

Need for remeasurements of nuclear magnetic dipole moments

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The need for a reassessment of nuclear magnetic dipole moments is prompted by recent experiments on the ground-state hyperfine structure in highly charged hydrogenlike systems which are sufficiently sensitive to probe QED effects. This work gives an overview of the magnetic dipole moments for the nuclei of interest, i.e., ^{165}Ho , $^{185,187}\text{Re}$, $^{203,205}\text{Tl}$, ^{207}Pb , and ^{209}Bi . It is found that the present uncertainties in the nuclear magnetic dipole moment limit the interpretation of the accurate experimental hyperfine structures for these systems.

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I. INTRODUCTION

Recent studies of the ground-state hyperfine structure (HFS) in highly charged hydrogenlike systems point to the need for a reassessment of nuclear magnetic moments. The interest in these systems arises, since the recent experiments [1–4] are sufficiently sensitive to probe QED effects, which are about 0.5% for these systems [5–8]. To make use of the attainable experimental accuracy, it is essential that the nuclear magnetic moments used are reliable and accurate. Since the HFS is directly proportional to the nuclear magnetic dipole moment μ_I , any uncertainty in μ_I will directly affect the comparison. In order to provide a critical test of QED effects, the nuclear moment has to be known at least to a fraction of a percent.

In addition, the HFS is sensitive to details in the nuclear charge and magnetization distributions, the so-called “Breit-Rosenthal” and “Bohr-Weisskopf” effects. Whereas the charge distribution is, in general, sufficiently well known not to disturb the analysis, the Bohr-Weisskopf effect is quite uncertain. In some cases, it may be useful to reverse the analysis, and use instead the calculated QED effects as input to provide information about the distribution of magnetization in the nucleus [3,9]. A reliable nuclear moment is, of course, still essential.

We review here the tabulated values for the nuclei of interest, i.e., ^{165}Ho , $^{185,187}\text{Re}$, $^{203,205}\text{Tl}$, ^{207}Pb , and ^{209}Bi , together with the reference nuclei ^1H , ^{23}Na , and ^{199}Hg . Section II contains a brief discussion about methods to determine nuclear magnetic dipole moments. Section III contains, for each nucleus of interest, the result and a discussion about how it was obtained. Summarizing in Sec. IV, we find that the present uncertainties in the magnetic moment limit the interpretation of the accurate experimental hyperfine structures for these systems.

II. MEASURED QUANTITIES

The HFS in the $1s$ ground state of hydrogenlike ions arises from an interaction between the magnetic dipole moments of the electron and the nucleus. Values for the nuclear moments can be found in several tabulations, where many values have an origin in measurements carried out in the 1950s and 60s, often using the technique of nuclear magnetic

resonance (NMR). Somewhat later, other methods, such as atomic beam magnetic resonance (ABMR) and optical pumping (OP), have also been used. Several methods are described in the 1976 compilation by Fuller [10]; more detailed treatments of nuclear moment determination can be found, e.g., in the classic textbooks by Ramsey [11] and Kopfermann [12], and in the works by Lindgren [13] and Nierenberg and Lindgren [14]. The most recent tabulation of nuclear magnetic dipole moments is the one in the 8th edition from 1996 of *Table of Isotopes* by Firestone [15]. This table was based on the 1989 compilation by Raghavan [16], except for values which have since been updated in the *Nuclear Data Sheets*. The work by Raghavan was preceded by the compilation in the 7th edition from 1978 of *Table of Isotopes* by Lederer and Shirley [17].

The nuclear magnetic dipole moment can be written as

$$\mu_I = g_I I \mu_N,$$

where g_I is the nuclear g factor, I is the nuclear spin, $\mu_N = e\hbar/2m_p$ is the nuclear magneton, e is the elementary charge, \hbar is the Planck constant divided by 2π , and m_p is the proton mass. It is sometimes practicable to use the quantity

$$g'_I = g_I \mu_N / \mu_B = g_I m_e / m_p,$$

where $\mu_B = e\hbar/2m_e$ is the Bohr magneton and m_e is the electron mass. (Note that Refs. [10,18–21] use an opposite sign convention for g'_I .) In all measurements of nuclear magnetic dipole moments an external magnetic field B_0 is applied, and the measured quantity is always proportional to $\mu_I B_0$. In, e.g., NMR measurements, the Larmor frequency

$$\nu_L = \mu_I B_0 / \hbar I = g_I \mu_N B_0 / \hbar$$

is measured, whereas in direct measurements using ABMR the quantity $2g'_I \mu_B B_0$ is measured.

