Phase-noise influence on coherent transients and hole burning

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Resonant excitation of an inhomogeneously broadened ensemble of two-level atoms (TLA) by a stochastic field with phase noise is theoretically investigated. Free-induction decay (FID), hole burning (HB), and transient nutation (TN) are studied. We consider two kinds of driving fields, one with a free walking phase and another with the phase locked in a limited domain. It is shown that the resonant excitation behavior depends strongly on the noise property. Noise induced by a walking phase gives a simple contribution to the dephasing time, T_2, of two-level atoms whereas phase locking qualitatively changes the laser-atom interaction. In the latter case, it is shown that even when the central part of the driving field spectrum is narrower than homogeneous absorption line of the TLA, the wide, low intensity wings of the spectrum (sidebands produced by the locked phase noise), have a strong effect on the FID, TN, and HB induced by the central, narrow part of the spectrum. The influence of sidebands on photon echoes is also discussed. [S1050-2947(98)06210-6]

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I. INTRODUCTION

The laser and other sources of coherent radiation can be considered as oscillators with compensated damping (i.e., where the gain equals the losses). If the state of this oscillator is stable, then any external noise influencing the parameters of the oscillator gives only the background and does not affect the delta-like, sharp spectrum of the output field. Meanwhile, it is known (see, for example, Ref. [1]) that the phase of the laser field is in a state of indifferent equilibrium (i.e., it is unstable). This means that any weak external force may shift the phase without resistance. This is a basic source of spectral line broadening [2]. Below we consider a single mode laser. As was shown in Ref. [2], for example, mechanical vibrations and thermal fluctuations of the cavity length and refraction index are usually the dominant source of line broadening of single mode lasing rather than the limit set by spontaneous emission. Because of the phase shift by the random force, the deltalike spectrum becomes Lorentzian with a finite width. This width depends on phase dispersion and correlation time τ_c of the noise that induces the phase shift. The random walk of the phase or phase diffusion process does not have any selected reference point, since the phase of the field can shift far from the initial value by small jumps accumulated in a large random phase shift. Technical locking of the device phase near some reference point results in essential narrowing of the field spectrum (a discussion of the phase locking process is presented in the Appendix). Random phase jumps (less than $\pi/2$) near the reference phase give the wide background and do not affect the central part, which remains a delta function. Only the walk of the reference phase makes the central part broad. As the processes of laser phase jump and reference phase jump are usually different, the width of the central part and background are also different. The former is determined by the device that locks the phase, and the latter by the laser output spectrum or amplifier.

Stabilized sources of coherent irradiation are commonly used for ultrahigh resolution spectroscopy. Optical transients and hole burning are some of the methods of coherent spectroscopy that require a narrow laser linewidth. Therefore the influence of the field spectrum on the transient response signal of resonant absorbers is of interest. Usually one expects that, when the driving field spectrum is narrower than the absorption linewidth, this field can be reliably considered as monochromatic. Using a model of phase-locked noise, we show that the latter condition on the narrow, sharp part of the field spectrum is insufficient. The weak and wide sidebands, caused by the phase-locked noise, significantly change the saturation and transients of the excited quantum system.

Resonant interaction of an atomic system with fluctuating classical fields has been already studied extensively [3–55] (the list is not exhaustive). This paper is not aimed at providing a review on this topic. We do not consider quantum noise, as the linewidth of a single mode laser well above threshold is not appreciably affected by spontaneous emission [2]. As mentioned above, we consider only semiclassical sources of laser line broadening such as vibration, thermal fluctuation, and index fluctuation. Real lasers can exhibit a variety of fluctuations in phase, frequency, and amplitude. Almost all previous analyses of noisy laser-atom interactions have been based on several models of classical fluctuating fields. Among them are phase diffusion field (PDF), chaotic field (CF) or Gaussian noise irradiation, random jump processes of the frequency, phase or amplitude, and shot noise description of the fluctuating phase or frequency.

Phase diffusion field has a constant real amplitude but its phase is a Wiener-Levy process [8,9,17,24,25,30,36, 41,51,52]. The phase diffusion model is based on the formal analogy between the position of the particle performing a Brownian motion and the random phase $\alpha(t)$ of the field.

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The velocity of the particle corresponds to the random frequency of the field $\omega(t)=\dot{\alpha}(t)$, which is a Gaussian white-noise process with delta correlation

$$\langle \omega(t)\omega(t')\rangle = 2\nu_{\mathrm{PDF}}\delta(t-t'). \tag{1.1}$$

Even if the name PDF is strictly appropriate only in the case when the frequency is a white noise and the phase is determined by a purely diffusive Wiener-Levy process, it is commonly used to indicate the more general case of finite correlation time when the frequency noise is an Ornstein-Uhlenbeck process [56,57]. The power spectrum of the field in a generalized PDF model depends on the correlation time of its frequency, evolving continuously from a Lorentzian to a Gaussian profile when the correlation time increases from zero to infinity [32,35,33,41,47]. Meanwhile, a random frequency process produces phase velocity fluctuations and as a result the phase itself changes continuously, increasing or decreasing gradually in a random way. It is impossible to select the reference phase within frequency fluctuating models as the phase walk is not bounded by any condition. Moreover, any abrupt, discontinuous change of the phase is beyond PDF and generalized PDF models. For this reason it is more appropiate to describe the locked phase field or bounded phase walk processes by the random phase jump model. Random jump processes of the frequency, phase, or amplitude of the field were considered in [3–6,10,13, 15,21,37,43–45,48,52] as they allow convenient and very flexible manipulation of interaction parameters, permitting nonperturbative examination of the noisy laser-atom interaction. While the question ''what is the origin of the noise, the frequency fluctuation or the phase fluctuation?'' is rather philosophical, the close relation between the correlated jump model and diffusion model, was shown in [6,58].

The shot-noise model [48] of phase fluctuations assumes that the instantaneous phase $\alpha(t)$ of the electromagnetic field consists of a sum of statistically independent pulses

$$\alpha(t)=\sum_{i=1}^{n}\alpha_i h(t-t_i), \tag{1.2}$$

where $h(t-t')$ is a causal pulse-shape function $[h(t)=0$ for $t<0]$, generated at a random time t_i with amplitude α_i. The correlation function of the shot-noise phase is equivalent to the Wiener-Levy correlation function of the phase-diffusion model. The important difference between the shot noise and PDF model is that $\alpha(t)$ is not a Gaussian stochastic process. In Ref. [48] it was shown that shot noise is strongly related to the correlated phase jump process as both models lead to the same Burshtein-Chapman-Kolmogorov-Smoluchowski equation [5,6,10,21,43,48].

Chaotic field (CF) or Gaussian noise irradiation [11,12,14,19,20,24,25,29–31,33,34,36,40] assumes that the amplitude and phase are random but their fluctuations are considered without introducing the amplitude-phase decomposition. The amplitude of the CF is a random Gaussian process:

$$E(t)=E_x(t)+iE_y(t), \tag{1.3}$$

$$\langle E(t)\rangle = 0. \tag{1.4}$$

A stochastic model of the chaotic field is described in terms of Langevin equation

$$\dot{E}(t)=-bE(t)+F_E(t) \tag{1.5}$$

where $F_E(t)$ is a random force

$$\langle F_E(t)F_{E*}(t)\rangle = 2b\langle |E|^2\rangle\delta(t-t'). \tag{1.6}$$

This model is closely related to the multimode free-running lasing. Therefore we do not consider the CF model in this paper.

II. PHASE JUMP MODELS

We consider the random phase $\alpha(t)$ determined by the density of a Markovian conditional probability

$$\varphi(\alpha_0,t_0|\cdots|\alpha_{n-1},t_{n-1}|\alpha_n,t_n)=\varphi(\alpha_{n-1},t_{n-1}|\alpha_n,t_n), \tag{2.1}$$

where $\alpha_0,\ldots,\alpha_n$ are successive values of phase at the moments of time $t_0<\cdots<t_n$. Condition (2.1) is a fundamental property of a Markovian process. The theory of phase relaxation is well developed for a stationary, discontinuous Markovian process [6]. Therefore we take the stationary conditional probability

$$\varphi(\alpha_0,t_0|\alpha,t)=\varphi(\alpha_0,0|\alpha,t-t_0), \tag{2.2}$$

$$\int \varphi(\alpha_0)\varphi(\alpha_0,t_0|\alpha,t)d\alpha_0=\varphi(\alpha), \tag{2.3}$$

where

$$\varphi(\alpha)=\lim_{t-t_0\to\infty}\varphi(\alpha_0,t_0|\alpha,t) \tag{2.4}$$

is a probability density that does not depend on the previous history of the process. It describes the probability of finding the phase α at any cross section of the process.

After Burshtein [6] we consider the discontinuous process of phase change. The random value of phase $\alpha(t)$ is constant inside each time interval (t_i,t_{i+1}) and jumps stepwise at the end of it. This time interval has a Poisson distribution

$$dW(t_{i+1}-t_i)=\exp\left[-\frac{t_{i+1}-t_i}{\tau_0(\alpha_i)}\right]\frac{dt_{i+1}}{\tau_0(\alpha_i)}, \tag{2.5}$$

where $\tau_0(\alpha_i)$ is a mean dwell time between jumps, generally depending on the value of α_i inside the time interval. The conditional probability of a phase jump from value β to value α is given by the function $f(\beta|\alpha)$. When the dwell time does not depend on phase value, the density of conditional probability of discontinuous Markovian process obeys the forward Kolmogorov-Feller equation [6,43,45,48,59]

$$\frac{\partial}{\partial t}\varphi(\alpha_0,t_0|\alpha,t)=-\frac{1}{\tau_0}\varphi(\alpha_0,t_0|\alpha,t)+\frac{1}{\tau_0}\int\varphi(\alpha_0,t_0|\beta,t)f(\beta|\alpha)d\beta \tag{2.6}$$

with the initial condition

$$\varphi(\alpha_0,t_0|\alpha,t_0)=\delta(\alpha-\alpha_0). \tag{2.7}$$

Solution of this equation describes the normalized, stationary density of the conditional probability if the relations

$$\int f(\beta|\alpha)d\alpha=1, \tag{2.8}$$

$$\int \varphi(\alpha_0)f(\alpha_0|\alpha)d\alpha_0=\varphi(\alpha) \tag{2.9}$$

are valid. The random process $\alpha(t)$ is completely determined by the functions $\varphi(\alpha)$, $f(\alpha_0|\alpha)$ and dwell time τ_0.

