

## Optimal quantum codes for preventing collective amplitude damping

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Collective decoherence is possible if the distance between quantum bits is smaller than the effective wavelength of the noise field. Collectivity in the decoherence helps us to devise more efficient quantum codes. We present a class of optimal quantum codes for preventing collective amplitude damping to a reservoir at zero temperature. It is shown that two quantum bits (qubits) are enough to protect one bit quantum information, and approximately  $L + \frac{1}{2} \ln_2(\pi L/2)$  qubits are enough to protect  $L$ -qubit information when  $L$  is large. For preventing collective amplitude damping, these codes are much more efficient than the previously discovered quantum error correcting or avoiding codes. [S1050-2947(98)08410-8]

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### I. INTRODUCTION

In quantum computation or communication systems, it is essentially important to maintain coherence of a quantum system [1]. In reality, however, decoherence due to interaction with a noisy environment is inevitable [2]. It is discovered that quantum redundant coding is the most efficient way to combat decoherence. Until now, many kinds of quantum errors correcting or preventing codes have been devised [3–15]. Quantum error-correcting codes (QECCs) cover a large range of decoherence, and they are very powerful in noise suppression for large quantum systems. However, for small systems, QECCs are rather costly of quantum computing resources [16]. To protect one quantum bit (qubit) of information from general single-qubit errors, one needs at least five qubits [6]. Apart from the QECCs, there are alternate quantum codes, such as the quantum error preventing or avoiding codes [14–20], which combat decoherence with specific noise modes, but have the advantage of being more efficient to implement, especially for small quantum systems. Quantum error-preventing codes are based on the quantum Zeno effect, and therefore useful with quadratic noise [14,15,21]. Quantum error-avoiding codes (QEACs) make use of collectivity in the decoherence [17–20]. For combatting collective decoherence, they are a better choice.

Collective decoherence is an ideal circumstance, which is possible if the qubits couple to the same environment, and the separations between them are smaller than the effective wavelength of the noise field. For collective decoherence, there are coherence-preserving states. In QEACs, arbitrary input states are encoded into superpositions of the coherence-preserving states. To avoid general collective decoherence, one needs at least four qubits to encode one qubit of information [19]. Nevertheless, with specific noise models, more efficient QEACs can be devised [18,20]. For example, a two-bit QEAC has been devised for eliminating the dissipation that can be transformed into collective phase damping by some techniques [18].

The dominant noise process in many quantum computa-

tion or communication systems is described by amplitude damping, such as the radiative decay [22–25]. In this paper, we propose a class of optimal QEACs for preventing collective amplitude damping to a reservoir at zero temperature. These codes are much more efficient than those devised in the presence of general collective decoherence or in the presence of independent amplitude damping [20,7]. For example, we need only two qubits to encode one qubit information, and approximately  $L + \frac{1}{2} \ln_2(\pi L/2)$  qubits to encode  $L$ -qubit information when  $L$  is large. A QEAC with a high efficiency has two advantages. On the one hand, it costs few additional quantum-computing resources. This is remarkable, since quantum-computing resources are very stringent [26,27]. On the other hand, to encode a bit of information, an efficient QEAC needs only a small number of qubits, and therefore is much easier to implement in practice. QEACs are based on collective decoherence. Collective decoherence is most possible for the closely spaced adjacent qubits. Cooperative effects in the amplitude damping of two trapped ions have been observed experimentally [28]. In our proposal, two qubits subject to collective amplitude damping are enough for protecting one qubit of information.

The paper is arranged as follows: First we derive the master equation for collective amplitude damping. In the derivation, the explicit condition for collective decoherence is obtained. Then, from the master equation, we show that there are many collective dark states, which are subject to no collective amplitude damping. In the whole  $2^L$ -dimensional Hilbert space of  $L$  qubits, the collective dark states span a subspace of dimensions  $\binom{L}{[L/2]}$ , where  $[L/2]$  indicates the minimum round number no less than  $L/2$ . For some small  $L$ , the codes are explicitly constructed. The two-bit code is of special interest, and we further discuss its possible physical implementation.