A. Shielding of the magnetic field

The external magnetic field induces a diamagnetic current density in the electron cloud surrounding the nucleus. This leads to an induced magnetic field $B'(0)$ at the nucleus opposing the external field, so that the internal field at the nucleus becomes

$$B = B_0 - B'(0) = B_0[1 - B'(0)/B_0] = B_0(1 - \sigma).$$

We here introduce a *magnetic shielding constant* σ , which cannot be determined by varying the magnetic field because of the proportionality between B and B_0 . To allow for this diamagnetic effect the observed nuclear magnetic dipole moment must thus be multiplied by $(1 - \sigma)^{-1}$. A value of the nuclear magnetic dipole moment which is not corrected for the magnetic shielding is called *uncorrected*, and will here be denoted by μ'_I , to differ from the *corrected* (or bare) value μ_I .

The problem of magnetic shielding in atoms was first considered by Lamb [22], who found that, for a spherical electron distribution

$$\sigma = \frac{B'(0)}{B_0} = -\frac{e}{3m_e c^2} V(0), \quad (1)$$

where c is the speed of light in vacuum and $V(0)$ is the electrostatic potential at the nucleus produced by the electrons. For a closed-subshell system we find

$$V(0) = -\frac{e}{4\pi\epsilon_0} \sum_a (2j_a + 1) \left\langle \frac{1}{r} \right\rangle_a,$$

where the summation is over all atomic subshells a . For hydrogenlike systems we find $\sigma = \alpha^2 Z/3$, where α is the fine-structure constant and Z is the atomic number. Using an effective screening radius $r \approx a_0/Z^{1/3}$, together with the Thomas-Fermi model for electron densities, Lamb was able to obtain an approximate relation $\sigma \approx (-0.319 \times 10^{-4}) Z^{4/3}$. In addition, he applied the Hartree model for a number of systems, and found shielding factors that are between 10% and 20% smaller. In 1950, Dickinson [23] used available Hartree and Hartree-Fock calculations for a number of atoms and ions to obtain a table of σ values for neutral atoms. More recently, Johnson and co-workers [24–27] studied the magnetic shielding factors using relativistic wave functions, as well as a proper relativistic operator. The use of a relativistic operator was found to be the dominant effect, and led to an increase of σ by nearly 50% for $Z \approx 80$. Their first calculations [24,25] were based on relativistic Hartree-Fock-Slater (RHFS) wave functions for a number of closed-subshell atoms and ions. In later work [26,27], they included also corrections from the relativistic random-phase approximation (RPA) for a number of closed-shell systems. The results for the atomic systems considered here are denoted by σ_a and given in Table I.

B. Chemical shifts

NMR measurements are usually performed on molecules in an aqueous solution. The magnetic field is then also shielded by the chemical environment, i.e., the molecular compound and the water. The variation of σ is called *the chemical shift*. Ramsey [28,29] found that the shielding factor for molecules, σ_m , can be split into two terms:

$$\sigma_m = \sigma_{\text{dia}} + \sigma_{\text{hf}}. \quad (2)$$

TABLE I. Diamagnetic shielding factors σ_a for atomic systems. The values are obtained by three different procedures based on nonrelativistic Hartree and Hartree-Fock calculations (H/HF) for neutral atoms [23], the relativistic Hartree-Fock-Slater (RHFS) electron theory for closed-subshell systems [24,25], and the relativistic random-phase approximation (RPA) for closed-shell systems [26,27], respectively. In addition, values for neutral Na, Ho, and Tl are also given in the RHFS column, these values are obtained from the table of spherical average diamagnetic corrections for neutral atoms, calculated by Lin, Johnson, and Feiock, and quoted in the compilation by Fuller [10].

System	H/HF ^a	RHFS	RPA
Na	0.000 629	0.000 6491 ^b	
Na ⁺		0.000 6426 ^c	0.000 6322 ^d
Ho	0.007 56	0.010 65 ^b	
Re ⁷⁺		0.013 56 ^c	0.013 31 ^e
Hg	0.009 65	0.015 87 ^c	0.015 77 ^e
Tl	0.009 82	0.016 36 ^b	
Tl ⁺		0.016 36 ^c	0.016 24 ^e
Pb	0.009 98	0.020 55 ^c	
Pb ²⁺		0.016 86 ^c	0.016 74 ^e
Bi ³⁺		0.017 39 ^c	0.017 27 ^{e,f}

^aDickinson [23].

^bLin, Johnson, and Feiock, quoted by Fuller [10].

^cFeiock and Johnson [25].

^dJohnson, Kolb, and Huang [27].

^eJohnson [31].