Classification of the phase noise was first introduced by Burshtein [6]. He defined the function $f(\alpha_0|\alpha)$ as

$$f(\alpha_0|\alpha)=f(\alpha-\xi\alpha_0)=f(\xi\alpha_0-\alpha), \tag{2.10}$$

where the parameter ξ characterizes the correlation between two successive values of phase.

(1) When $\xi=0$ the random process is uncorrelated

$$f(\alpha_0|\alpha)=\varphi(\alpha) \tag{2.11}$$

since the phase α does not depend on its value prior to the jump. Here each jump reproduces the stationary distribution of phase $\varphi(\alpha)$.

(2) Phase jump is anticorrelated when $\xi=-1$. A two-state jump process $\alpha=\pm a$ (telegraph noise) with transition probability of

$$f(\alpha_0|\alpha)=\delta(\alpha+\alpha_0) \tag{2.12}$$

satisfies this condition.

(3) When $\xi\rightarrow1$, the phase jump is a correlated process. The function $f(\alpha-\alpha_0)$ is even and its width gives the phase shift value after the jump. When this width is zero,

$$f(\alpha_0|\alpha)=\delta(\alpha-\alpha_0) \tag{2.13}$$

the process is ineffective. A small width specifies a process with small phase jumps from the initial value α_0. For this process, the Kolmogorov-Feller equation (2.5) is reduced to the Fokker-Plank equation [6,58]. Solution of the latter describes the phase diffusion when $1-\xi\ll1$. By means of small jumps, the phase of the field can go very far from the initial value.

We introduce a new addition to the phase noise classification. Whereas the function $f(\alpha_0|\alpha)$ describes the correlation between successive jumps, the stationary distribution function $\varphi(\alpha)$ specifies the reference point of phase. When

$$\varphi(\alpha)=\text{const}, \tag{2.14}$$

$$\int \varphi(\alpha)d\alpha=1, \tag{2.15}$$

the reference point is absent as all values of the phase have equal probability and it is not possible to select some particular reference phase. If the function

$$\varphi(\alpha)=\varphi(\alpha-\alpha_0)=\varphi(\alpha_0-\alpha) \tag{2.16}$$

is centered near the value α_0, then the latter is a reference point, as the phase walks randomly near it. The phase has a mean value of

$$\langle\alpha\rangle=\int \alpha\varphi(\alpha-\alpha_0)d\alpha=\alpha_0, \tag{2.17}$$

whereas for the process with phase distribution (2.14), we have $\langle\alpha\rangle=0$ since the value of α_0 is indefinite.

III. PHASE DIFFUSION FIELD

The correlated phase jump model ($\xi=1$) without a reference point $\varphi(\alpha)=\text{const}$ [see Eqs. (2.14) and (2.15)] is a good description of the phase diffusion process when the jump size is small. This model was developed in [5,6] to analyze the influence of noise on atom evolution in a resonant field. The Rabi oscillation of the population difference, transition probability, and luminescence of the atom excited by the field with correlated [5,6] and uncorrelated [3,4] phase jumps were considered. We apply this model to a theoretical study of polarization transients and hole burning in the inhomogeneous spectrum of impurity-ion crystals excited by phase-noise field.

To define the parameters of the phase diffusion field $E(t)=E_0e^{i\omega t+i\alpha(t)}$ we consider the field correlation function

$$\langle E(t)E^*(t_0)\rangle=E_0^2K(t-t_0)e^{i\omega(t-t_0)}, \tag{3.1}$$

$$K(t-t_0)=\langle e^{i\alpha(t)-i\alpha(t_0)}\rangle$$
$$=\int\int e^{i(\alpha-\alpha_0)}\varphi(\alpha_0)\varphi(\alpha_0,t_0|\alpha,t)d\alpha d\alpha_0. \tag{3.2}$$

Multiplying the Kolmogorov-Feller equation (2.6) on $e^{i(\alpha-\alpha_0)}\varphi(\alpha_0)$ and integrating the result over α and α_0, we get

$$\frac{\partial}{\partial t}K(t-t_0)=-\frac{1}{\tau_0}(1-\langle e^{i\theta}\rangle)K(t-t_0), \tag{3.3}$$

$$\langle e^{i\theta}\rangle=\int e^{i\theta}f(\theta)d\theta, \tag{3.4}$$

where $\theta=\alpha-\alpha_0$. Fourier transform of its solution

$$K(t)=\exp(-t/\tau_1), \tag{3.5}$$

$$\frac{1}{\tau_1}=(1-\langle\cos\theta\rangle)\frac{1}{\tau_0} \tag{3.6}$$

gives the Lorentzian power spectrum of the field

$$S(\omega')=\text{Re}\frac{1}{\pi}\int_0^\infty\langle E(t)E^*(0)\rangle e^{-i\omega' t}dt$$
$$=\frac{|E_0|^2}{\pi}\frac{\tau_1}{1+(\omega-\omega')^2\tau_1^2}. \tag{3.7}$$

When, for example, the phase jump size has a Gaussian distribution

$$f(\theta)=\frac{1}{a\sqrt{2\pi}}\exp\left(-\frac{\theta^2}{2a^2}\right) \tag{3.8}$$

then the half width of the power spectrum is

$$\nu=\frac{1}{\tau_1}=\frac{1-\exp(-a^2/2)}{\tau_0}. \tag{3.9}$$

For a process with a small jump size ($a\ll 1$), the half width is reduced to

$$\nu=\frac{a^2}{2\tau_0} \tag{3.10}$$

and the effective dwell time τ_1 becomes much longer than τ_0. Therefore the field spectral width ν is much smaller than the phase jump frequency $\nu_0=1/\tau_0$ when the phase changes by small jumps. With growing jump size ($a\gg 1$), the conditional probability $f(\theta)$ tends to a uniform one and finally it coincides with the phase distribution $\varphi(\alpha)=\text{const}$. As a result, the correlated process approaches the uncorrelated process and the effective dwell time τ_1 becomes equal to the real dwell time τ_0. Thus the increase of the phase jump size leads to an increase in width of the field spectrum from the value ν (3.10) to ν_0.

To calculate the two-level atom response to such phase fluctuations, we first use Burshtein's [5,6] equation for the partial density matrix $\hat{\rho}(\alpha)$:

$$\frac{d\hat{\rho}(\alpha)}{dt}=-\frac{i}{\hbar}[\hat{\mathcal{H}}(\alpha),\hat{\rho}(\alpha)]-\frac{1}{\tau_0}\hat{\rho}(\alpha)+\frac{1}{\tau_0}\int\hat{\rho}(\beta)f(\beta|\alpha)d\beta+\hat{R}(\hat{\rho}(\alpha)), \tag{3.11}$$

where $\hat{\mathcal{H}}(\alpha)$ is a Hamiltonian of the two-level atom excited during a dwell time τ_0 by the field portion of phase α and $\hat{R}(\hat{\rho}(\alpha))$ is an operator that describes the density matrix relaxation induced by internal interactions. We have to find the mean phase difference of the field and induced atomic polarization, as just this value characterizes the field absorption. This phase difference is defined by a variable

$$\sigma_{12}(\alpha)=\rho_{12}(\alpha)\exp(-i\omega t-i\alpha+ikz), \tag{3.12}$$

where 1 (2) denotes ground (excited) state and k is a field wave number. Its mean value, as well as the mean value of the population difference

$$\langle\sigma_{12}\rangle=\int\sigma_{12}(\alpha)d\alpha, \tag{3.13}$$

$$\overline{w}=\int[\rho_{22}(\alpha)-\rho_{11}(\alpha)]d\alpha \tag{3.14}$$

satisfy the equations

$$\dot{\overline{w}}=i\chi(\langle\sigma_{12}\rangle-\langle\sigma_{21}\rangle)-\frac{1}{T_1}(\overline{w}-w_0), \tag{3.15}$$

$$\langle\dot{\sigma}_{12}\rangle=\langle\dot{\sigma}_{21}\rangle^*=\left(i\Delta-\frac{1}{T_2}-\frac{1}{\tau_1}\right)\langle\sigma_{12}\rangle+i\frac{\chi}{2}\overline{w}, \tag{3.16}$$

which are derived from Eq. (3.11) by simple averaging. Here T_1 and T_2 are relaxation times of population difference and polarization, respectively; $\Delta=\omega_0-\omega$ is a detuning parameter from the atomic resonant frequency ω_0; $\chi=2\mu E_0/\hbar$ is the Rabi frequency, and $\mu=\mu_{12}=\mu_{21}$ is a dipole transition matrix element. We also define w_0 as the thermal equilibrium population difference.

Equations (3.15) and (3.16) are reduced to the conventional Bloch equations by the substitution

$$\overline{u}=\langle\sigma_{12}+\sigma_{21}\rangle;\quad \overline{v}=-i\langle\sigma_{12}-\sigma_{21}\rangle. \tag{3.17}$$

Their solution gives the mean value of polarization in the instant reference frame linked rigidly to the field phase and hence allows calculation of the field absorption. Phase noise results in additional dephasing of polarization in this frame, as its decay rate is modified as

$$\frac{1}{T_{2m}}=\frac{1}{T_2}+\frac{1}{\tau_1}. \tag{3.18}$$

Therefore the correlated phase walk leads to broadening of the absorption line in the same way as fluctuation of the resonant frequency.

