Quantum communications: Tetrat coding

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This work shows that an orthogonal tetrat basis element made up of two photons can be generated with two optical parametric down-converters and identified with a multiport detection system for coincidences. The transmission gain over the usual coding in bits is $g=2$. The information is assigned to each photon pair using a *single* conjugate beam of the parametric down-conversion. This system also demonstrates a way to get a *pair* of disentangled polarized photons by handling just *one* photon of an entangled pair. [S1050-2947(98)05010-0]

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Quantum communication concepts and derived experimental schemes have been developed with potential applicability in areas such as cryptography, teleportation, and quantum computing. The concept of quantum *dense* coding [1] may increase a communication channel capacity and has been demonstrated recently in the laboratory by Mattle *et al.* [2] using a basis of distinguishable *trit* elements instead of *bits*. The trit experiment [2] was achieved through the generation of a basis of four orthogonal polarization states (entangled Bell states) from a nonlinear crystal and the use of a detection scheme able to identify three distinct outcomes within the four elements of the basis. The coding density was $d=2$.

The density d is here defined as the *number of polarization states in the wave function* divided by the *number of photons associated with the wave function* and depends only on the quantum state of the light, regardless of the way this state was generated. The theoretical transmission gain g, defined by $g \equiv n(\text{bits})/m(\text{trits})$, using the trit basis instead of a bit state was $g=1.58$, assuming identical information being transmitted: $2^n=3^m$. Five trits may then replace eight bits in the representation of a given character.

The term *dense* coding has been associated in the literature with a specific communication *protocol* of Bennett and Wiesner (BW) [1] in which only *one* photon of a conjugated pair travels between the communicating parts, Alice and Bob. Alice prepares an Einstein-Podolsky-Rosen state and sends only one of the particles to Bob. He performs a unitary operation on this particle that, entangled with the particle kept with Alice, places the two-particle system in one of the so-called Bell states. After this, he sends his particle to Alice, who, having both particles, determines which operation Bob applied.

Differently from BW's procedure, in the scheme presented in this work *both* particles are simultaneously coded and sent from the emitter to the receptor system. In this way, classical-like quaternary bases (tetrats) are generated and identified by photon detectors. Some fundamental differences exist between this quaternary basis and a classical one: The photon pair correlation created in the down-conversion process *cannot* be generated by a classical field; photons of arbitrary polarization cannot be cloned [3]. Consequently, practical applications such as quantum cryptography using quaternary bases can be easily implemented. A practical quaternary communication basis has the advantage, over BW's scheme, of a better communication gain. However, the price paid to have all four elements of this basis in use ($g=2$) is a reduction in the code *density* d and in the possibility to teleport information.

Basically, this work has a twofold motivation: (i) to devise a quantum emitter able to achieve a maximum communication gain g and (ii) to demonstrate how to disentangle an entangled photon pair. It is shown that the superposition of probability amplitudes for emission in two nonlinear crystals undergoing type-II down-conversion makes possible the construction of a tetrat basis constituted of experimentally *distinguishable* elements at the average coding density $d=1.5$ and transmission gain $g=2$. This quantum scheme [4] is different from the local dense coding scheme proposed by BW [1] also because two photon sources are used to produce the basis elements. The constructed *indistinguishability* of the photon sources makes possible the inhibition of certain emissions of the two-crystal system and the generation of polarized states, which is not possible with the single-crystal scheme and unitary transformations on a single trajectory.

A tetrat basis can easily be constructed with intense classical light beams. However, these bases will have a vanishingly small coding density d because the number of photons in each coded beam is very high. Decreasing the number of photons in each beam down to a few photons, *no* classical source could produce the necessary time correlation between photons to achieve the potentiality presented by down-conversion light sources that emit photon pairs. Photons from a classical source have Mandel's parameter $Q \geqslant 0$, while a twin photon source has $Q<0$. This is a crucial point that excludes arbitrarily weak classical sources as generators of a tetrat basis made up of two photons.

