined by

$$\xi_1=\xi_\epsilon+\frac{(2E_s)^{1/3}}{2}(z-r),$$

$$\xi_2=\xi_\epsilon+\frac{(2E_s)^{1/3}}{2}(z+r), \tag{A10}$$

$$\xi_\epsilon=-\frac{2\epsilon}{(2E_s)^{2/3}}.$$

In Eqs. (A8) and (A9), Ai(ξ) and Bi(ξ) are the regular and irregular Airy functions [47]. Substituting Eq. (A8) into Eq. (A2) and making use of the differential equation satisfied by each Airy function to replace the second derivative terms [47], we obtain, finally,

$$y_\epsilon=-\pi(2E_s)^{1/3}[\mathrm{Ai}'(\xi_\epsilon)\mathrm{Ci}'(\xi_\epsilon)-\xi_0\mathrm{Ai}(\xi_\epsilon)\mathrm{Ci}(\xi_\epsilon)]. \tag{A11}$$

2. Expressions for M_{-10}

For $n=-1$ and $m=0$, Eq. (38) gives

$$M_{-10}=\frac{iE_0E_s}{2(2\pi i)^{1/2}\omega^3}\int_0^\infty\frac{d\tau}{\tau^{3/2}}\exp\left[i(\epsilon-\omega)\tau-i\frac{E_s^2\tau^3}{24}\right]\times\left[1-e^{-i\omega\tau}+\frac{\omega\tau}{2i}(1+e^{-i\omega\tau})\right]. \tag{A12}$$

Using Eqs. (58) and (62) for the stationary Green's function for $r=0$ as well as the expression

$$\frac{\partial G_s(r,\epsilon)}{\partial\epsilon}=i\int_0^\infty\tau d\tau e^{i\epsilon\tau}G_s(r,\tau), \tag{A13}$$

we may rewrite Eq. (A12) as

$$M_{-10}=-\frac{i\pi E_0E_s}{\omega^3}\left\{G_s(0,\epsilon-\omega)-G_s(0,\epsilon-2\omega)-\frac{\omega}{2}\left(\frac{\partial G_s}{\partial\epsilon}(0,\epsilon-\omega)+\frac{\partial G_s}{\partial\epsilon}(0,\epsilon-2\omega)\right)\right\}. \tag{A14}$$

Through use of Eq. (A8), one may express Eq. (A14) in terms of Airy functions. However, one must employ the expressions for the Green's functions with nonzero values of $\mathbf{r}$ and take the limit as $\mathbf{r}\to0$. For simplicity, one may take $r=z$ and then take the limit as $z\to0$. Thus

$$\lim_{z\to0}[G_s(z,\epsilon-\omega)-G_s(z,\epsilon-2\omega)]=\lim_{z\to0}\left\{-\frac{1}{2z}[\mathrm{Ai}(\xi_1)\mathrm{Ci}'(\xi_2)-\mathrm{Ai}'(\xi_1)\mathrm{Ci}(\xi_2)]+\frac{1}{2z}[\mathrm{Ai}(\overline{\xi_1})\mathrm{Ci}'(\overline{\xi_2})-\mathrm{Ai}'(\overline{\xi_1})\mathrm{Ci}(\overline{\xi_2})]\right\}, \tag{A15}$$

where [cf. Eq. (A10)]

$$\xi_1=\xi_{\epsilon-\omega},$$
$$\xi_2=\xi_{\epsilon-\omega}+(2E_s)^{1/3}z,$$
$$\overline{\xi_1}=\xi_{\epsilon-2\omega}, \tag{A16}$$
$$\overline{\xi_2}=\xi_{\epsilon-2\omega}+(2E_s)^{1/3}z.$$

In the limit $z\to0$, we may expand Ci(ξ_2) and Ci(ξ_2) about ξ_1 and Ci($\overline{\xi_2}$) and Ci′($\overline{\xi_2}$) about $\overline{\xi_1}$. Using also the fact that [47]

$$\mathrm{Ai}(\xi)\mathrm{Ci}'(\xi)-\mathrm{Ai}'(\xi)\mathrm{Ci}(\xi)=\pi^{-1}, \tag{A17}$$

we obtain

$$G_s(0,\epsilon-\omega)-G_s(0,\epsilon-2\omega)=+\frac{(2E_s)^{1/3}}{2}[J_{\epsilon-\omega}-J_{\epsilon-2\omega}], \tag{A18}$$

where we have defined

$$J_\epsilon\equiv\mathrm{Ai}'(\xi_\epsilon)\mathrm{Ci}'(\xi_\epsilon)-\xi_\epsilon\mathrm{Ai}(\xi_\xi)\mathrm{Ci}(\xi_\epsilon), \tag{A19}$$

where ξ_ϵ is defined by Eq. (A10). Finally, the derivative terms in Eq. (A14) may be obtained by differentiating Eq. (A8) and taking the limit $r\to0$:

$$\lim_{r\to0}\frac{\partial G_s}{\partial\epsilon}(r,\epsilon)=\lim_{r\to0}(2E_s)^{-1/3}\mathrm{Ai}(\xi_1)\mathrm{Ci}(\xi_2)=(2E_s)^{-1/3}\mathrm{Ai}(\xi_\epsilon)\mathrm{Ci}(\xi_\epsilon). \tag{A20}$$