IV. RANDOM PHASE FIELD WITH A REFERENCE POINT: HOLE BURNING AND POLARIZATION TRANSIENTS

As an example of a random phase field with a reference point, we consider the well-known phase telegraph noise (PTN) model. The phase changes instantly between two values a and $-a$, whereas its mean value is zero and just this value is the reference phase of the field. Oscillations of population difference and luminescence of a two-level atom excited by the field with PTN were considered in [45,48]. We extend this analysis to hole burning and polarization transients.

As the transition probability of PTN is described by expression (2.12), the Kolmogorov-Feller equation (2.6) is reduced to

$$\frac{\partial}{\partial t}\varphi(\alpha_0,0|\alpha,t)=-\frac{1}{\tau_0}\varphi(\alpha_0,0|\alpha,t)+\frac{1}{\tau_0}\varphi(\alpha_0,0|-\alpha,t). \tag{4.1}$$

For the initial condition

$$\varphi(\alpha_0,0|\alpha,0)=\delta(\alpha-\alpha_0) \tag{4.2}$$

its solution is

$$\varphi(\alpha_0,0|\alpha,t)=\frac{1}{2}\delta_{\alpha,\alpha_0}\left[1+\exp\left(-\frac{2t}{\tau_0}\right)\right]+\frac{1}{2}\delta_{\alpha,-\alpha_0}\left[1-\exp\left(-\frac{2t}{\tau_0}\right)\right]. \tag{4.3}$$

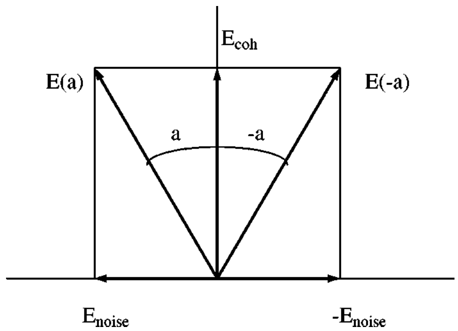

FIG. 1. Phase diagram of phase-telegraph noise field. $E_{\rm coh}$ is a coherent component with a constant phase and amplitude. $E_{\rm noise}$ is a noise component with phase jumping randomly over $\pm\pi$.

Taking into account that the equilibrium distribution function of PTN is

$$\varphi(\alpha_0)=\tfrac{1}{2}\delta_{\alpha_0,a}+\tfrac{1}{2}\delta_{\alpha_0,-a} \tag{4.4}$$

one can calculate a two-dimensional distribution function for this process

$$\psi(\alpha_0,0|\alpha,t)=\varphi(\alpha_0)\varphi(\alpha_0,0|\alpha,t) \tag{4.5}$$

and then an autocorrelation function

$$\langle E(t)E^*(0)\rangle=E_0^2e^{i\omega t}\left\{\cos^2a+\exp\left(-\frac{t}{\tau_c}\right)\sin^2a\right\}, \tag{4.6}$$

where $\tau_c=\tau_0/2$ is the correlation time of PTN. The power spectrum of the driving field is given by the Fourier spectrum of the function (4.6)

$$S(\omega')=\frac{E_0^2}{\pi}\left\{\cos^2a\quad\delta(\omega'-\omega)+\frac{\tau_c\sin^2a}{1+(\omega'-\omega)^2\tau_c^2}\right\}. \tag{4.7}$$

The PTN field can be considered as consisting of two fields, one is coherent with an amplitude $E_{\rm coh}=E_0\cos\alpha$ and the other is random with a constant amplitude $E_{\rm noise}=E_0|\sin\alpha|$ and fluctuating phase $\pm\pi/2$, the total phase jump being π (see Fig. 1). Therefore its spectrum contains two lines, the sharp deltalike line and broad Lorentzian one. The ratio of the integral intensities of these fields is given by

$$\frac{I_{\rm noise}}{I_{\rm coh}}=\tan^2a, \tag{4.8}$$

$$I_i=\int_{-\infty}^{\infty}S_i(\omega')d\omega', \tag{4.9}$$

where i denotes *noise* or *coh* and S_i is the spectrum of i field component. When, for example, $a=\pi/4$, both fields have the same integral power. However, the power of the coherent field $E_{\rm coh}$ is stored in an infinitely narrow frequency band, whereas the power of noise component is spread over a broad line with half width (HW) of $\gamma=1/\tau_c$. We realize that the phase of the coherent field also walks in a real experiment due to instability of the reference oscillator. For example, the linewidths of stabilized lasers used in an ultrahigh resolution spectroscopy of solids are in the range 300–2000 Hz [60–63]. As this width is much smaller than homogeneous absorption linewidth of most impurity ions in solids, the field is considered to be coherent and one can drop its contribution to T_2 [see Sec. III, Eq. (3.18)]. Here we assume that the phase of the reference oscillator undergoes a diffusion process.

If the spectrum of a power source with unlocked phase has, for example, a 1-MHz half width, then because of phase locking to the phase of the reference oscillator, the output spectrum shows two components: a sharp one with HW of say 1000 Hz and a broad one with HW of 1 MHz. When the integral intensities of both components are equal ($a=\pi/4$), the spectral power density of the sharp component differs strongly from that of the wide component. The wide component is 30 dB less intense than the narrow one.

Below we show that this weak, broad component strongly influences the two-level atom saturation by the coherent component, whereas saturation by the wide component is unaffected by the coherent part. The resultant saturation is seen to be a two-component line. The narrow part of the line is produced by combined effect of the fields $E_{\rm coh}$ and $E_{\rm noise}$ on two-level atoms while the second, wide part is burnt only by incoherent field $E_{\rm noise}$.

To describe this saturation, we use the Burshtein equation (3.11) as in a previous section. First we calculate the two-level atom response to the coherent field $E_{\rm coh}$, i.e., the following mean values

$$\overline{u}_{\rm coh}+i\overline{v}_{\rm coh}=2\langle\rho_{12}(\alpha)\rangle\exp(-i\omega t+ikz), \tag{4.10}$$

$$\overline{w}=\langle\rho_{22}(\alpha)-\rho_{11}(\alpha)\rangle \tag{4.11}$$

[compare them with expressions (3.12)–(3.14), (3.17)]. They are defined in the reference frame related to the phase of field $E_{\rm coh}$. Then the Burshtein equation takes the form

$$\dot{x}(W_{\rm noise},\tau)=-(\hat{L}_0+W_{\rm noise}\hat{L}_1)x(W_{\rm noise},\tau)+x(-W_{\rm noise},\tau)+\varphi(W_{\rm noise})\hat{\Lambda}, \tag{4.12}$$

$$\dot{x}(-W_{\rm noise},\tau)=-(\hat{L}_0-W_{\rm noise}\hat{L}_1)x(-W_{\rm noise},\tau)+x(W_{\rm noise},\tau)+\varphi(-W_{\rm noise})\hat{\Lambda}, \tag{4.13}$$

where

$$x=\begin{bmatrix}u_{\rm coh}\\ v_{\rm coh}\\ w\end{bmatrix};\ \hat{L}_0\begin{bmatrix}t_2 & z & 0\\ -z & t_2 & -W_{\rm coh}\\ 0 & W_{\rm coh} & t_1\end{bmatrix};$$

$$\hat{L}_1=\begin{bmatrix}0&0&1\\0&0&0\\-1&0&0\end{bmatrix};\ \hat{\Lambda}=w_0(t_1-1)\begin{bmatrix}0\\0\\1\end{bmatrix}. \tag{4.14}$$

Variables $u_{\rm coh}$, $v_{\rm coh}$, and w are defined relative to the phase of field $E_{\rm coh}$, i.e., $w=\rho_{22}(\alpha)-\rho_{11}(\alpha)$, $u_{\rm coh}+iv_{\rm coh}=2\rho_{12}(\alpha)\exp(-i\omega t+ikz)$. Two dimensionless parameters,

$W_{\mathrm{coh}}=\chi\tau_0\cos a$ and $W_{\mathrm{noise}}=\chi\tau_0\sin a$, correspond to Rabi frequencies of the coherent and incoherent fields; $z=\Delta\tau_0$ is a dimensionless detuning and time τ is chosen in units τ_0;

$$t_{1,2}=1+\frac{\tau_0}{T_{1,2}}. \tag{4.15}$$

We define two variables that are symmetric and asymmetric combinations of the previous ones, i.e.,

$$\overline{x}=x(W_{\mathrm{noise}})+x(-W_{\mathrm{noise}}), \tag{4.16}$$

$$x_A=x(W_{\mathrm{noise}})-x(-W_{\mathrm{noise}}). \tag{4.17}$$

The first is the averaged density matrix. These variables obey the equations

$$\dot{\overline{x}}=-(\hat{L}_0-1)\overline{x}-W_{\mathrm{noise}}\hat{L}_1x_A+\hat{\Lambda}, \tag{4.18}$$

$$\dot{x}_A=-(\hat{L}_0+1)x_A-W_{\mathrm{noise}}\hat{L}_1\overline{x}. \tag{4.19}$$

By Laplace transformation (LT)

$$\overline{x(p)}=\int_0^\infty e^{-p\tau}\overline{x(\tau)}d\tau \tag{4.20}$$

the differential equations (4.18) and (4.19) are converted to algebraic equations

$$(\hat{L}_0+p-1)\overline{x(p)}+W_{\mathrm{noise}}\hat{L}_1x_A(p)=\frac{\hat{\Lambda}}{p}+\overline{x(0)}, \tag{4.21}$$

$$W_{\mathrm{noise}}\hat{L}_1\overline{x(p)}+(\hat{L}_0+p+1)x_A(p)=x_A(0), \tag{4.22}$$

where $\overline{x(0)}$ and $x_A(0)$ are initial values of the variables $\overline{x(\tau)}$ and $x_A(\tau)$.