Figure 1 shows the *emitter* setup together with a Hong-Ou-Mandel (HOM) quantum interferometer [6] aligned after the emitter output. Two nonlinear crystals of susceptibility $\chi^{(2)}$, 1 and 2, are pumped by the same single-mode laser of bandwidth $\Delta\omega \ll 2\pi c/L$, where L is the separation distance between crystals [5]. The degenerate down-converted beams are along the trajectories a and b. A controllable phase difference ϕ between the down-converted beams at the crystal positions can be set by a suitable displacer along either down-converted line between the two crystals; line a is then

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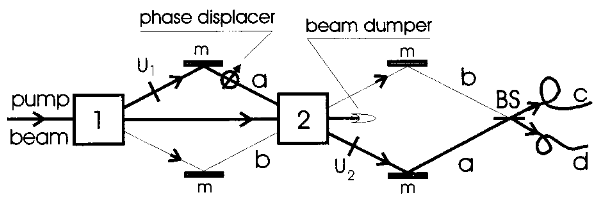

FIG. 1. Emitter setup to generate a tetrat basis. Crystals 1 and 2 produce entangled polarization states that can be implemented by wave retarders U_1 and U_2 placed along the beam *a*. The states generated are transformed by the beam splitter (BS) and launched into optical fibers along trajectories *c* and *d*.

chosen. This phase difference could also be introduced by a displacer on the pump beam between crystal 1 and crystal 2, generating similar states.

First, both crystals are cut and oriented with respect to the pump beam according to Garuccio's prescription [7] to generate an entangled photon state [8], symmetric under polarization and spatial transformations:

$$|\Psi^{(+)}\rangle_j = \frac{1}{\sqrt{2}}\,(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b) \quad (j=1,2). \tag{1}$$

The pump laser intensity is assumed to be weak in the sense that the probability of emission for more than one photon pair from each crystal is low. As the down-conversion efficiency is quite low, the probability for *simultaneous* emission of a photon pair in both crystals is also very low. This state $|\Psi^{(+)}\rangle$ is a phase-matched state defined at the crossing of the two emission cones [7]. The finite size of the illuminated region in the crystal produces divergences on each beam such that they overlap transversely in a finite region [9].

Assuming a situation of indistinguishability of the photon pair source in the detection process, the state generated is a *superposition* of each state $|\psi_1\rangle$ and $|\psi_2\rangle$ produced by each down-converter and transformed by the unitary transformations represented by the wave plates U_1, U_2 and the phase displacer ϕ. The two possible processes, exclusive in generating photons,

$$|\psi_1\rangle = U_2 e^{i\phi_a} U_1 \alpha |\Psi^{(+)}\rangle_1 \quad \text{or} \quad |\psi_2\rangle = U_2 \beta |\Psi^{(+)}\rangle_2, \tag{2}$$

have to be superposed and lead to the set of wave functions

$$|\zeta\rangle = U_2(e^{i\phi_a} U_1 \alpha |\Psi^{(+)}\rangle_1 + \beta |\Psi^{(+)}\rangle_2), \tag{3}$$

where α and β are proportional to the laser intensity and the efficiency of each crystal. Each sequence of unitary transformations $U_2 e^{i\phi_a} U_1$ is equivalent to a single effective unitary transformation.

Due to the negligible field intensities from crystal 1 when they are superposed on crystal 2, it should be understood that stimulated processes are to be neglected. The *indistinguishability* of the photon source is what produces interference effects between the two down-converters. In addition to that, the simultaneous presence of *two* photon pairs in crystal 2 is *neglected*, either originating from the same crystal or one pair from crystal 1 and the other pair from crystal 2.