### II. MASTER EQUATION FOR COLLECTIVE AMPLITUDE DAMPING

We start by deriving the master equation for collective amplitude damping. Amplitude damping of the qubits is caused by the interaction with noisy environment. The qubits are described by the spin-1/2 operators  $\vec{s}_i$ , and the environ-

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ment is modeled by a bath of oscillators with infinite degrees of freedom. The Hamiltonian for amplitude damping of  $L$  qubits in the interaction picture has the following form (setting  $\hbar = 1$ )

$$H_I(t) = \sum_{l=1}^L \sum_k [g_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}_l} e^{-i(\omega_{\vec{k}} - \omega_0)t} s_l^+ a_{\vec{k}} + \text{H.c.}], \quad (1)$$

where  $a_{\vec{k}}$  is the annihilation operator of the bath mode  $\vec{k}$ , and  $\omega_{\vec{k}}$  and  $\omega_0$  denote frequencies of the bath mode  $\vec{k}$  and of the qubits, respectively. The symbol  $\vec{r}_l$  indicates the site of the  $l$  qubit, and  $g_{\vec{k}}$  is the coupling coefficient. Under the Born-Markov approximation, the general form of the master equation with the interaction Hamiltonian  $H_I(t)$  is expressed as [29]

$$\frac{d}{dt} \rho(t) = - \int_0^\infty d\tau \text{tr}_B \{ [H_I(t), [H_I(t-\tau), \rho(t) \otimes \rho_B]] \}, \quad (2)$$

where  $\rho_B$  is the bath density operator, and  $\rho(t)$  denotes the reduced density operator of the qubits in the interaction picture. Suppose that the bath is at zero temperature. This is the case in many circumstances, such as for the radiative decay or for the loss process [22–25]. Substituting Hamiltonian (1) into Eq. (2), we obtain the following master equation for spatially correlated amplitude damping:

$$\begin{aligned} \frac{d}{dt} \rho(t) = & i \sum_{i,j=1}^L \delta_{ij} [s_j^+ s_i^-, \rho(t)] + \frac{1}{2} \sum_{i,j=1}^L \{ \gamma_{ij} [2s_i^- \rho(t) s_j^+ \\ & - s_j^+ s_i^- \rho(t) - \rho(t) s_j^+ s_i^-] \}, \end{aligned} \quad (3)$$

where the spatially correlated damping coefficients  $\gamma_{ij}$  and Lamb shifts  $\delta_{ij}$  are defined, respectively, by

$$\gamma_{ij} = \sum_k [2\pi |g_{\vec{k}}|^2 \delta(\omega_{\vec{k}} - \omega_0) e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)}], \quad (4)$$

$$\delta_{ij} = \sum_k \left[ |g_{\vec{k}}|^2 \frac{1}{\omega_{\vec{k}} - \omega_0} e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)} \right]. \quad (5)$$

In the continuum limit, the summations of Eqs. (4) and (5) become integrals, and the principal value should be taken of the integral of Eq. (5). The main contributions to the summations of Eqs. (4) and (5) come from the modes  $\vec{k}$  that satisfy  $\omega_{\vec{k}} \approx \omega_0$ . Suppose  $d$  is the maximum separation between the qubits, and  $v_0$  is the velocity of the noise field around  $\omega_{\vec{k}} = \omega_0$ , i.e.,  $v_0 = \omega_{\vec{k}} / |\vec{k}|_{\omega_{\vec{k}} = \omega_0}$ . If  $d$  and  $v_0$  satisfy the condition

$$d \ll \frac{v_0}{\omega_0}, \quad (6)$$

in Eqs. (4) and (5)  $e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)} \approx 1$ , and then  $\gamma_{ij}$  and  $\delta_{ij}$  are independent of the qubit index. In this circumstance, we denote  $\gamma_{ij} = \gamma_0$ ,  $\delta_{ij} = \delta_0$ , and  $S^\pm = \sum_{l=1}^L s_l^\pm$ . Equation (3) is thus simplified to

$$\begin{aligned} \frac{d}{dt} \rho(t) = & i \delta_0 [S^+ S^-, \rho(t)] + \frac{\gamma_0}{2} [2S^- \rho(t) S^+ - S^+ S^- \rho(t) \\ & - \rho(t) S^+ S^-]. \end{aligned} \quad (7)$$

This is the master equation for collective amplitude damping, which is obtained under condition (6). The term  $v_0/\omega_0$  in Eq. (6) defines the effective wavelength of the noise field. This expression for the effective wavelength is derived using the Born-Markov approximation, and holds in the case of amplitude damping. For other sources of decoherence, the expression for the effective wavelength may have a different form [30]. Condition (6) may be satisfied in practice for some sources of decoherence. For example, in the ion trap quantum computer, a fundamental limit to internal state decoherence is given by the radiative decay. For this source of decoherence,  $v_0$  is estimated by the velocity of light, and the typical value of the separations of ions (qubits) has the order of a few  $\mu\text{m}$ , then Eq. (6) requires that  $\omega_0 \ll 10^{14}$  Hz. For some hyperfine transitions, it is possible to meet this condition [27].