Solution of these equations gives the Laplace transform of the averaged density matrix

$$\overline{x(p)}=\{(\hat{L}_0+p-1)-W_{\mathrm{noise}}^2\hat{L}_1(\hat{L}_0+p+1)^{-1}\hat{L}_1\}^{-1}\times\left\{\frac{\hat{\Lambda}}{p}+\overline{x(0)}-W_{\mathrm{noise}}\hat{L}_1(\hat{L}_0+p+1)^{-1}x_A(0)\right\}. \tag{4.23}$$

Before switching on the field $E(t)$, the value $x_A(0)$ was zero and the averaged density matrix was $\overline{x(0)}=\hat{\Lambda}/(t_1-1)$. By substituting $\overline{t_{1,2}}=t_{1,2}+p-1$, expression (4.23) can be rewritten as

$$\overline{x(p)}=[\overline{\hat{L}_0}-W_{\mathrm{noise}}^2\hat{L}_1(\overline{\hat{L}_0}+2)^{-1}\hat{L}_1]^{-1}\frac{\hat{\Lambda}\overline{t_1}}{p(\overline{t_1}-p)}, \tag{4.24}$$

where

$$\overline{\hat{L}_0}=\begin{bmatrix}\overline{t_2} & z & 0\\ -z & \overline{t_2} & -W_{\mathrm{coh}}\\ 0 & W_{\mathrm{coh}} & \overline{t_1}\end{bmatrix}. \tag{4.25}$$

Calculation of the $\overline{v}_{\mathrm{coh}}(p)$ component of the column vector $\overline{x(p)}$ gives the result

$$\overline{v}_c(p)=\frac{w_0\overline{t_1}}{pt_{1m}}\frac{W_{\mathrm{coh}}Bt_{2m}}{z^2+W_{\mathrm{coh}}^2(t_{2m}/t_{1m})+Bt_{2m}\overline{t_2}}, \tag{4.26}$$

where

$$t_{1m}=\overline{t_1}+(W_{\mathrm{noise}}^2/D)(\overline{t_1}+2)(\overline{t_2}+2),$$

$$t_{2m}=\overline{t_2}+(W_{\mathrm{noise}}^2/D)(\overline{t_2}+2)^2, \tag{4.27}$$

$$B^{-1}=1+(W_{\mathrm{noise}}^2/D)\overline{t_2},$$

$$D=z^2(\overline{t_1}+2)+W_{\mathrm{coh}}^2(\overline{t_2}+2)+(\overline{t_1}+2)(\overline{t_2}+2)^2. \tag{4.28}$$

If the value $W_{\mathrm{noise}}=0$ is taken, then the expression (4.26) reduces to the solution of the conventional Bloch equations. The stationary solution of equations (4.12) and (4.13) is obtained from the limit

$$(\overline{v}_{\mathrm{coh}})_{st}=\lim_{p\to 0}p\overline{v}_{\mathrm{coh}}(p). \tag{4.29}$$

It contains the information about the hole burnt into the inhomogeneous spectrum of an ensemble of two-level atoms by an infinitely long driving field pulse.

We consider the noise with a short correlation time given the inequality $\tau_0\ll T_1,T_2,\chi_{\mathrm{coh}}^{-1},\chi_{\mathrm{noise}}^{-1},\Delta^{-1}$, where $\chi_{\mathrm{coh}}=\chi\cos a$ and $\chi_{\mathrm{noise}}=\chi\sin a$ are Rabi frequencies of the coherent and incoherent fields. Then we can approximate the values D, B, t_{2m}, and t_{1m} when $p=0$ by $D\approx 8$; $B\approx 1$ and

$$(t_{1,2})_m=\frac{\tau_0}{T_{1,2}}+\frac{W_{\mathrm{noise}}^2}{2}. \tag{4.30}$$

For these conditions, expression (4.29) takes the form

$$(\overline{v}_{\mathrm{coh}})_{\mathrm{st}}=\frac{w_0}{T_1}\cdot\frac{\chi_{\mathrm{coh}}(\Gamma_u/\Gamma_w)}{\Delta^2+\chi_{\mathrm{coh}}^2(\Gamma_u/\Gamma_w)+(\Gamma_u/T_2)}, \tag{4.31}$$

where

$$\Gamma_u=\frac{1}{T_2}+\Gamma,\quad \Gamma_w=\frac{1}{T_1}+\Gamma,\quad \Gamma=\chi_{\mathrm{noise}}^2\tau_c. \tag{4.32}$$

The variable $(\overline{v}_{\mathrm{coh}})_{\mathrm{st}}$ describes the absorption of the field E_{coh}. Moreover it is proportional to the deviation of the stationary saturated population difference from the unperturbed value: $w_0-\overline{w}_{\mathrm{st}}$, as

$$\overline{w}_{\text{st}}=\frac{w_0}{T_1\Gamma_w}\left\{1-\frac{\chi_{\text{coh}}^2(\Gamma_u/\Gamma_w)}{\Delta^2+\chi_{\text{coh}}^2(\Gamma_u/\Gamma_w)+(\Gamma_u/T_2)}\right\}. \tag{4.33}$$

When $\Gamma\gg1/T_{1,2}$ (or $\chi_{\text{noise}}^2\tau_cT_{1,2}\gg1$), the hole half width for strong saturation ($\chi_{\text{coh}}^2\gg\Gamma_w/T_2$) approaches the Rabi frequency of the coherent field χ_{coh}, as the rates Γ_u and Γ_w become equal. This behavior is markedly different from that of saturation of solids by a monochromatic field, i.e., by the field E_{coh} without noise E_{noise}. The latter burns a hole with HW of $\chi_{\text{coh}}\sqrt{T_1/T_2}$, which is much larger than χ_{coh} (since $T_1\gg T_2$). The associated free induction decay (FID) after a long saturating pulse with phase noise has a decay rate

$$\Gamma_{\text{FID}}=\frac{1}{T_2}+\sqrt{\frac{\Gamma_u}{T_2}+\chi^2\frac{\Gamma_u}{\Gamma_w}}. \tag{4.34}$$

When $\Gamma_u\approx\Gamma_w$ (or $\Gamma\gg1/T_{1,2}$), this rate becomes anomalously slow as in experiments [60,64,65].

We now consider the transient nutation (TN) excited by coherent field E_{coh} when the incoherent field is present. The induced coherent polarization per particle is given by

$$P_{\text{coh}}(t)=\frac{\mu}{2}(\overline{u}_{\text{coh}}+i\overline{v}_{\text{coh}})e^{i\omega t-ikz}, \tag{4.35}$$

where the overbar denotes an average over the noise states. We consider an ensemble of particles with an *inhomogeneous* spectrum. The ensemble response to the driving field is

$$\langle P_{\text{coh}}(t)\rangle_{\text{ens}}=\frac{\mu}{2}\langle\overline{u}_{\text{coh}}+i\overline{v}_{\text{coh}}\rangle_{\text{ens}}e^{i\omega t-ikz}, \tag{4.36}$$

where $\langle\ \rangle_{\text{ens}}$ denotes an average over the inhomogeneous spectrum

$$\langle P_{\text{coh}}(t)\rangle_{\text{ens}}=\int P_{\text{coh}}(t,\omega_0)\Phi(\omega_0)d\omega_0 \tag{4.37}$$

and $\Phi(\omega_0)$ is the particle number distribution over the inhomogeneous spectrum. If the driving field frequency ω falls at the center ω_c of the symmetric spectrum, then $\overline{v}_{\text{coh}}(\Delta,t)$ becomes an even function of Δ, and $\overline{u}_{\text{coh}}(\Delta,t)$ is odd since its Laplace transform is proportional to resonant detuning $z=\Delta\tau_c$, i.e.,

$$\overline{u}_{\text{coh}}(p)=-\frac{w_0\overline{t}_1}{pt_{1m}}\frac{zW_{\text{coh}}}{z^2+W_{\text{coh}}^2(t_{2m}/t_{1m})+Bt_{2m}\overline{t}_2}. \tag{4.38}$$

In the limit of infinite inhomogeneous width, expression (4.24) simplifies to

$$\langle P_{\text{coh}}(t)\rangle_{\text{ens}}=i\frac{\mu}{2}\Phi(\omega_c)e^{i\omega t-ikz}\int\overline{v}_{\text{coh}}(\omega_0,t)d\omega_0. \tag{4.39}$$

The Laplace transform of function $\overline{v}_{\text{coh}}(\omega_0,t)$ is given by Eq. (4.26). We consider time scales of the function $\overline{v}_{\text{coh}}(\omega_0,t)$ that are longer than τ_c. Therefore it is possible to restrict the analysis to small $p(|p|\ll1)$. This approximation corresponds to neglecting the fast decaying (at rate $1/\tau_c$) part of the solution. For this condition, Eq. (4.26) is reduced to

$$\overline{v}_{\text{coh}}(\overline{p})=\frac{w_0}{p}\frac{\chi_{\text{coh}}(\Gamma_u+\overline{p})(1/T_1+\overline{p})}{(\Gamma_w+\overline{p})[\Delta^2+(\Gamma_u+\overline{p})(1/T_2+\overline{p})]+\chi_{\text{coh}}^2(\Gamma_u+\overline{p})} \tag{4.40}$$

where $\overline{p}=p/\tau_0$.