The emitter is then composed of the *two* crystals *and* wave plates necessary to code the tetrat basis. The two crystals can then be seen as an interferometer with an internal structure that can be set by wave plates and the phase shifter. These optical elements can be interpreted as boundary conditions that define an electric field to which a photon pair belongs. A unitary transformation is performed *outside* the interferometer by the wave plate U_2. In this sense, this system is akin to a four-way routing Mach-Zehnder interferometer, where one of the internal arms contains a phase displacer and outside, at the input, a polarization rotator is set. At the two Mach-Zehnder outputs, polarizing beam splitters are followed by two detectors. Through manipulation of the rotator and the phase shifter, a single photon can be sent to a chosen one of the four detectors. For a given setting of the phase shifter, the *single* photon output state is a single polarization state with ''classical'' code density $d=1$.

In order to ensure indistinguishability of the photon source the photon pair trajectories after crystal 1 have to merge together in crystal 2. However, one *cannot* say that photons from crystal 1 are suffering transformations on crystal 2 because this crystal only acts as a passive medium when, eventually, a photon pair passes through it. Although similarities can be found with the Mach-Zehnder interferometer, some difference exists, for example, (a) the down-converter system performs a ''routing'' operation on the entangled photon *pair* at once for each setting of the wave plates; (b) the unitary transformation U_2, the last component of the emitter, is performed *after* crystal 2 and acts only on photons *a* and when any emitted *b* photon already left crystal 2; and (c) this coding is processed in a *nonlocal* photon source or by a ''*quantum*'' *Bob* (Bob is equivalent to sender).

For $|\alpha| = |\beta|$ and considering any phase difference in α and β included in ϕ_a, the choice $\phi_a = \phi_1 = 2n\pi$ and $\alpha = \beta$, together with $U_1 = U_2 = 1$, leads to the first normalized wave function or the first element of a possible basis

$$|\zeta^{(1)}\rangle = \frac{1}{\sqrt{2}}\,(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b). \tag{4}$$

With U_2 being a half wave plate oriented with the optic axis vertical to the plane defined by the horizontal trajectories *a* and *b*, $U_2(90°)$, a normalized state is established

$$|\zeta^{(2)}\rangle = \frac{1}{\sqrt{2}}\,(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b) \equiv |\Psi^{(-)}\rangle. \tag{5}$$

Each of these two elements $|\zeta^{(1)}\rangle$ and $|\zeta^{(2)}\rangle$ is coded with density $d=2$. When $U_2 = U_2(45°)$ represents a half wave plate with the optical axis at 45° from the vertical, $\phi_a = \phi_3 = 2n\pi$, and $U_1 = U_1(90°)$ one obtains the third normalized element

$$|\zeta^{(3)}\rangle = U_2(45°)(U_1|\Psi^{(+)}\rangle_1 + |\Psi^{(+)}\rangle_2) = |1_V\rangle_a |1_V\rangle_b. \tag{6}$$

With $U_2 = U_2(45°)$, $U_1 = U_1(90°)$, and $\phi_a = \phi_4 = (2n+1)\pi$ one obtains a fourth tetrat element

$$|\zeta^{(4)}\rangle = U_2(45°)(e^{i\phi_4}|\Psi^{(-)}\rangle_1 + |\Psi^{(+)}\rangle_2) = |1_H\rangle_a|1_H\rangle_b. \tag{7}$$

Elements $|\zeta^{(3)}\rangle$ and $|\zeta^{(4)}\rangle$, however, are coded at a lower density $d=1$ and therefore the average code density is $d=(2+2+1+1)/4=1.5$ [10].

These quantum superpositions resulted in *suppression* of certain crystal emissions, due to the establishment of field coherence between both crystals. Singly polarized states cannot be constructed from unitary transformations applied on one output of a single type-II down-converter [1,2] and were made possible in this scheme through this interference effect. This coherence between the crystals is an aspect of the phenomenon ''induced coherence without stimulated emission'' [11]. A good enhancement and suppression of emission in down-converters was experimentally demonstrated by Herzog *et al.* [12]. In the scheme presented here, this interference resulted in the singly polarized states $|\zeta^{(3)}\rangle$ and $|\zeta^{(4)}\rangle$. This also demonstrates a way to get a *pair* of simultaneously polarized photons by handling just *one* photon of an entangled pair.