### III. COLLECTIVE DARK STATES

In the language of quantum trajectories [31], the system evolution described by the master equation (7) is represented by an ensemble of wave functions that propagate according to the effective Hamiltonian

$$H_{\text{eff}} = -\delta_0 S^+ S^- - \frac{i}{2} \gamma_0 S^+ S^-, \quad (8)$$

interrupted at random times by quantum jumps. A quantum jump takes place in the time interval  $[t, t+dt)$ , with probability

$$P(t) = \langle \Psi(t) | \gamma_0 S^+ S^- | \Psi(t) \rangle dt, \quad (9)$$

leading to a wave function collapse according to

$$|\Psi(t+dt)\rangle = c' \sqrt{\gamma_0} S^- |\Psi(t)\rangle, \quad (10)$$

where  $c'$  is a normalization constant. From Eqs. (8) and (9), it follows that if a initial state satisfies

$$S^- |\Psi(0)\rangle = 0, \quad (11)$$

it remains unchanged during the effective evolution, and is subject to no quantum jumps at any time. All the states satisfying Eq. (11) are called collective dark states. Coherence between these states is perfectly preserved during collective amplitude damping. It can also be seen from Eqs. (8) and (9) that no other states except those satisfying Eq. (11) remain unchanged during the effective evolution and quantum jumps.

To obtain all the collective dark states, we notice that  $\vec{S} = \sum_{l=1}^L \vec{s}_l$  is expressed as a sum of  $L$  spin- $\frac{1}{2}$  operators. From the angular momentum theory [32],  $S^{(x)}$ ,  $S^{(y)}$ , and  $S^{(z)}$  can be chosen as three generators of the  $\text{su}(2)$  algebra. The irreducible representation of the  $\text{su}(2)$  algebra in the two-dimensional Hilbert space  $H_{1/2}$  of a single qubit is denoted by  $D_{1/2}$ , then  $D_{1/2}^{\otimes L}$  defines an  $L$ -fold tensor product represen-

tation of the  $\text{su}(2)$  algebra in the whole  $2^L$ -dimensional Hilbert space  $H_{1/2}^{\otimes L}$  of  $L$  qubits. The representation  $D_{1/2}^{\otimes L}$  is reducible, and it can be decomposed into a series of irreducible representations of the  $\text{su}(2)$  algebra, such as

$$D_{1/2}^{\otimes 2} = D_{1/2} \otimes D_{1/2} = D_1 \oplus D_0. \quad (12)$$

Suppose  $D_{1/2}^{\otimes 2l}$  has the decomposition  $D_{1/2}^{\otimes 2l} = \bigoplus_{j=0}^l n_j(2l) D_j(2l)$ , where  $D_j(2l)$  denotes the  $(2j+1)$ -dimensional irreducible representations of the  $\text{su}(2)$  algebra in the state space of  $2l$  qubits, and  $n_j(2l)$  is the multiplicity of  $D_j(2l)$  in the decomposition; then we have the following recursion relations [setting  $n_{-1}(2l) = n_{l+1}(2l) = n_{l+2}(2l) = 0$ ]:

$$\begin{aligned} D_{1/2}^{\otimes 2l+1} &= D_{1/2}^{\otimes 2l} \otimes D_{1/2} \\ &= \bigoplus_{j=(1/2)}^{l+(1/2)} [n_{j+(1/2)}(2l) + n_{j-(1/2)}(2l)] D_j(2l+1), \end{aligned} \quad (13)$$

$$\begin{aligned} D_{1/2}^{\otimes 2l+2} &= D_{1/2}^{\otimes 2l+1} \otimes D_{1/2} \\ &= \bigoplus_{j=0}^{l+1} [2n_j(2l) + n_{j-1}(2l) + n_{j+1}(2l)] D_j(2l+2). \end{aligned} \quad (14)$$