^fBařtuř *et al.* [46].

The first term is essentially the Lamb correction, discussed above, with contributions mainly from the inner electron shells. This part is thus expected to be relatively insensitive to the chemical environment.

The second term is often called the high-frequency term, and it is explained in terms of a temperature-independent paramagnetism which is expected in molecules having excited electronic states close to the ground state. This paramagnetic term is quite sensitive to the chemical environment, since it is mainly caused by the outer electrons and can also depend on the concentration. It is difficult to evaluate except for simple molecular systems, since it depends on excitations energies in the molecule. For most elements, the chemical shift seems to be of the order of 10^{-3} or 10^{-4} , but can sometimes be larger. Shifts up to 1.3% have been observed in Co compounds [12,13]. When NMR results are used solely for calibration, as discussed below, any chemical shift for the reference nucleus cancels. For ABMR and optical pumping measurements performed on free atoms or ions, of course, no correction for chemical shift is needed, only the correction for the diamagnetic shielding.

C. Reported quantities

An absolute determination of the applied magnetic field B_0 is very difficult. Therefore, especially in the case of NMR, B_0 may be calibrated by a simultaneous measurement for a known nucleus, and the result is then often reported as a ratio of Larmor frequencies for the nucleus of interest and the reference nucleus. The value of an uncorrected magnetic dipole moment is then given by

TABLE II. Recommended values of the nuclear magnetic dipole moments discussed in this paper. The diamagnetic corrections $(1 - \sigma_a)^{-1}$ are obtained with use of the σ_a values in Table I, discussed in more detail in Sec. III.

Isotope	Method	Ref.	Measured quantity	μ'/μ_N	Ionization	$(1 - \sigma_a)^{-1}$	μ/μ_N
²³ Na	ABMR	[18]	$g'_I = 0.804\,6108(8) \times 10^{-3}$	2.216\,082(2)	0	1.000\,639(5)	2.217\,499(11)
¹⁶⁵ Ho	ABMR	[21]	$g'_I = 0.642\,99(80) \times 10^{-3}$	4.1322(51)	0	1.010\,76(20)	4.1767(53)
¹⁸⁵ Re	NMR	[39]	$\nu/\nu(^{23}\text{Na}) = 0.851\,14(9)$	3.1439(3)	7+	1.013\,49(13)	3.186(3) ^a
¹⁸⁷ Re	NMR	[39]	$\nu/\nu(^{23}\text{Na}) = 0.859\,87(9)$	3.1761(3)	7+	1.013\,49(13)	3.219(3) ^a
¹⁹⁹ Hg	OP	[36]	$\mu'_I/\mu'_p = 0.178\,2706(3)$	0.497\,8698(8)	0	1.016\,02(5)	0.505\,847(26)
²⁰³ Tl	ABMR	[41]	$g'_I = 17.375(14) \times 10^{-4}$	1.5952(13)	0	1.016\,63(21)	1.6217(13)
²⁰⁵ Tl	ABMR	[41]	$g'_I = 17.549(14) \times 10^{-4}$	1.6111(13)	0	1.016\,63(21)	1.6379(13)
²⁰⁷ Pb	ABMR	[44]	$g'_I = 6.314(15) \times 10^{-4}$	0.5797(14)	0	1.020\,98(21)	0.5918(14) ^b
²⁰⁹ Bi	NMR	[45]	$\nu/\nu(^2\text{H}) = 1.046\,84(5)$	4.039\,10(19)	3+	1.017\,57(6)	4.110(4) ^a

^aSubject to a chemical shift. It must be emphasized that the error can be larger.

^bSee the discussion in Sec. III G.

$$\mu'_a = \mu'_p (\nu_a/\nu_b) (\nu_b/\nu_p) (I_a/I_p),$$

where the subscript a refers to the nucleus under consideration, b to a reference standard, and p to the proton. The value used for the proton moment is given in Sec. III A. In ABMR measurements, a possible way to determine B_0 is to measure a quantity depending on $g_J B_0$, where g_J is the electronic g factor, and the determination of g'_I can then be reduced to a measurement of the ratio of g'_I/g_J , but the reported quantity is g'_I . The value of an uncorrected magnetic dipole moment is obtained by using the relation

$$\mu'_I = g'_I I \frac{m_p}{m_e} \mu_N$$

where the proton-electron mass ratio is $m_p/m_e = 1836.152\,701(37)$ [30].