Averaging over the inhomogeneous spectrum

$$\langle\overline{v}_{\text{coh}}(\overline{p})\rangle_{\text{ens}}=\Phi(\omega_c)\int\overline{v}_{\text{coh}}(\omega_0,\overline{p})d\omega_0 \tag{4.41}$$

gives the Laplace transform of the ensemble's $\langle\overline{v}_{\text{coh}}(\overline{p})\rangle_{\text{ens}}$ component

$$\langle\overline{v}_{\text{coh}}(\overline{p})\rangle_{\text{ens}}=\frac{\pi\Phi(\omega_c)w_0\chi_{\text{coh}}(1/T_1+\overline{p})}{p\sqrt{\chi_{\text{coh}}^2+(\Gamma_w+\overline{p})(1/T_2+\overline{p})}}\sqrt{\frac{\Gamma_u+\overline{p}}{\Gamma_w+\overline{p}}}. \tag{4.42}$$

When $T_1\to\infty$, we can use the approximation applied in [66] and obtain

$$\langle\overline{v}_{\text{coh}}(t)\rangle_{\text{ens}}=\pi\Phi(\omega_c)w_0\chi_{\text{coh}}J_0(\chi_mt)\exp\left(\frac{1/T_2+\Gamma}{2}t\right), \tag{4.43}$$

where $\chi_m=\chi_{\text{coh}}\sqrt{1-\zeta^2}$; $\zeta=(1-\chi_{\text{noise}}^2\tau_cT_2)/2\chi_{\text{coh}}T_2$ and $J_0(x)$ is the zeroth-order Bessel function.

The transient nutation signal (4.36) with amplitude (4.43) demonstrates an additional decay rate

$$\Gamma_{\text{TN}}=\frac{1}{2}\left(\frac{1}{T_2}+\Gamma\right), \tag{4.44}$$

which is intensity dependent as $\Gamma=\chi_{\text{noise}}^2\tau_c$. This dependence is qualitatively similar to the observed one in Ref. [67]

$$\Gamma_{\text{exp}}=\alpha+\beta\chi, \tag{4.45}$$

where $\alpha\approx1/2T_2$ and $\beta=2.4\times10^{-2}$ (sample No. 1). Both decay rates grow with excitation intensity increase, however, one is square and the other is linearly dependent on the Rabi frequency. The linear dependence of Γ_{TN} is described in Ref. [68] by a different model of the phase-locked noise, where the phase distribution near the reference phase is Lorentzian.

To clarify this effect, we reconstruct from Laplace transforms $\bar{u}_{coh}(p)$, $\bar{v}_{coh}(p)$ and $\bar{w}(p)$ the modified Bloch equations

$$\dot{\bar{u}}_{coh} = -\Delta \bar{v}_{coh} - \Gamma_u \bar{u}_{coh}, \tag{4.46}$$

$$\dot{\bar{v}}_{coh} = \Delta \bar{u}_{coh} + \chi_{coh}\bar{w} - \Gamma_v \bar{v}_{coh}, \tag{4.47}$$

$$\dot{\bar{w}} = -\chi_{coh}\bar{v}_{coh} - \Gamma_w \bar{w} + \frac{w_0}{T_1}, \tag{4.48}$$

which are valid for the frequency range of $|\Delta| \ll 1/\tau_c$ when $t \gg \tau_c$. Modified relaxation rates are

$$\Gamma_u = \frac{1}{T_2} + \Gamma, \quad \Gamma_v = \frac{1}{T_2}, \quad \Gamma_w = \frac{1}{T_2} + \Gamma. \tag{4.49}$$

Then we consider the two-level atom (TLA) as a spin $S = 1/2$ in a constant magnetic field $\mathbf{H}_0$. Interaction of the S_z spin component with this field (z axis is parallel to $\mathbf{H}_0$) leads to energy splitting of the TLA. The field $\mathbf{H}_1(t)$, rotating in the x-y plane is similar to resonant excitation when the rotation frequency is equal to the TLA transition frequency. In a reference frame rotating with field $\mathbf{H}_1(t)$, spin $\mathbf{S}$ does not see the field $\mathbf{H}_0$ and therefore it undergoes precession around field $\mathbf{H}_1$. The result is the resonant spin-flip or resonant TLA transitions. This simple picture is well known in nuclear magnetic resonance [69]. Within this picture it is easy to explain TLA behavior in the two fields, E_{coh} and E_{noise}. Since the $\bar{w}$ component is equivalent to the S_z spin component, $\bar{u}_{coh}$ is equivalent to the S_x component (which is in phase with the coherent field in the rotating frame), and $\bar{v}_{coh}$ is equivalent to the S_y component, we can represent TLA interaction with fields E_{coh} and E_{noise} as shown in Fig. 2. We

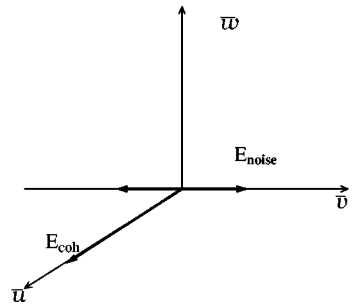

FIG. 2. Two-level atom interaction with coherent and incoherent fields. These fields induce precession of the components that are transverse to them. The E_{coh} field induces change of the $\bar{w}$ and $\bar{v}_{coh}$ components. The E_{noise} field causes random change of the $\bar{w}$ and $\bar{u}_{coh}$ components.

see that the E_{coh} field induces a resonant transition since it is transverse to the $\bar{w}$ component. The effective spin undergoes a precession around E_{coh} and its projection on the $\bar{w}$ axis is changed. Field E_{noise} induces a resonant transition for the same reason. Moreover E_{noise} induces the change of $\bar{u}_{coh}$ component in a random way, as it is transverse to this component also. Thus the noise field E_{noise} contributes to the relaxation of the $\bar{u}_{coh}$ and $\bar{w}$ components, i.e., to the Γ_u and Γ_w rates. Since this field is parallel to $\bar{v}_{coh}$ component, it has no effect on this component and the Γ_v rate remains unchanged.

Equations (4.46)–(4.48) with modified relaxation rates (4.49) describe TLA saturation by the coherent component of the field when, in addition, the incoherent component induces relaxation and saturation (the noise saturates TLA with a rate $\Gamma = \chi^2_{noise}\tau_c$). Modified equations (4.46)–(4.48) are valid over a frequency range that is smaller than the noise band $\gamma = 1/\tau_c$. Over a frequency range comparable with γ, we must use the exact solution (4.24) of equations (4.12) and (4.13). Then, for example, the Laplace transform of TLA population difference takes the form

$$\bar{w}(p) = \frac{w_0}{p}\{1-(W_{coh}/t_{1m})F_1(p)\}\{1-(W_{noise}/\bar{t}_1)F_2(p)\}, \tag{4.50}$$

$$F_1(p) = \frac{W_{coh}t_{2m}}{z^2 + W^2_{coh}(t_{2m}/t_{1m}) + Bt_{2m}\bar{t}_2}, \tag{4.51}$$

$$F_2(p) = \frac{W_{noise}(\bar{t}_2+2)}{z^2 + W^2_{noise}(\bar{t}_2+2)/\bar{t}_1 + W^2_{coh}(\bar{t}_2+2)/(\bar{t}_1+2) + (\bar{t}_2+2)^2}. \tag{4.52}$$

It consists of two components. The first (narrow) describes TLA saturation by the field E_{coh}, and the second (broad) is burnt by the field E_{noise}. The narrow component is described by the term with function $F_1(p)$. Relevant modified Bloch equations include decay rates (4.49). The wide component is described by the term with function $F_2(p)$. If we put $E_{coh} = 0$ and $E_{noise} \neq 0$, then it is possible to reconstruct from Eq. (4.50) other modified Bloch equations

$$\dot{u}_{\rm noise} = -\Delta v_{\rm noise} - \left(\frac{1}{T_2} + \gamma\right) u_{\rm noise}, \quad (4.53)$$

$$\dot{v}_{\rm noise} = \Delta u_{\rm noise} + \chi_{\rm noise} w_{\rm noise} - \left(\frac{1}{T_2} + \gamma\right) v_{\rm noise}, \quad (4.54)$$

$$\dot{w}_{\rm noise} = -\chi_{\rm noise} v_{\rm noise} - \frac{w_{\rm noise} - w_0}{T_1}, \quad (4.55)$$

which describe TLA saturation by incoherent field $E_{\rm noise}$. We introduced here the variables $u_{\rm noise}$, $v_{\rm noise}$, $w_{\rm noise}$ to distinguish them from those defined in Eqs. (4.10) and (4.11). It should be emphasized that the absorption of field $E_{\rm noise}$ is not described by $\overline{u}_{\rm coh}$, $\overline{v}_{\rm coh}$ components of the Bloch vector, although the TLA population difference $\overline{w}$ is correct. The latter saturates as

$$\overline{w} = w_{\rm noise} = w_{\rm st} + (w_0 - w_{\rm st}) e^{-\Gamma t}, \quad (4.56)$$

where

$$\Gamma = \frac{\chi^2_{\rm noise}\gamma}{\Delta^2 + \gamma^2}, \quad w_{\rm st} = w_0 \frac{\Delta^2 + \gamma^2}{\Delta^2 + \gamma^2 + \chi^2_{\rm noise}\gamma T_1}. \quad (4.57)$$

Here, the rate Γ is defined over an extended frequency range, comparable to that defined in Eq. (4.32). Below we derive the TLA response for the combined field $E_{\rm coh} + E_{\rm noise}$ and show the validity of Eqs. (4.53)–(4.55).