Substituting Eq. (A18) and (A20) into Eq. (A14), we obtain finally an expression for M_{-10} in terms of Airy functions, which are convenient for numerical evaluation:

$$M_{-10}=-i\frac{\pi E_0E_s}{\omega^3}\left\{\frac{(2E_s)^{1/3}}{2}[J_{\epsilon-\omega}-J_{\epsilon-2\omega}]-\frac{\omega}{2(2E_s)^{1/3}}[\mathrm{Ai}(\xi_{\epsilon-\omega})\mathrm{Ci}(\xi_{\epsilon-\omega})+\mathrm{Ai}(\xi_{\epsilon-2\omega})\mathrm{Ci}(\xi_{\epsilon-2\omega})]\right\}, \tag{A21}$$

where J_ϵ is defined by Eq. (A19).

APPENDIX B: RELATION BETWEEN $\psi_f^{(+)}$ AND $\psi_f^{(-)}$

Although for calculation of the photodetachment matrix element we need the solution $\psi_f^{(-)}$, it was more convenient for us to calculate first $\psi_f^{(+)}$. Therefore we need to establish the relationship between these two solutions of our nonstationary problem for an electric field of the form in Eq. (1). We write first Eq. (8) in a more general form

$$\psi_{\mathbf{p}E_0}^{(+)}(\mathbf{r},t)=\psi_{\mathbf{p}E_0}^{(0)}(\mathbf{r},t)+\int_{-\infty}^t dt'\int d\mathbf{r}'\times G_0^{(+)}(\mathbf{r},t;\mathbf{r}',t';E_0)V(\mathbf{r}')\psi_{\mathbf{p}E_0}^{(+)}(\mathbf{r}',t'), \tag{B1}$$

where the retarded Green's function is given by Eq. (15) and $\psi^{(0)}_{\mathbf{p}E_0}\equiv\psi_0$ by Eqs. (9)–(12). (Note that our slight change of notation in this appendix from that used in the main text is meant to indicate explicitly the dependence on the electron's momentum **p** and the laser field amplitude E_0.) In the stationary theory, the ''minus'' solution $\psi_f^{(-)}$ can be obtained from the ''plus'' solution $\psi_f^{(+)}$ by performing three operations: complex conjugation, $\mathbf{p}\to-\mathbf{p}$, and $t\to-t$. However, in a nonstationary problem, time reversal changes the Hamiltonian. Therefore, there is no general relationship between ''plus'' and ''minus'' solutions. However, in our problem the Hamiltonian does not change if the operation $t\to-t$ is performed together with $E_0\to-E_0$, as can be seen from Eq. (1). In particular, it can be verified directly that $\psi^{(0)}_{\mathbf{p}E_0}$, given by Eqs. (9)–(12), does not change if we perform the following four operations: complex conjugation, $\mathbf{p}\to-\mathbf{p}$, $t\to-t$, and $E_0\to-E_0$.

Applying now all these operations to Eq. (B1), and using Eq. (15) for the retarded propagator, we obtain

$$\psi^{(+)*}_{-\mathbf{p}-E_0}(\mathbf{r},-t)=\psi^{(0)}_{\mathbf{p}E_0}(\mathbf{r},t)+\int_{-\infty}^{-t}dt'\int d\mathbf{r}'\,\frac{i}{[2\pi i(t+t')]^{3/2}}\times\exp[-iI_{-E_0}(\mathbf{r},-t,\mathbf{r}',t')]V(\mathbf{r}')\times\psi^{(+)*}_{-\mathbf{p}-E_0}(\mathbf{r}',t'). \tag{B2}$$

Introducing a new integration variable $t''=-t'$ and using the following property of the action [cf. Eq. (20)],

$$I_{-E_0}(-t,-t'')=-I_{E_0}(t,t''), \tag{B3}$$

we obtain

$$\psi^{(+)*}_{-\mathbf{p}-E_0}(\mathbf{r},-t)=\psi^{(0)}_{\mathbf{p}E_0}(\mathbf{r},t)+\int_t^{\infty}dt''\int d\mathbf{r}'\,\frac{i}{[-2\pi i(t''-t)]^{3/2}}\times\exp[iI_{E_0}(\mathbf{r},t,\mathbf{r}',t'')]V(\mathbf{r}')\psi^{(+)*}_{-\mathbf{p}-E_0}(\mathbf{r}',-t''). \tag{B4}$$