III. RESULTS

The determination of the nuclear magnetic dipole moments for the nuclei, ¹⁶⁵Ho, ^{185,187}Re, ^{203,205}Tl, ²⁰⁷Pb, and ²⁰⁹Bi, is reviewed below in Secs. III D–III H. First, however, we discuss, in Secs. III A–III C the nuclear magnetic dipole moments of ^{1,2}H, ²³Na, and ¹⁹⁹Hg, which are used as reference nuclei in some cases. All corrected results are obtained with use of the σ_a values in Table I, the RPA values are used wherever available, and half the difference between the RHFS and RPA values for a certain system is assigned as a uncertainty for the RPA value, since the dominant corrections are assumed to be included in the RPA values. Other corrections enter in higher orders of perturbation theory, and are expected to be smaller [31]. For neutral Na, Ho, and Tl spherical average diamagnetic corrections, calculated by Lin, Johnson, and Feiock and quoted in the compilation by Fuller [10], have been used. In the case of results achieved by NMR the corrected values are connected with an extra estimated uncertainty of one part in 10^3 to consider the unknown chemical shift. It must be emphasized, however, that the error can be larger.

The results are also compared with the values in the 8th edition of *Table of Isotopes* [15]. These values are said to be ‘‘corrected for diamagnetic shielding wherever applicable.’’ Although no table of σ values is given, they can be assumed

to be taken from the work by Feiock and Johnson [24,25] based on RHFS calculations. These values were used by both Raghavan [16] and by Lederer and Shirley in the 7th edition of *Table of Isotopes* [17]. No consideration of the chemical shift has, however, been applied in these compilations, except for the case of $Z=1$, discussed below. The present status of the nuclear magnetic dipole moments discussed are summarized in Table II.

A. ^{1,2}H

The magnetic dipole moments for the proton and the deuteron are given by Cohen and Taylor in the most recent update of fundamental physical constants [30]. The number for the bare proton moment is $\mu_p = 2.792\,847\,386(63)\mu_N$, and that for the shielded proton moment is $\mu'_p = 2.792\,775\,642(64)\mu_N$, where the latter is obtained for a spherical pure water sample at 25 °C, corresponding to $\sigma_{\text{H}_2\text{O}} = 25.689(15) \times 10^{-6}$. Raghavan [16] refers to this value, whereas the 7th edition of *Table of Isotopes* [17] refers to an older value [32]. Cohen and Taylor also give the formula for the magnetic shielding from the electron on the proton in a hydrogen atom, in addition to Lamb’s correction $\alpha^2/3$, obtained from Eq. (1), this relation includes reduced mass, relativistic and radiative corrections:

$$\mu_p(\text{H}) = \mu_p \left[1 - \alpha^2 \left[\frac{1}{3} - \frac{m_p m_e}{2(m_p + m_e)^2} \right] + \frac{\alpha^2(m_p + 3m_e)m_e a_p}{6(m_p + m_e)^2(1 + a_p)} + \dots \right],$$

where $a_p = \mu_p/\mu_N - 1$ is the proton magnetic moment anomaly. This relation yields $\sigma_H = 17.7329 \times 10^{-6}$, corresponding to $\mu_p(\text{H}) = 2.792\,797\,861(63)\mu_N$.

The ratio for the magnetic dipole moments of the deuteron and the proton is given to be

$$\mu_d/\mu_p = 0.307\,012\,2035(51),$$

giving

$$\mu_d = 0.857438\,230(24)\mu_N.$$

According to Kaminker *et al.* [33], in the case of NMR, the ratio of the proton and deuteron g factors is related to the ratio of the resonance frequencies by the formula

$$g_p/g_d \approx \nu_p/\nu_d [1 - (\sigma_d - \sigma_p)],$$

in which σ_d and σ_p are the deuteron and proton screening constants. Neronov and Barzakh [34] give $\sigma_d - \sigma_p = 15.0 \times 10^{-9}$, yielding $\nu_d/\nu_p = 0.153\,506\,1041(26)$. This frequency ratio is used as reference in the NMR measurements of the nuclear magnetic dipole moments in ^{207}Pb and ^{209}Bi and also in ^{23}Na , which in turn serves as reference for the nuclear magnetic dipole moments in $^{185,187}\text{Re}$.