Equations (13) and (14), together with Eq. (12), determine the decomposition of  $D_{1/2}^{\otimes L}$  with an arbitrary  $L$ . In the decomposition of  $D_{1/2}^{\otimes L}$ , there are  $n_j(L)$   $(2j+1)$ -dimensional irreducible representations  $D_j(L)$ , whose representation spaces are denoted by  $H_j^{(m)}(L)$ , where  $m=1,2,\dots$ , and  $n_j(L)$ , respectively. The whole  $2^L$ -dimensional Hilbert space  $H_{1/2}^{\otimes L}$  of  $L$  qubits splits into a series of orthogonal subspaces  $H_j^{(m)}(L)$  according to the decomposition of  $D_{1/2}^{\otimes L}$ . In every subspace  $H_j^{(m)}(L)$ , the Casimir operator  $\tilde{S}^2 = (S^{(x)})^2 + (S^{(y)})^2 + (S^{(z)})^2$  has the eigenvalue  $j(j+1)$ . The subspace  $H_j^{(m)}(L)$  is of  $2j+1$  dimensions, whose basis vectors can be chosen as the eigenvectors  $|j, m_j\rangle_m$  of the operator  $S^{(z)}$ , where  $m_j = -j, -j+1, \dots, j$ . In each space  $H_j^{(m)}(L)$ , the lowest-weight state  $|j, -j\rangle_m$  satisfies the condition  $S^{(-)}|j, -j\rangle_m = 0$ , and no other states have this property. Hence there is one and merely one collective dark state in each subspace  $H_j^{(m)}(L)$ , and the dark states in different subspaces are orthogonal to each other. The total number  $N(L)$  of orthogonal collective dark states is therefore just the number of the irreducible representations in the decomposition of  $D_{1/2}^{\otimes L}$ , i.e., the total number  $N(L) = \sum_j n_j(L)$ . From Eqs. (13) and (14), we obtain the following recursion equations about  $N(L)$ :

$$N(2l+1) = 2N(2l) - n_0(2l), \quad (15)$$

$$N(2l+2) = 2N(2l+1), \quad (16)$$

where  $n_0(2l)$  is the multiplicity of the one-dimensional irreducible representations in the decomposition of  $D_{1/2}^{\otimes L}$ , and is known to be  $n_0(2l) = (2l)!/[l!(l+1)!]^{-1}$  [19]. Substituting

this into Eqs. (15) and (16), we obtain  $N(L) = \binom{L}{[L/2]}$ , where  $[L/2]$  indicates a minimum round number of no less than  $L/2$ . The quantum error avoiding codes are obtained by encoding arbitrary input states into superpositions of the collective dark states. The encoding space is of  $N(L)$  dimensions; thus the optimal  $L$ -bit quantum code has the efficiency

$$\eta(L) = \frac{1}{L} \ln_2 N(L) = \frac{1}{L} \ln_2 \binom{L}{[L/2]}. \quad (17)$$

If  $L$  is large,  $\eta(L)$  is approximated by  $1 - (1/2L) \ln_2(\pi L/2)$ , which approaches 1 very rapidly. Hence, in the presence of collective amplitude damping, these codes are much more efficient than the previously-discovered quantum error correcting or avoiding codes.

#### IV. EXPLICIT CONSTRUCTION OF THE $L$ -BIT CODES WITH SOME SMALL $L$

The orthogonal collective dark states obtained in Sec. III can be chosen as a set of basis vectors for the encoding space. To construct the codes explicitly, we need only express the collective dark states in the computational basis, whose basis vectors are the coeigenstates of the operators  $s_1^z, s_2^z, \dots$ , and  $s_L^z$ . The two eigenstates of the operator  $s_j^z$ , with the eigenvalues  $\pm \frac{1}{2}$ , are denoted by  $|1\rangle$  and  $|0\rangle$ , respectively. The collective dark states and the computational basis vectors are connected by the Clebsch-Gordan coefficients [32]. Here, we explicitly construct the optimal  $L$ -bit QEACs with  $L=2,3,4$ . These codes are simple and involve only a small number of qubits, and at the same time have notably high efficiencies, so they are an ideal choice of quantum codes in the presence of collective amplitude damping.

