Time reversal of atomic wave-packet dynamics by a phase-modulated standing wave

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Atoms interacting with a standing wave are diffracted into even multiples of photon momentum. We argue that the spreading of the atomic momentum is analogous to the fanning out of Bloch vectors of an ensemble of two-level systems in coherent optics. It is shown that a continuous, time-proportional phase increment of the standing wave reverses the dynamics of the atoms and generates momentum echoes. Monte Carlo wave-function simulations are performed to see how the scheme works for realistic atomic models, taking spontaneous emission into account. Application of the echo formation to lifetime measurements of atoms is proposed. [S1050-2947(98)05709-6]

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Many experiments in optics and interferometry with atoms [1–5] take advantage of wave properties of the atomic center of mass. These developments are significant in both fundamental science and technological advances. One of the prominent examples of this is the diffraction of atoms with a standing wave [6]. In this experiment a well-collimated beam of two-level atoms crosses the standing wave, and it is observed that the atoms are diffracted into partial waves that have momentum equal to even multiples of photon momentum $\hbar k$, where k is the wave vector of the standing wave. In the present paper we consider this effect in a way that is analogous to a well-known phenomenon in coherent optics. More specifically, we will relate the spreading of momentum to the fanning out of Bloch vectors of a system of two-level atoms (or spin $\frac{1}{2}$'s), and then devise a scheme to bring the momentum back to the initial distribution, thereby generating a momentum "echo." Recently, it has been shown that the momentum distribution of an atom moving in a sinusoidally phase-modulated standing wave is dynamically localized [7,8]. Here, we report that a *linear* phase modulation leads to the time reversal of the atomic dynamics and the revival of momentum.

A standing-wave laser along Ox may be expressed as $E(x,t)=2\hat{\boldsymbol{\varepsilon}}_0\cos(kx+\varphi)\cos\omega t$, where φ, ω, X, and $\hat{\boldsymbol{\varepsilon}}_0$ are, respectively, the phase, the frequency, the polarization, and the amplitude of the laser. Let $\Delta\omega$ be the detuning and $\Omega=-\mathbf{d}\cdot\hat{\boldsymbol{\varepsilon}}_0/\hbar$ be the Rabi frequency, $\mathbf{d}$ being the dipole moment of the atom. When $\Delta\omega\gg\Omega$ with spontaneous emission ignored, the dynamics of the atom along Ox may be described by the Hamiltonian

$$H=\frac{p_x^2}{2m}+\frac{\hbar\Delta\omega}{2}+U(x), \tag{1}$$

where m is the atomic mass and $U(x)$ is the optical potential given by

$$U(x)=\hbar\,\frac{[\Omega\,\cos(kx+\varphi)]^2}{\Delta\omega}=\hbar\,\frac{\Omega^2}{2\Delta\omega}\,[1+\cos 2(kx+\varphi)]. \tag{2}$$

If the interaction time with the standing wave is much less than the recoil time $t_{\rm rec}\equiv 2m/(\hbar k^2)$ (i.e., in the Raman-Nath regime), the kinetic energy term may be dropped. Within this regime, for the initial atomic wave function given as a plane wave the atomic momentum spreads linearly with time and is given by

$$\Delta p=\hbar k\,\frac{\Omega^2 t}{\sqrt{2}\Delta\omega}. \tag{3}$$

A similar expression for the exact resonance case is derived by Cook and Bernhardt [9]. Or, if initially the atom is described by a Gaussian wave packet

$$\psi(x,0)=(2\pi\sigma^2)^{-1/4}\exp\left[ik_0x-\left(\frac{x-x_0}{2\sigma}\right)^2\right], \tag{4}$$

where $\hbar k_0$ is the atomic momentum, x_0 is the center of the wave packet, and σ is the spread of atomic position, the momentum spread is instead

$$\Delta p=\hbar\left[\frac{1}{4\sigma^2}+\frac{t^2k^2\Omega^4}{\Delta\omega^2}\left(\frac{1}{2}-\frac{e^{-4k^2\sigma^2}}{8}+\frac{e^{-4k^2\sigma^2}\cos 4kx_0}{8}-\frac{e^{-8k^2\sigma^2}\cos 4kx_0}{2}\right)\right]^{1/2}. \tag{5}$$

Note that in the limit $\sigma\to\infty$, Eq. (5) reduces to Eq. (3) as expected. Because of the finite width of the initial wave packet, Eq. (5) is not linear for very small times. It is linear in the regime $1/(4\sigma^2)\ll t^2k^2\Omega^4/(\Delta\omega^2)$.