B. ^{23}Na

The two most recent measurements of the nuclear magnetic dipole moment in ^{23}Na , were done by Lutz in 1967 using NMR [35], and by Beckmann, Böklen, and Elke in 1974 using ABMR [18]. The NMR measurement was carried out on NaCl in heavy water, and the result for a vanishing concentration of the alkali salt was

$$\nu(^{23}\text{Na})/\nu(^2\text{H}) = 1.723\,1746(4),$$

which corresponds to an uncorrected

$$\mu_I' = 2.216\,2168(5)\mu_N.$$

Furthermore, the shift of the resonance line as function of the concentration was found to be small. The diamagnetic corrections obtained by Johnson and co-workers [24–27] for Na^+ are 0.000 642 6 and 0.000 632 2 from RHFS and RPA calculations, respectively. Assigning half the difference between the RHFS and RPA values as an uncertainty to the RPA value yields

$$(1 - \sigma_a)^{-1} = 1.000\,632\,6(52).$$

This correction, and an assumed chemical shift uncertainty of one part in 10^3 , yields a corrected

$$\mu_I = 2.218(2)\mu_N,$$

where the accuracy is limited by our knowledge of the chemical shift. The nuclear magnetic dipole moment in ^{23}Na has been used as a reference for various NMR determinations of nuclear magnetic dipole moments, including those in $^{185,187}\text{Re}$ discussed below, but for that purpose, the uncorrected value can be used directly.

The nuclear magnetic dipole moment in the 8th edition of *Table of Isotopes* [15] $\mu_I = 2.217\,520(2)\mu_N$ is taken from the compilation by Raghavan [16]. This value is based on the ABMR experiment yielding

$$g_I' = 0.804\,6108(8) \times 10^{-3},$$

which corresponds to an uncorrected

$$\mu_I' = 2.216\,082(2)\mu_N.$$

In the ABMR case, the diamagnetic correction should be evaluated for neutral Na. Fuller [10] quotes a RHFS result by Lin, Johnson, and Feiock: $\sigma_a = 0.000\,649\,1$, indicating a con-

tribution of about 0.000 006 5 from the $3s$ valence electron. Adding this contribution to the RPA value for the diamagnetic correction for Na^+ gives an estimate

$$(1 - \sigma_a)^{-1} = 1.000\,639(5).$$

Applying this correction gives

$$\mu_I = 2.217\,499(11)\mu_N,$$

An updated calculation of the shielding factor for Na could reduce the uncertainty in this magnetic moment determination.

C. ^{199}Hg

The tabulated nuclear magnetic dipole moment in ^{199}Hg is based on a 1961 optical pumping measurement by Cagnac [36]. The main result quoted in that work is a ratio

$$\mu_I'(^{201}\text{Hg})/\mu_I'(^{199}\text{Hg}) = -1.107\,416(5).$$

In the text, additional results are presented, including the ratio

$$\mu_I'(^{199}\text{Hg})/\mu_p' = 0.178\,2706(3),$$

with the remark that the diamagnetic correction is not included, ‘‘which according to Lamb’s calculation [22] is 0.96%.’’ The ^{199}Hg nucleus has been used as a reference in the optical pumping determination of the Pb magnetic moments considered below, but, for that purpose, the uncorrected value can be used directly.

The diamagnetic correction for neutral Hg obtained by Johnson and co-workers [24–27] includes also relativistic effects and is considerably larger than Lamb’s values. The tabulated value in the 8th edition of *Table of Isotopes* [15], $\mu_I = 0.505\,885\,5(9)\mu_N$, is based on the RHFS value for σ_a giving $1/(1 - \sigma) = 1.0161$, corresponding to $\sigma_a = 0.015\,87$. Including RPA corrections leads to a slightly modified value of 0.015 77. Assigning half the difference as an uncertainty gives

$$(1 - \sigma_a)^{-1} = 1.016\,02(5),$$

and the corrected

$$\mu_I = 0.505\,847(26)\mu_N,$$

where the uncertainty due to the diamagnetic correction is about 30 times larger than the experimental uncertainty.

D. ^{165}Ho

The nuclear magnetic dipole moment values for ^{165}Ho in the most recent tabulations are all derived from the ABMR result by Haberstroh, Moran, and Penselin [19], $g_I' = 6.370(70) \times 10^{-4}$, corresponding to an uncorrected $\mu_I' = 4.094(45)\mu_N$. This result was reevaluated by Dankwort and Ferch [20], including the influence of higher fine-structure levels and the deviation from LS coupling. The result then was $g_I' = 6.423(42) \times 10^{-4}$, corresponding to an uncorrected $\mu_I' = 4.128(27)\mu_N$. There is a reference to Dankwort and Ferch in the compilation by Raghavan [16],

and a diamagnetic correction of $1/(1-\sigma) = 1.0108$ gives the corrected value $4.173(27)\mu_N$.

The next reevaluation based on the experimental data of Ref. [19] was done by Nachtsheim [21], and his result was

$$g'_I = 0.642\,99(80) \times 10^{-3},$$

corresponding to an uncorrected

$$\mu'_I = 4.1322(51)\mu_N.$$

Nachtsheim clearly stated that his value for the magnetic moment is not corrected for diamagnetism.