The states $\{|\zeta^{(j)}\rangle\}$ with $j=1-4$ constitute a set of four orthogonal bosonic states or a workable tetrat basis where each element can be identified by the detection scheme. The ''receptor'' system used in Ref. [2] is adequate for the task. The two down-converters and wave retarders constitute the *emitter*, whose output is transformed by the HOM interferometer. Optical fibers that preserve polarization [13] can be used to guide the photon pairs to the *detector system*. The beam splitter (BS) of the HOM interferometer is set in such a way that the phase difference between the optical path of the two trajectories *a* and *b*, from the crystals to the BS, is close to zero. The spatial operation performed on the wave function by the HOM interferometer will depend on the symmetry of the spin part of the wave function. The state $|\zeta^{(1)}\rangle$, symmetric in the exchange of the polarization or spin variables, will be transformed by the HOM interferometer leading to

$$|\zeta^{(1)}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b)$$

$$\underset{\text{HOM}}{\Rightarrow} \frac{1}{\sqrt{2}}(|H\rangle_c|V\rangle_c - |V\rangle_d|H\rangle_d), \tag{8}$$

where $\hat{a}$, $\hat{b}$, $\hat{c}$, and $\hat{d}$ are annihilation operators for photons along the *a*, *b*, *c*, and *d* trajectories and the basic transformation of the HOM interferometer may be described [14] by $\hat{a}=t\hat{c}-r\hat{d}$ and $\hat{b}=r\hat{c}+t\hat{d}$, for an asymmetric beam splitter, real transmissivity t, and reflectivity r and such that $r=t=1/\sqrt{2}$. Coincidence counts will then appear between any of the D_H and D_V detectors or any of the $D_{H'}$ and $D_{V'}$ detectors. Similarly, the state $|\zeta^{(2)}\rangle$ will be modified, under action of the HOM interferometer, to

$$|\zeta^{(2)}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_c|V\rangle_d - |V\rangle_c|H\rangle_d), \tag{9}$$

giving coincidence counts between any of the D_H and $D_{V'}$ detectors or any of the $D_{H'}$ and D_V detectors. The states $|\zeta^{(3)}\rangle$ and $|\zeta^{(4)}\rangle$ are both symmetric in the spin variables and then also symmetric under the spatial operation introduced by the HOM interferometer. $|\zeta^{(3)}\rangle$ transforms to

$$|\zeta^{(3)}\rangle = \frac{1}{\sqrt{2}}(|2_V\rangle_c|0_V\rangle_d - |0_V\rangle_c|2_V\rangle_d), \tag{10}$$

generating *two* photons at each trajectory *c* and *d* that can be detected by coincidences between D_{V1} and D_{V2} or $D_{V'1}$ and $D_{V'2}$. Analogously, $|\zeta^{(4)}\rangle$ transforms to

$$|\zeta^{(4)}\rangle = \frac{1}{\sqrt{2}}(|2_H\rangle_c|0_H\rangle_d - |0_H\rangle_c|2_H\rangle_d), \tag{11}$$

which generates two photons at each trajectory *c* and *d* that can be identified by coincidences between D_{H1} and D_{H2} or $D_{H'1}$ and $D_{H'2}$. Two photon states in a given trajectory could be detected with a better efficiency using single detectors able to identify two photons. These detectors, although existing in the experimental stage in the laboratory are not yet available commercially.

An experimentally identifiable set of four orthogonal states constituted of superpositions of singly polarized and entangled polarization states is then demonstrated at the code density $d=1.5$ and gain $g=2$. The use of four tetrat elements, which cannot be cloned, to replace eight bits in a cryptographic system, for example, indicate a possible concrete application for this quaternary emitter.

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Ref. [8]. ''Quasi-phase-matched'' crystals eliminate the need of compensating crystals.

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