V. ABSORPTION OF THE FIELD WITH PHASE NOISE

The mean value of polarization in the reference frame linked rigidly to the field phase is described by $\overline{u}$ and $\overline{v}$ functions defined in Eq. (3.17). We need not derive the equations for them, as there is a relation between $\overline{u}$, $\overline{v}$, and $\overline{u}_{\rm coh}$, $\overline{v}_{\rm coh}$ components, i.e.,

$$\overline{u} = \overline{u}_{\rm coh} \cos a + v_A \sin a; \quad \overline{v} = \overline{v}_{\rm coh} \cos a - u_A \sin a, \quad (5.1)$$

where u_A and v_A are asymmetric combinations of partial Bloch-vector components [see Eqs. (4.16) and (4.17)]

$$u_A = u_{\rm coh}(a) - u_{\rm coh}(-a); \quad v_A = v_{\rm coh}(a) - v_{\rm coh}(-a). \quad (5.2)$$

The latter are related to the mean value of the Bloch vector according to Eqs. (4.21)–(4.24) as

$$x_A(p) = -W_{\rm noise}(\hat{\overline{L}}_0 + 2)^{-1} \hat{L}_1 \overline{x}(p). \quad (5.3)$$

Only the mean value of polarization (calculated in the instantaneous reference frame) provides reliable information about the field absorption coefficient (proportional to $\overline{v}$ component). Substitution of Eq. (5.3) into Eq. (5.1) gives its Laplace transform

$$\overline{v}(p) = \frac{w_0}{p}\Bigg\{ B \frac{\overline{t}_1}{t_{1m}} F_1(p) \cos a + \left[1 - \frac{2(\overline{t}_1 + \overline{t}_2 + 2) W_{\rm coh} B F_1(p)}{(\overline{t}_1 + 2)(\overline{t}_2 + 2) t_{1m}}\right] F_2(p) \sin a \Bigg\}, \quad (5.4)$$

where functions $F_1(p)$ and $F_2(p)$ are defined in Eqs. (4.51) and (4.52). The TLA response consists of two parts, one with a narrow spectrum [function $F_1(p)$] and another with a wide spectrum [function $F_2(p)$]. If we put $\chi_{\rm coh} = 0$ and $\chi_{\rm noise} \neq 0$, then the wide component coincides with a solution of modified Bloch equations (4.53)–(4.55) for the $u_{\rm noise}$, $v_{\rm noise}$, $w_{\rm noise}$ variables. There is only one difference, an extra $\sin a$ function which comes from the phase averaging of power absorption. Below we clarify the origin of this difference.

When the driving field

$$E(t) = 2E_0 \cos(\omega t + \alpha) \quad (5.5)$$

has a random phase α, we cannot use the common expression for field absorption. We must start from first principles and derive a modified one. It is well known that power absorption takes place when there is a phase difference between the field and TLA response. Power absorption by a unit volume is described by [70]

$$A = \frac{\omega}{2\pi}\int_{t=0}^{t=T} E(t)\, dP(t), \quad P(t) = N(\mu_{12}\rho_{21} + \mu_{21}\rho_{12}), \quad (5.6)$$

where $P(t)$ is the TLA polarization; N is the number of TLAs per unit volume and $T = 2\pi/\omega$ is the field oscillation period. There are two possibilities of defining TLA polarization. The first one is

$$P(t) = \mu N[u \cos(\omega t + \alpha) - v \sin(\omega t + \alpha)], \quad (5.7)$$

where $u + iv = 2\sigma_{12}$ and σ_{12} is defined in a reference frame linked with the instant phase of the field [see Eq. (3.12)]. Substitution of this polarization into Eq. (5.6) and calculation of the integral over t gives the result (it is assumed that the phase α has not changed during the field oscillation period T since $\tau_c \gg T$)

$$A = -\omega\mu E_0 N v. \quad (5.8)$$

Then the phase average

$$\langle A\rangle_\alpha = -\omega\mu E_0 N \langle v\rangle_\alpha \quad (5.9)$$

gives an expression for the field absorption. Since v is defined in the instant reference frame, the mean value $\langle v\rangle_\alpha$ coincides with $\overline{v}$. Therefore the $\overline{v}(t)$ function calculated from Eq. (5.4) can be used directly in the expression for field absorption (5.9). We see that the result coincides with the usual expression for power absorption of the field with constant phase (5.8).

The second way to define polarization is

$$P(t) = \mu N(u_{\rm coh} \cos \omega t - v_{\rm coh} \sin \omega t), \quad (5.10)$$

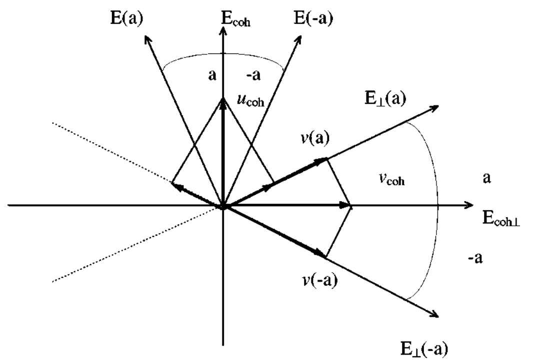

FIG. 3. Phase diagram of the noise field and Bloch-vector components. E_{coh}, $E(\pm a)$ represent phases of these two fields. $E_{\perp}(\pm a)$ and $E_{\mathrm{coh}\,\perp}$ indicate vectors with phases shifted by $\pi/2$ relative to the corresponding fields. u_{coh}, v_{coh}, and $v(\pm a)$ are Bloch-vector components defined in reference frames linked to the phase of the coherent field E_{coh} and random-phase field $E(\pm a)$, respectively.

where u_{coh} and v_{coh} are defined relative to the reference phase. For this case, calculation of the integral (5.6) gives

$$A=-\omega\mu E_0 N(v_{\mathrm{coh}}\cos\alpha-u_{\mathrm{coh}}\sin\alpha). \tag{5.11}$$

Phase average of the latter gives a new expression for power absorption

$$\langle A\rangle_{\alpha}=-\omega\mu E_0 N(\langle v_{\mathrm{coh}}\cos\alpha\rangle_{\alpha}-\langle u_{\mathrm{coh}}\sin\alpha\rangle_{\alpha}). \tag{5.12}$$

For phase telegraph noise, one can calculate these average functions in the following way. Consider the phase diagram of the field and Bloch-vector components u_{coh}, v_{coh}, $v(\pm a)$, presented in Fig. 3. Here E_{coh} denotes the coherent component of the field; $E(a)$ and $E(-a)$ show the relative phases of the field for the two states; $E_{\mathrm{coh}\perp}$, $E_{\perp}(a)$, and $E_{\perp}(-a)$ are vectors with the $\pi/2$ phase shift relative to E_{coh}, $E(a)$, and $E(-a)$, respectively. The Bloch-vector component u_{coh} is in phase with E_{coh} whereas v_{coh}, $v(a)$, and $v(-a)$ are phase shifted relative to E_{coh}, $E(a)$, and $E(-a)$, respectively. Projection of the v_{coh} component on $E_{\perp}(\pm a)$ directions does not change with phase shift and therefore

$$\langle v_{\mathrm{coh}}\cos\alpha\rangle_{\alpha}=[v_{\mathrm{coh}}(a)+v_{\mathrm{coh}}(-a)]\cos a=\overline{v}_{\mathrm{coh}}\cos a. \tag{5.13}$$

Meanwhile, projection of the u_{coh} component on these directions changes the sign and hence

$$\langle u_{\mathrm{coh}}\sin\alpha\rangle_{\alpha}=[u_{\mathrm{coh}}(a)-u_{\mathrm{coh}}(-a)]\sin a=u_A\sin a. \tag{5.14}$$

Substitution of Eqs. (5.13) and (5.14) into Eq. (5.12) gives the result

$$\langle A\rangle_{\alpha}=-\omega\mu E_0 N(\overline{v}_{\mathrm{coh}}\cos a-u_A\sin a) \tag{5.15}$$

coinciding with that obtained by substitution of Eq. (5.1) in Eq. (5.9).

Thus, we have shown that the power absorption of the field with phase telegraph noise for an ensemble of two-level particles with a *homogeneous* absorption spectrum is described by Eq. (5.15) or Eq. (5.9). Taking the limit (4.29) of function (5.4) on the p variable, one gets an exact expression for power absorption:

$$\langle A\rangle_{\alpha}=-Nw_0\frac{\omega}{\hbar}\left\{\frac{\Gamma_1}{\Gamma_w}B\chi_{\mathrm{coh}}F_1(\Delta)+\chi_{\mathrm{noise}}F_2(\Delta)\times\left[1-\frac{\gamma(\gamma+\Gamma_1+\Gamma_2)}{\Gamma_w(\gamma+\Gamma_1)(\gamma+\Gamma_2)}B\chi_{\mathrm{coh}}F_1(\Delta)\right]\right\}, \tag{5.16}$$

where

$$F_1(\Delta)=\frac{\chi_{\mathrm{coh}}\Gamma_u}{\Delta^2+\chi_{\mathrm{coh}}^2(\Gamma_u/\Gamma_w)+B\Gamma_u\Gamma_v},$$

$$F_2(\Delta)=\frac{\chi_{\mathrm{noise}}(\gamma+\Gamma_2)}{\Delta^2+\chi_{\mathrm{coh}}^2 g+(\gamma+\Gamma_2)^2+\chi_{\mathrm{noise}}^2(\gamma+\Gamma_2)/\Gamma_1},$$

$$\Gamma_{1,2}=\frac{1}{T_{1,2}},\quad B^{-1}=1+\frac{\Gamma\Gamma_2}{(\gamma+\Gamma_1)(\gamma+\Gamma_2)},\quad g=\frac{\gamma+\Gamma_2}{\gamma+\Gamma_1},$$

and decay rates are defined in explicit form without approximation

$$\Gamma_u=\frac{1}{T_2}+g\Gamma,\quad \Gamma_v=\frac{1}{T_2},\quad \Gamma_w=\frac{1}{T_1}+\Gamma, \tag{5.17}$$

$$\Gamma=\frac{\chi_{\mathrm{noise}}^2(\gamma+\Gamma_2)}{\Delta^2+\chi_{\mathrm{coh}}^2 g+(\gamma+\Gamma_2)^2}. \tag{5.18}$$