However, after a long chain of indirect references, it is this uncorrected value of Nachtsheim that occurs as a *corrected* value in the 8th edition of *Table of Isotopes* [15], taken from Peker [37], who in turn made a reference to Burghardt *et al.* [38]. Burghardt *et al.* gave μ_I to be $4.132(5)\mu_N$, with references to Nachtsheim and Haberstroh, Moran, and Penselin, but without any mention of diamagnetic corrections.

Assigning an assumed uncertainty of 2×10^{-4} to the average diamagnetic correction for neutral Ho quoted by Fuller [10] yields

$$(1 - \sigma_a)^{-1} = 1.010\,76(20).$$

Applying this correction gives

$$\mu_I = 4.1767(53)\mu_N,$$

where the accuracy is limited by the experimental uncertainty. We thus suggest that this be the new recommended value for the magnetic moment of Ho.

E. $^{185,187}\text{Re}$

The nuclear magnetic dipole moments in ^{185}Re and ^{187}Re were measured by Alder and Yu [39] using NMR on an aqueous solution of NaReO_4 , with the magnetic dipole moment in ^{23}Na as the reference. Their results were

$$\nu(^{185}\text{Re})/\nu(^{23}\text{Na}) = 0.851\,14(9)$$

and

$$\nu(^{187}\text{Re})/\nu(^{23}\text{Na}) = 0.859\,87(9).$$

This gives, by using the uncorrected NMR value for ^{23}Na given in Sec. III B, the uncorrected

$$\mu'_I(^{185}\text{Re}) = 3.1439(3)\mu_N$$

and

$$\mu'_I(^{187}\text{Re}) = 3.1761(3)\mu_N.$$

The corrected values in the 8th edition of *Table of Isotopes* [15] are $\mu_I(^{185}\text{Re}) = 3.1871(3)\mu_N$ and $\mu_I(^{187}\text{Re}) = 3.2197(3)\mu_N$. These values have been taken from the compilation by Raghavan [16], where a diamagnetic correction of $1/(1-\sigma) = 1.0138$ has been applied to these values. In the absence of calculated magnetic shielding constants for the ReO_4^- molecule, the best approximation to the diamagnetic correction is the one for the closed-shell system Re^{7+} ,

since the seven $5d$ and $6s$ valence electrons of Re can be expected to be delocalized within the covalent bonding. The diamagnetic corrections obtained by Johnson and co-workers [24–27] for Re^{7+} are 0.013 56 and 0.013 31 from RHFS and RPA calculations, respectively. Assigning half the difference between the RHFS and RPA values as an uncertainty to the RPA value yields

$$(1 - \sigma_a)^{-1} = 1.013\,49(13).$$

This correction, and an assumed chemical shift uncertainty of one part in 10^3 , yields

$$\mu_I(^{185}\text{Re}) = 3.186(3)\mu_N$$

and

$$\mu_I(^{187}\text{Re}) = 3.219(3)\mu_N,$$

where the accuracy is limited by our knowledge of the chemical shift. However, even with this relatively large uncertainty, the experimental results for the HFS in the ground states of $^{185,187}\text{Re}^{75+}$ indicate that the magnetization is localized at considerably larger radii than the nuclear charge [3], unless the chemical shift for this many-valence system is significantly larger than assumed.

F. $^{203,205}\text{Tl}$

The most precise measurement of the nuclear magnetic dipole moments in ^{203}Tl and ^{205}Tl were carried out by Baker and Burd [40] using NMR on an aqueous solution of CH_3COOTl . Their results were

$$\nu(^{203}\text{Tl})/\nu(^1\text{H}) = 0.571\,391\,45(4)$$

and

$$\nu(^{205}\text{Tl})/\nu(^1\text{H}) = 0.577\,011\,73(4),$$

which give the uncorrected

$$\mu'_I(^{203}\text{Tl}) = 1.595\,7681(1)\mu_N$$

and

$$\mu'_I(^{205}\text{Tl}) = 1.611\,4643(1)\mu_N.$$

The corrected values in the 8th edition of *Table of Isotopes* [15] and in Raghavan's compilation [16] are $\mu_I(^{203}\text{Tl}) = 1.622\,257\,87(12)\mu_N$ and $\mu_I(^{205}\text{Tl}) = 1.638\,214\,61(12)\mu_N$. These values include a diamagnetic correction of $1/(1-\sigma) = 1.0166$ corresponding to the RHFS value $\sigma_a = 0.016\,36$ for Tl^+ obtained by Johnson and co-workers [24–27]. Including RPA corrections, they found a slightly reduced value 0.016 24. Again assigning half the difference as an uncertainty yields