The momentum spread is due to the spatial inhomogeneity of the optical potential, given by $\cos[2(kx+\varphi)]$. Moreover, the rate of the momentum spread depends on the strength of the potential, $\Omega^2/\Delta\omega$. Analogous spread occurs for the Bloch vectors, following an application of a $\pi/2$ pulse in the presence of inhomogeneity due to a distribution of resonance frequencies. Here, too, the spread of precessing angles $\Delta\theta(t)$ is linear with respect to time, $\Delta\theta(t)=(\omega_f$

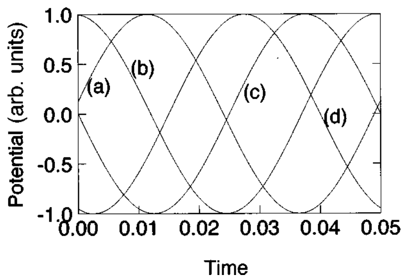

FIG. 1. Potential (arbitrary units) at four different positions along Ox over the interaction time $T=0.05$ (in units of recoil time). Positions considered are (a) 0.01, (b) 0.13, (c) 0.25, and (d) -0.15 (in units of optical wavelength).

$-\omega_s)t$, the rate $\omega_f-\omega_s$ being the difference in coupling strengths between the fastest and the slowest precessing components. Thus we have an apparent analogy between the fanning out of Bloch vectors and the atomic momentum spreading.

A usual photon (or spin) echo is generated by an application of a π pulse in the middle of free precession. A π pulse reverses the sign of the Hamiltonian, and hence effectively the sign of time in the evolution operator $U(t)=\exp(-iHt/\hbar)\rightarrow U(-t)=\exp(iHt/\hbar)$. Likewise, an atomic momentum echo can be generated by reversing the sign of the Hamiltonian in Eq. (1), which amounts to reversing the sign of the potential if the kinetic energy operator is dropped. [The constant terms in Eqs. (1) and (2) do not affect dynamics.] It can be achieved by either reversing the sign of the detuning or shifting the phase of the laser by $\pi/2$ suddenly. This scheme has been described in a previous paper [10]. We introduce in the present paper a scheme that reverses the sign of the potential not suddenly but with a *continuous* phase modulation of the standing wave.

Let $\varphi(t)$ be the phase of the laser incremental linearly with time: $\varphi(t)=n\pi t/T$, where n is an integer and T is the total interaction time with the standing wave. Then the potential reads

$$U_n(x,t)=\hbar\,\frac{\Omega^2\cos^2(kx+n\pi t/T)}{\Delta\omega}. \tag{6}$$

The force on an atom is given by the potential gradient. So if an atom spends an equal amount of time in positive and negative potential gradients, the net effect of the potential becomes null. Hence, at the end of the interaction the atomic momentum will come back to the initial distribution and a momentum echo will form. In Fig. 1 we show the potential at a few arbitrarily chosen points x for $n=1$. It shows that for all x the potential oscillates sinusoidally once over T, and thus there will be one momentum echo at $t=T$. In general, for $n>1$ there will be n momentum echoes over T.

A common feature of the echo techniques is that they are effective only on the reversible processes responsible for inhomogeneous broadening of the atomic transitions. In real situations homogeneous broadening effects are also present. The atom optical analogs of the ''homogeneous'' effects we now consider are the atomic recoil thus far ignored and spontaneous emission.