Examples of absorption spectra [Eq. (5.16)] are shown in Fig. 4. They consist of two lines; a narrow one sitting on a broad one. The broad line arises from the incoherent component of the field. Its half width

$$\Gamma_{\mathrm{incoh}}\approx\sqrt{\gamma^2+\chi_{\mathrm{noise}}^2\gamma T_1} \tag{5.19}$$

may become much larger than the half width of the broad component of the field spectrum γ. For example, when $\chi_{\mathrm{noise}}/(2\pi)=\chi_{\mathrm{coh}}/(2\pi)=200$ kHz, $\gamma/(2\pi)=1$ MHz and $T_1=4$ msec, the value $\Gamma_{\mathrm{incoh}}/(2\pi)=31.7$ MHz becomes comparable with frequency tuning (Stark shift [71]) of resonant atoms or exciting beam frequency shift in photon echo experiments. Therefore atoms removed from resonance with the central field component are still excited by the broad component of the field. This gives a contribution Γ to the T_2 and T_1 relaxation times measured by two-pulse and three-pulse echoes. For example, when the frequency shift is 25 MHz, this contribution to decay rates $1/T_2$ and $1/T_1$ is 64 Hz for the above conditions and 1.6 kHz when $\chi_{\mathrm{noise}}/(2\pi)=\chi_{\mathrm{coh}}/(2\pi)=1$ MHz. For this reason, one has to be careful in the interpretation of the intensity dependence of T_2, since the broad component of the excitation field spectrum can give a contribution similar to the instantaneous diffusion effect.

Our consideration is not related to the photon-echo induced by delayed incoherent pulses [46,49,50,53,54]. In these studies an incoherent optical pulse is produced by a

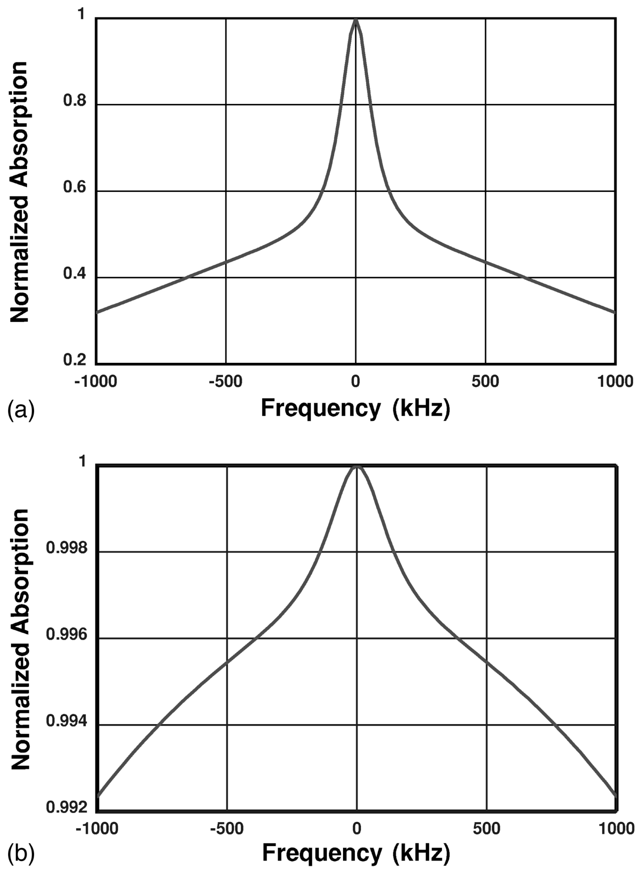

FIG. 4. Phase-telegraph-noise field absorption of a *homogeneously* broadened line vs resonant tuning (in kHz). TLA relaxation times are $T_1=4$ msec and $T_2=15$ μsec. Half width of the broad component of the field spectrum $\gamma/(2\pi)$ is 1 MHz. Rabi frequencies of the coherent and noise components of the field are equal to (a) $\chi_{\rm coh}/(2\pi)=\chi_{\rm noise}/(2\pi)=6$ kHz and (b) $\chi_{\rm coh}/(2\pi)=\chi_{\rm noise}/(2\pi)=100$ kHz.

dye ''laser'' with only one mirror or a light-emitting diode. Therefore the irradiation arises as enhanced spontaneous emission with statistical properties close to the chaotic field (see Introduction). Since the pulse sequence is realized by optical delay of a single pulse, both pulses are correlated in phase in spite of their incoherent property.

We consider excitation with a single mode laser with locked phase. Pulses of length t_p are, in effect, produced by fast frequency tuning of the CW output beam or resonant frequency of the atoms. Thus a certain group of the atoms are in resonance with the field only during these frequency tuning periods and out of resonance between tuning periods. As the phase of the field fluctuates, there is no correlation between excitation pulses although the reference phase is assumed to be fixed. When the pulse duration t_p is shorter than dwell time of the phase jump $\tau_c (t_p \ll \tau_c)$, each pulse can be considered coherent and atom excitation is described by the Bloch equations with relaxation and phase fluctuation being neglected. The phase of the two-pulse echo signal depends on the relation between phases of the pulses. For example, the phase difference between the second pulse and echo signal is $\alpha_{21}+\frac{3}{2}\pi$, where α_{21} is the phase difference between second and first pulses. The echo amplitude does not depend on this difference. As was shown above, this amplitude acquires an additional damping due to the saturation of the broad ''incoherent'' hole. The latter process is described by Bloch equation with T_1 and T_2 modified according to Eqs. (5.17) and (5.18). The same considerations are true for the stimulated photon echo.

The accumulated photon echo is sensitive to the relative phases of the exciting pulses. As was shown in [55,72,73], one can even get zero signal at a particular value of the phase difference of the exciting pulses, which may be helpful in photon echo data erasure. We do not consider this effect in detail. We only show that there is additional damping caused by saturation of the ''incoherent'' hole when an accumulated-stimulated echo is produced by frequency tuning and excitation by a single mode laser with locked phase.

VI. FID AFTER SATURATION BY THE FIELD WITH PHASE-TELEGRAPH NOISE

We consider an ensemble of particles (with an *inhomogeneous* absorption spectrum) excited by the field

$$E(t)=E_0\exp(i\omega t+i\alpha) \tag{6.1}$$

whose phase α undergoes a random telegraph process. Excitation is switched on at $t=-\infty$. It is assumed that saturation reaches a stationary state before time $t=0$. The hole burnt into the inhomogeneous spectrum is described by the population difference (4.50)

$$\overline{w}(\Delta)=w_0\left[1-\frac{\chi_{\rm coh}}{\Gamma_w}F_1(\Delta)\right]\left[1-\frac{\chi_{\rm noise}}{\Gamma_1}F_2(\Delta)\right]. \tag{6.2}$$

At time $t=0$, the field frequency ω is shifted instantly to $\omega+\Omega$:

$$E(t)=E_0\exp(i\omega t+i\Omega t+i\alpha). \tag{6.3}$$

We assume that the frequency shift Ω is much larger than hole width, and consider saturated particles to be removed from excitation at $t=0$. Two response fields appear in the sample, one at frequency $\omega+\Omega$ and the other at frequency ω, i.e.,

$$E_r(t)=E_1(t)e^{i(\omega+\Omega)t}+E_2(t)e^{i\omega t}. \tag{6.4}$$

The beam reaching the detector is the sum of the laser field (6.3) and emitted sample field (6.4). Since the optical detector is a square law device, we must calculate the intensity present at the detector, which is

$$I(t)=c\varepsilon_0E_s(t)E_s^*(t), \tag{6.5}$$

where $E_s(t)=E(t)+E_r(t)$. Now $E_r \ll E_0$ for any optically thin sample, so the term, proportional to $|E_r|^2$ may be ignored and

$$I(t)=I_0+I_1(t)+I_2(t), \tag{6.6}$$

where

$$I_0 = c\varepsilon_0 E_0^2,$$

$$I_1(t) = c\varepsilon_0\{E_0 E_1^*(t)e^{i\alpha} + \text{c.c.}\},$$

$$I_2(t) = c\varepsilon_0\{E_0 E_2^*(t)e^{i\Omega t + i\alpha} + \text{c.c.}\}.$$

The last term on the right-hand side of Eq. (6.6) contains the FID signal of particles removed from excitation and second term corresponds to transient nutation signal of particles tuned to resonance at $t=0$. We consider the FID signal that is due to oscillations on the carrier frequency Ω.