$$(1 - \sigma_a)^{-1} = 1.016\,51(6).$$

The slight modification in σ_a is negligible compared to the assumed chemical shift uncertainty of one part in 10^3 , which leads to the values

$$\mu_I(^{203}\text{Tl}) = 1.622(2)\mu_N$$

and

$$\mu_I(^{205}\text{Tl}) = 1.638(2)\mu_N.$$

A more direct determination of the nuclear magnetic dipole moments in ^{203}Tl and ^{205}Tl is provided by Fowler's ABMR measurement [41], giving

$$g'_I(^{203}\text{Tl}) = 17.375(14) \times 10^{-4}$$

and

$$g'_I(^{205}\text{Tl}) = 17.549(14) \times 10^{-4},$$

which correspond to the uncorrected

$$\mu'_I(^{203}\text{Tl}) = 1.5952(13)\mu_N$$

and

$$\mu'_I(^{205}\text{Tl}) = 1.6111(13)\mu_N.$$

The RHFS value $\sigma_a = 0.01636$ for neutral Tl, quoted by Fuller [10], coincides with the value for Tl^+ , indicating negligible contributions from the $6p_{1/2}$ valence electron. Assigning an uncertainty of 2×10^{-4} , which corresponds to about twice the size of the RPA correction for the Tl ion, gives

$$(1 - \sigma_a)^{-1} = 1.01663(21).$$

Applying this correction gives

$$\mu_I(^{203}\text{Tl}) = 1.6217(13)\mu_N$$

and

$$\mu_I(^{205}\text{Tl}) = 1.6379(13)\mu_N$$

where the accuracy is limited by the experimental uncertainty.

G. ^{207}Pb

Recent tabulations quote two measurements of the nuclear magnetic dipole moment in ^{207}Pb : A 1969 optical pumping experiment by Gibbs and White [42], and a 1971 NMR measurement by Lutz and Stricker [43]. In addition, a third measurement was performed by Brenner [44] using ABMR, but its result does not seem to have had much impact. Both tabulated values include a diamagnetic shielding correction of 0.01686, evaluated by Feiock and Johnson [25] in the RHFS model for Pb^{2+} . The RPA calculation [31] gives a slightly smaller value $\sigma_a = 0.01674$. For the optical pumping experiment it would, however, be more appropriate to use the correction factor for neutral Pb, where the shielding is, in fact, considerably more important. The RHFS value by Feiock and Johnson [25] is 0.02055, indicating that the two valence electrons in the closed $(6p_{1/2})^2$ subshell significantly perturb the inner electrons. No RPA value for Pb is available, but a reduction of about 0.00010, as for the Pb^{2+} case, gives a first estimate. The correction for neutral Pb may, however, be relatively uncertain: there is a strong mixing between the closed $(6p_{1/2})^2$ subshell and the nearby $(6p_{3/2})^2$ configuration, and for neutral Pb we use

$$(1 - \sigma_a)^{-1} = 1.02098(21)$$

(half the difference between the RHFS and the RPA values has been assigned as an uncertainty to σ_a). The optical pumping result [42] was

$$\mu'_I(^{207}\text{Pb})/\mu'_I(^{199}\text{Hg}) = 1.14960(4).$$

Combining this result with the uncorrected value for ^{199}Hg , given in Sec. III C, leads to

$$\mu'_I = 0.572351(20)\mu_N$$

for ^{207}Pb . This optical pumping result is slightly outside the error bars of the recent, but less accurate, ABMR result [44]:

$$g'_I = 6.314(15) \times 10^{-4},$$

which corresponds to the uncorrected

$$\mu'_I = 0.5797(14)\mu_N.$$

Both these measurements are subject to the same diamagnetic shielding from the neutral Pb atoms. Applying this correction to the optical pumping value gives

$$\mu_I = 0.58436(12)\mu_N,$$

somewhat larger than the tabulated value, for which the Pb^{2+} shielding factor was applied. For the ABMR result the corrected value becomes

$$\mu_I = 0.5918(14)\mu_N.$$

The NMR measurement [43] is, in fact, reported as a measurement of "the shielding of lead ions by water." The value of the shielding is given in the abstract but the value of the magnetic moment is *not* given there. This measurement was carried out on $\text{Pb}(\text{NO}_3)_2$ in heavy water, and the result of Ref. [43] is given as

$$\nu(^{207}\text{Pb})/\nu(^2\text{H}) = 1.35888(2)$$

after an extrapolation to vanishing concentration of lead nitrate. The result corresponds to the uncorrected

$$\mu'_I = 0.582563(9)\mu_N.$$

Applying the diamagnetic correction for Pb^{2+} ,

$$(1 - \sigma_a)^{-1} = 1.01702(6),$$

and an assumed chemical shift uncertainty of one part in 10^3 gives the corrected

$$\mu_I = 0.5925(6)\mu_N.$$

The slightly different value in the most recent *Table of Isotopes* [15], $\mu_I = 0.592583(9)\mu_N$, is based on the RHFS value for the magnetic shielding, and does not consider the chemical shift.