In the case of two qubits, the encoding space is of two dimensions. The two codewords are given by

$$|j=0, m_j=0\rangle = \frac{1}{\sqrt{2}} (|10\rangle - |01\rangle), \quad (18)$$

$$|j=1, m_j=-1\rangle = |00\rangle, \quad (19)$$

which are sufficient to encode one qubit of information. The efficiency is  $\frac{1}{2}$ .

In the case of three qubits, the encoding space is of three dimensions. The codewords read

$$\left| j = \frac{1}{2}, m_j = -\frac{1}{2} \right\rangle_1 = \frac{1}{\sqrt{6}} (|001\rangle + |100\rangle - 2|010\rangle), \quad (20)$$

$$\left| j = \frac{1}{2}, m_j = -\frac{1}{2} \right\rangle_2 = \frac{1}{\sqrt{2}} (|001\rangle - |100\rangle), \quad (21)$$

$$\left| j = \frac{3}{2}, m_j = -\frac{3}{2} \right\rangle = |000\rangle. \quad (22)$$

The efficiency of this code is  $\frac{1}{3} \ln_2 3$ . At least one qubit of information can be encoded.

In the case of four qubits, the encoding space is of six dimensions. The codewords are

$$|j=0, m_j=0\rangle_1 = \frac{1}{2} (|10\rangle - |10\rangle)(|10\rangle - |01\rangle), \quad (23)$$

$$|j=0, m_j=0\rangle_2 = \frac{1}{\sqrt{3}} [|0011\rangle + |1100\rangle - \frac{1}{2} (|01\rangle + |10\rangle)(|01\rangle + |10\rangle)], \quad (24)$$

$$|j=1, m_j=-1\rangle_1 = \frac{1}{\sqrt{2}} (|10\rangle - |01\rangle)|00\rangle, \quad (25)$$

$$|j=1, m_j=-1\rangle_2 = \frac{1}{\sqrt{2}} |00\rangle(|10\rangle - |01\rangle), \quad (26)$$

$$|j=1, m_j=-1\rangle_3 = \frac{1}{2} [(|01\rangle + |10\rangle)|00\rangle - |00\rangle(|01\rangle + |10\rangle)], \quad (27)$$

$$|j=2, m_j=-2\rangle = |0000\rangle, \quad (28)$$

respectively. The efficiency of this code is  $\frac{1}{4}(1 + \ln_2 3)$ . At least two qubits of information can be encoded.

The two-bit code is of special interest. It costs the least number of qubits, and therefore has a good chance to be first implemented. We further give the encoding and decoding for this code. Let  $H'$  denote the joint operation  $H\sigma_x$ , where  $\sigma_x$  is the flip operation, and  $H$  the Hadamard transformation

$[|0\rangle \rightarrow (|0\rangle + |1\rangle)/\sqrt{2}, |1\rangle \rightarrow (|0\rangle - |1\rangle)/\sqrt{2}]$ . The symbols  $C_{ij}$  and  $C_{ij}(H')$  denote the controlled-Not and the controlled- $H'$  operations, respectively, where the first subscript of  $C_{ij}$  or  $C_{ij}(H')$  refers to the control bit and the second to the target. The controlled- $H'$  operation performs the joint transformation  $H'$  on the target bit if the control bit is in  $|1\rangle$ , and leaves the target bit unchanged if the control bit is in  $|0\rangle$ . The input state of a single qubit can be generally expressed as  $|\Psi(0)\rangle_1 = c_0|0\rangle + c_1|1\rangle$ . An ancillary qubit 2 is prepared in the state  $|0\rangle_2$ . The input state is encoded by the following operation:

$$|\Psi(0)\rangle_1|0\rangle_2 \xrightarrow{C_{21}C_{12}(H')} |\Psi_{\text{enc}}\rangle_{12} = c_0|00\rangle + \frac{c_1}{\sqrt{2}} (|10\rangle - |01\rangle). \quad (29)$$

The encoded state is subject to no collective amplitude damping, and afterwards it can be decoded by applying the same operation again in the reverse order, i.e.,

$$|\Psi_{\text{enc}}\rangle_{12} \xrightarrow{C_{12}(H')C_{21}} |\Psi(0)\rangle_1|0\rangle_2. \quad (30)$$

The controlled-Not and the controlled- $H'$  operations involved in the encoding and decoding have been demonstrated [26,27], and cooperative effects in amplitude damping of two trapped ions have been observed experimentally [28], so the proposed two-bit code has a good chance to be implemented in the near future experiment.

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