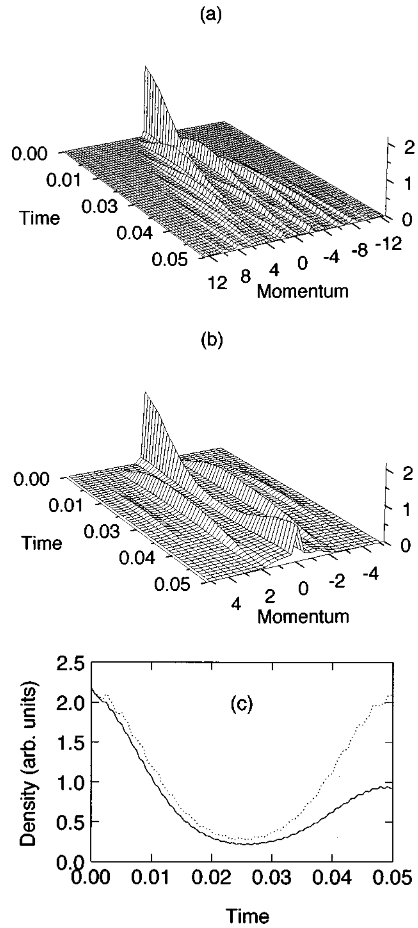

FIG. 2. Results of 1000 Monte Carlo wave-function simulations. (a) Evolution of the ground-state atomic density (arbitrary units) in momentum space in a standing wave without phase modulation. Atomic momentum is in units of photon momentum. (b) With linear phase modulation for $n=1$, showing the formation of a momentum echo. (c) Same as in (b) along $k=0$ (solid line). For comparison, we also show the result for the $\Gamma=0$ case (dotted line). The echo formation is not perfect due to the breakdown of the Raman-Nath approximation and spontaneous emission.

For subsequent figures we perform full Monte Carlo wave-function simulations [11–13] to see the effects of the atomic recoil and the decay of the upper state by spontaneous emission. In this method, between quantum jumps due to spontaneous emission the atom evolves according to the Schrödinger equation

$$i\hbar\,\frac{\partial}{\partial t}\,\Psi(x,t)=H\Psi(x,t),\quad \Psi(x,t)=\begin{pmatrix}\psi_e(x,t)\\ \psi_g(x,t)\end{pmatrix}, \tag{7}$$

with the non-Hermitian Hamiltonian

$$H=\frac{p_x^2}{2m}+\hbar\Delta\omega\sigma_z+2\hbar\Omega\,\cos(kx+\varphi)\sigma_x-i\hbar\,\frac{\Gamma}{2}\,\sigma_+\sigma_-, \tag{8}$$

were σ_α $(\alpha=x,y,z)$ are spin-$\frac{1}{2}$ operators for the atomic internal states, $\sigma_\pm=\sigma_x\pm i\sigma_y$, and Γ is the decay rate.

In Fig. 2(a) we show the atomic momentum distribution for the ground state without phase modulation, and in Fig.

2(b) with phase modulation taking $n=1$. The former figure shows the spreading of momentum, while the latter shows the formation of an echo at $t=T$. In these simulations we assume that initially the atomic beam is perfectly collimated, crosses the standing wave at a right angle, and is given by a Gaussian wave packet with a width $\sigma=0.5$. Other parameters are $\Delta\omega=10\,000$, $\Omega=3000$, $\Gamma=1500$, and $T=0.05$. [Time and frequency are given in units of t_{rec} and ω_{rec} $(=1/t_{\mathrm{rec}})$, and length in multiples of optical wavelength $\lambda=2\pi/k$.] Also, we assume that the standing-wave laser field is linearly polarized. We did 1000 trajectory simulations for these figures. As shown in Fig. 2(c), the echo formation, albeit not a perfect one, is clear. The imperfection is due, of course, to the atomic recoil and spontaneous emission, which are not reversed by the scheme.

Many applications require an accurate knowledge of the lifetime of atoms, and there are a number of methods available for precision lifetime measurements [14]. By performing a series of calculations, we found that the amplitude of the echo varies with Γ as $\sim\exp(-b\Gamma)$, where b is a quantity that depends on the parameters of the system under consideration. We also found that the amplitude varies with t as $\sim\exp(-\Gamma t)$. This also bears analogy with the exponential decay of echoes with a time constant T_2, the transverse relaxation time, in coherent optics and in magnetic resonance. Thus, a measurement of the peak height can serve as a new tool for measuring the Γ value. Details will be reported elsewhere.

In conclusion, we showed that a linear time-proportional phase increment of the standing wave causes the time reversal of atomic wave-packet dynamics, leading to wave-packet revivals or the formation of momentum echoes. The formation and the decay of the amplitude of echoes are parallel to those in coherent optics. The phenomenon is not only interesting in its own right, but can also serve as a starting point for new applications. Experiments with Rb atoms are underway to demonstrate the echo formation and its application to lifetime measurements.

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