It has been suggested [43] that an absolute shielding of $^{207}\text{Pb}^{2+}$ by D_2O of about 2% could explain the large discrepancy between the NMR and the optical pumping values. Gibbs, in a private communication quoted in the compilation

by Fuller [10], noted that the discrepancy “may be due to large interaction of 3P_1 -state with 3P_0 -state” in the case of the optical pumping measurement. The ABMR result agrees within the relatively large error bars with the NMR result, but not with the value from optical pumping. A renewed analysis of the optical pumping results using adequate atomic wave functions might be helpful, as would more precise ABMR measurements. This uncertainty concerning the Pb magnetic moment, presents, of course, a serious complication in the interpretation of the accurate measurements of the $1s$ hyperfine structure in H-like Pb.

H. ^{209}Bi

The nuclear magnetic dipole moment in ^{209}Bi was measured by Ting and Williams [45] using NMR on $\text{Bi}(\text{NO}_3)_3$ in heavy water, and their result was

$$\nu(^{209}\text{Bi})/\nu(^2\text{H}) = 1.046\,84(5),$$

which corresponds to an uncorrected

$$\mu_I' = 4.039\,10(19)\mu_N.$$

The corrected value in the 8th edition of *Table of Isotopes* [15] is $\mu_I = 4.1106(2)\mu_N$. These values have been taken from the compilation by Raghavan [16], where a diamagnetic correction of $1/(1-\sigma) = 1.0177$ was applied to this value. The magnetic shielding constants obtained by Johnson and co-workers [24–27] for Bi^{3+} are 0.01739 and 0.01727 from RHFS and RPA calculations, respectively. Assigning half the difference between the RHFS and RPA values as an uncertainty to the RPA value yields

$$(1 - \sigma_a)^{-1} = 1.017\,57(6).$$

In later work, Bařtuř *et al.* performed a nonrelativistic calculation for σ using Eq. (1) for Bi^{3+} and $\text{Bi}(\text{NO}_3)_3$, respectively, in order to investigate the effect on the valence electrons and of the other atoms in the molecule. They found a correction of about 0.1% of σ , much smaller than the difference between the RHFS and RPA values. The chemical shifts due to the solvent and the σ_{hf} term in Eq. (2) were, however, not considered. The assumed chemical shift uncertainty of one part in 10^3 dominates the final uncertainty, and yields the corrected magnetic moment

$$\mu_I = 4.110(4)\mu_N.$$

IV. CONCLUSION

For the nuclear magnetic dipole moments discussed here, only those in ^{165}Ho , $^{203,205}\text{Tl}$, and ^{207}Pb are obtained by direct measurements, whereas those in $^{185,187}\text{Re}$, and ^{209}Bi are measured with NMR and are therefore subject to unknown chemical shifts. Remeasurements of nuclear magnetic moments would be worthwhile for all these systems, but is most important for $^{185,187}\text{Re}$, $^{203,205}\text{Tl}$, ^{207}Pb , and ^{209}Bi . One possible method would be to perform the measurements on closed-shell ions in ion traps.

The determination of nuclear magnetic moments was an active field some years ago, in particular in the 1950s and 60s. Recent years have seen astonishing developments of both computing power and of techniques for precision measurements. In order to make full use of the accurate experiments on highly charged H-like systems—whether to test calculations of bound-state QED effects or to probe into nuclear structure—the nuclear magnetic moments must be known with better precision. In some cases, refined calculations of diamagnetic shielding factors would alleviate the problem. Nevertheless, a remeasurement of several nuclear magnetic moments with today’s experimental capacity and higher accuracy seems long overdue.

Note added in proof. Tomaselli *et al.* have in a recent paper [47] presented calculations of the Bohr-Weisskopf effect with the use of the dynamic correlation model for one-hole nuclei. Our revised values for the nuclear magnetic dipole moments in ^{165}Ho and ^{207}Pb , and the possibility of a chemical shift for $^{185,187}\text{Re}$, may modify their conclusions.

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