According to Reiss [37], the advanced Green's function corresponding to the ''minus'' solution can be written as

$$G^{(-)}(\mathbf{r},t,\mathbf{r}',t')=\frac{i\theta(t'-t)}{[2\pi i(t-t')]^{3/2}}\exp[iI(\mathbf{r},t,\mathbf{r}',t')]. \tag{B5}$$

This allows us to rewrite Eq. (B4) as

$$\psi^{(-)}_{\mathbf{p}E_0}(\mathbf{r},t)=\psi^{(0)}_{\mathbf{p}E_0}(\mathbf{r},t)+\int dt'\int d\mathbf{r}'\times G^{(-)}_{E_0}(\mathbf{r},t,\mathbf{r}',t')V(\mathbf{r}')\psi^{(-)}_{\mathbf{p}E_0}(\mathbf{r}',t'), \tag{B6}$$

where

$$\psi^{(-)}_{\mathbf{p}E_0}(\mathbf{r},t)=\psi^{(+)*}_{-\mathbf{p}-E_0}(\mathbf{r},-t). \tag{B7}$$

APPENDIX C: EXPRESSIONS FOR $\Pi_{\epsilon_f}(r)$

We discuss here methods for calculating $\Pi_{\epsilon_f}(r)$, which is defined by Eq. (64). Substituting Eq. (60) into Eq. (64), we have

$$\Pi_{\epsilon_f}(r)=\int_0^{\infty}d\tau e^{i\epsilon_f\tau}G_s(r,\tau)\left\{\frac{z}{2\omega}-\frac{z}{2i\omega^2\tau}[1-e^{-i\omega\tau}]+\frac{E_s}{2\omega^3}[1-e^{-i\omega\tau}]+\frac{E_s\tau}{4i\omega^2}[1+e^{-i\omega\tau}]\right\}, \tag{C1}$$

where the Green's function $G_s(r,\tau)$ is defined by Eq. (58). Noting from Eqs. (58) and (62) that

$$-2i\frac{\partial G_s}{\partial(x^2)}(r,\epsilon_f)=\int_0^{\infty}\frac{d\tau}{\tau}G_s(r,\tau)e^{i\epsilon_f\tau}, \tag{C2}$$

and using Eq. (A13), Eq. (C1) may be rewritten in terms of $G_s(r,\epsilon_f)$ as follows:

$$\Pi_{\epsilon_f}(r)=\frac{z}{2\omega}G_s(r,\epsilon_f)+\frac{z}{\omega^2}\left\{\frac{\partial G_s}{\partial(x^2)}(r,\epsilon_f)-\frac{\partial G_s}{\partial(x^2)}(r,\epsilon_f-\omega)\right\}+\frac{E_s}{2\omega^3}[G_s(r,\epsilon_f)-G_s(r,\epsilon_f-\omega)]-\frac{E_s}{4\omega^2}\left[\frac{\partial G_s}{\partial\epsilon_f}(r,\epsilon_f)+\frac{\partial G_s}{\partial\epsilon_f}(r,\epsilon_f-\omega)\right]. \tag{C3}$$

Equation (C3) may be more conveniently expressed in terms of Airy functions. From the Airy function representation for $G_s(r,\epsilon)$ given in Eq. (A8), we obtain [using Eq. (A9) and (A10)]

$$\frac{\partial G_s(r,\epsilon)}{\partial(x^2)}=\frac{1}{2r^2}\left\{-G_s(r,\epsilon)+\frac{(2E_s)^{1/3}}{2}\left[+\mathrm{Ai}'(\xi_1)\mathrm{Ci}'(\xi_2)-\left(\xi_\epsilon+\frac{(2E_s)^{1/3}}{2}z\right)\mathrm{Ai}(\xi_1)\mathrm{Ci}(\xi_2)\right]\right\}. \tag{C4}$$

The terms in Eq. (C3) involving $\partial G_s/\partial\epsilon_f$ may be expressed in terms of Airy functions using the first equality in Eq. (A20). Thus, Eqs. (A8), (A20), and (C4) allow us to express $\Pi_{\epsilon_f}(r)$ in terms of Airy functions. The apparent singularity in the limit $r\to 0$ for the third term in Eq. (C3) (involving a difference of Green's functions for two energies) may be treated as in Eqs. (A18) and (A19).

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