The response field $E_2(t)$ is driven by the free precessing polarizations of all particles excited before the frequency switch, i.e.,

$$E_2(t)\, e^{i\omega t} = -i\frac{\omega l}{\varepsilon_0 c}\int_{-\infty}^{\infty} P(\omega_0)\Phi(\omega_0)d\omega_0, \tag{6.7}$$

$$P(\omega_0) = \frac{\mu}{2}[u_{\text{coh}}(\alpha_0) + iv_{\text{coh}}(\alpha_0)]\exp(i\omega_0 t - t/T_2), \tag{6.8}$$

where α_0 is the field phase at switching time $t=0$. Substitution of Eqs. (6.7) and (6.8) into the $I_2(t)$ function gives

$$I_2(t) = \mu E_0 \omega l\, e^{-t/T_2}\{\langle\cos(\Delta t - \Omega t - \alpha)\, v_{\text{coh}}(\alpha_0)\rangle_{\text{ens}} + \langle\sin(\Delta t - \Omega t - \alpha)\, u_{\text{coh}}(\alpha_0)\rangle_{\text{ens}}\}, \tag{6.9}$$

where $\langle\ \rangle_{\text{ens}}$ denotes average over the inhomogeneous broadening. Since the phase α changes with time, we must average the intensity $I_2(t)$ using the phase transition probability (4.3):

$$\langle I_2(t)\rangle_\alpha = \int I_2(t)\varphi(\alpha_0,0|\alpha,t)\, d\alpha. \tag{6.10}$$

The result is

$$\langle I_2(t)\rangle_\alpha = \mu E_0\omega l\, e^{-t/T_2}[V(t) + U(t)], \tag{6.11}$$

$$V(t) = \langle[\cos(\Delta - \Omega)t\cos\alpha_0 + e^{-\gamma t}\sin(\Delta - \Omega t\sin\alpha_0]\, v_{\text{coh}}(\alpha_0)\rangle_{\text{ens}}, \tag{6.12}$$

$$U(t) = \langle[\sin(\Delta - \Omega)t\cos\alpha_0 - e^{-\gamma t}\cos(\Delta - \Omega)t\sin\alpha_0] \times u_{\text{coh}}(\alpha_0)\rangle_{\text{ens}}. \tag{6.13}$$

Bloch-vector components $u_{\text{coh}}(\alpha_0)$ and $v_{\text{coh}}(\alpha_0)$ are defined in a reference frame linked to the phase of the coherent part of the field E_{coh}. Their values are defined by the simple relations:

$$u_{\text{coh}}(\pm a) = \tfrac{1}{2}(\overline{u}_{\text{coh}} \pm u_A), \tag{6.14}$$

$$v_{\text{coh}}(\pm a) = \tfrac{1}{2}(\overline{v}_{\text{coh}} \pm v_A). \tag{6.15}$$

When an ensemble of two-level particles is excited by the field (6.1) at the center of the inhomogeneous line, then components $\overline{u}_{\text{coh}}(\Delta)$ and $v_A(\Delta)$ are odd functions of Δ, whereas $\overline{v}_{\text{coh}}(\Delta)$ and $u_A(\Delta)$ are even functions. This allows simplification of Eq. (6.11) in terms of the $\overline{u}_{\text{coh}}$, $\overline{v}_{\text{coh}}$, u_A, and v_A components as follows:

$$\langle I_2(t)\rangle_\alpha = \frac{1}{2}\mu E_0\omega l\, e^{-t/T_2}\left[\langle M_1(t)\rangle_{\text{ens}}\cos\Omega t - \frac{\alpha_0}{a}\langle M_2(t)\rangle_{\text{ens}}\sin\Omega t\right], \tag{6.16}$$

$$M_1(t) = (\overline{v}_{\text{coh}}\cos a - e^{-\gamma t}u_A\sin a)\cos\Delta t + (\overline{u}_{\text{coh}}\cos a + e^{-\gamma t}v_A\sin a)\sin\Delta t, \tag{6.17}$$

$$M_2(t) = (u_A\cos a + e^{-\gamma t}\overline{v}_{\text{coh}}\sin a)\cos\Delta t - (v_A\cos a - e^{-\gamma t}\overline{u}_{\text{coh}}\sin a)\sin\Delta t. \tag{6.18}$$

The FID intensity (6.16) consists of two terms, one (with $\sin\Omega t$ function) depends on the sign of α_0 while the other (with $\cos\Omega t$ function) does not. In FID experiments, the phase α_0 at the time of the frequency shift changes randomly from shot to shot. The probability of finding phases a or $-a$ is equal [see Eq. (4.4)]. Therefore averaging a large number of FID signal sweeps will cancel out the contribution of the term with the $\sin\Omega$ function. Then the average FID signal detected by the heterodyne technique is

$$\langle\langle I_2(t)\rangle\rangle_{\alpha,\alpha_0} = \tfrac{1}{2}\mu E_0\omega l\, e^{-t/T_2}\langle M_1(t)\rangle_{\text{ens}}\cos\Omega t, \tag{6.19}$$

$$M_1(t) = \overline{v}_m\cos\Delta t + \overline{u}_m\sin\Delta t, \tag{6.20}$$

$$\overline{v}_m = \overline{v}_{\text{coh}}\cos a - u_A\, e^{-\gamma t}\sin a, \tag{6.21}$$

$$\overline{u}_m = \overline{u}_{\text{coh}}\cos a + v_A\, e^{-\gamma t}\sin a, \tag{6.22}$$

where double brackets $\langle\langle\ \rangle\rangle_{\alpha,\alpha_0}$ denote averaging over α and α_0 phases. The effective components $\overline{u}_m$ and $\overline{v}_m$ differ from the $\overline{u}$ and $\overline{v}$ functions, defined in the instant reference frame [see Eq. (5.1)] by the exponent $e^{-\gamma t}$. Their stationary subcomponents are

$$\overline{u}_{\text{coh}} = -\frac{\Gamma_1}{\Gamma_w}\frac{\Delta\chi_{\text{coh}}w_0}{\Delta^2 + \Gamma_{\text{coh}}^2}, \tag{6.23}$$

$$\overline{v}_{\text{coh}} = B\frac{\Gamma_1}{\Gamma_w}\frac{\Gamma_u\chi_{\text{coh}}w_0}{\Delta^2 + \Gamma_{\text{coh}}^2}, \tag{6.24}$$

$$u_A = -\frac{\gamma\chi_{\text{noise}}w_0}{\Delta^2 + \Gamma_{\text{incoh}}^2}\left[1 - \frac{\gamma(\gamma + \Gamma_1 + \Gamma_2)}{\Gamma_w(\gamma + \Gamma_1)(\gamma + \Gamma_2)}B\chi_{\text{coh}}F_1(\Delta)\right], \tag{6.25}$$

$$v_A = -\frac{\Delta\chi_{\text{noise}}w_0}{\Delta^2 + \Gamma_{\text{incoh}}^2}\left[1 - \frac{\gamma(\Gamma_2 - \Gamma_1)}{\Gamma_w\Gamma_u(\gamma + \Gamma_1)}\chi_{\text{coh}}F_1(\Delta)\right], \tag{6.26}$$

where

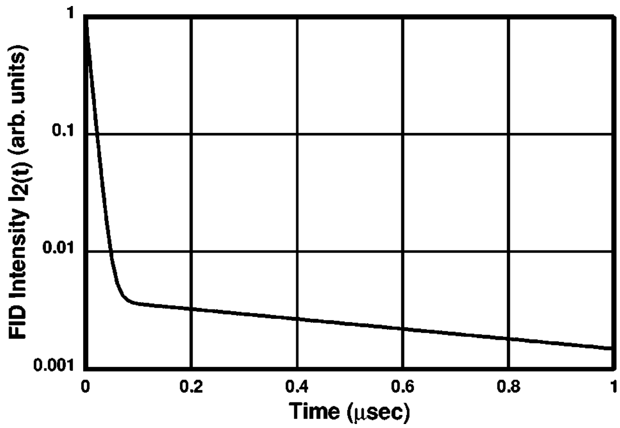

FIG. 5. FID vs time (in μsec). Parameters of the field and two-level atom relaxation times are the same as in Fig. 4 (case b).

$$\Gamma_{\mathrm{coh}}=\sqrt{B\Gamma_u\Gamma_v+\chi_{\mathrm{coh}}^2\frac{\Gamma_u}{\Gamma_w}}, \tag{6.27}$$

$$\Gamma_{\mathrm{incoh}}=\sqrt{(\gamma+\Gamma_2)^2+\chi_{\mathrm{coh}}^2 g+\chi_{\mathrm{noise}}^2\left(\frac{\gamma+\Gamma_2}{\Gamma_1}\right)}. \tag{6.28}$$

The main contribution to the FID signal is given by the $\overline{u}_m\sin\Delta t$ term. Averaging over the inhomogeneous line gives the following expression for the FID signal:

$$\langle\langle I_2(t)\rangle\rangle_{\alpha,\alpha_0}=-\frac{\pi\chi^2N\omega l w_0}{2\hbar\overline{\Delta}}\left\{\sin^2 a e^{-(\Gamma_n+\gamma)t}+\frac{\cos^2 a}{\Gamma_w T_1}e^{-\Gamma_c t}\right\}e^{-t/T_2}\cos\Omega t, \tag{6.29}$$

where $\overline{\Delta}$ is the inhomogeneous line half-width — assumed to be much larger than the widest hole in the spectrum. Here, to simplify this expression, we keep only the first largest terms in the expansion of the exponents' coefficients. The time dependence of Eq. (6.29) is shown in Fig. 5. The FID consists of a fast and slow component. The fast part is caused by the broad (''incoherent'') hole and the slow by the narrow (''coherent'') hole. The narrow hole demonstrates non-Bloch saturation [60,64,65].

VII. CONCLUSION

We considered resonant excitation of an ensemble of two-level atoms by a phase-noise field. A new equation of the field absorption (5.9), which does not depend on the type of random phase process, was derived. Then random-walk phase (phase diffusion model without reference point) and phase fluctuation in the limited domain (model of random phase locked near some reference point) were considered as examples of two different processes. We showed that interaction of a phase diffusion field with two-level atoms is described by a Bloch equation with modified dephasing rate

$$\frac{1}{T_{2m}}=\frac{1}{T_2}+\nu, \tag{7.1}$$

where ν is a half width of the (Lorentzian) field spectrum. Thus, the random walk of the phase produces only an additive broadening of the absorption line. As the saturation of this line is described by a Bloch model with modified T_{2m}, the saturation broadening also increases if T_1 is the longest relaxation time of TLA. It is easy to show that free induction decay and transient nutation are described by the same modified Bloch model.

Phase-telegraph noise is considered as an example of locked-phase noise. The power of this field is divided into two parts. One is compressed into a narrow (δ function) line and the other is spread over a broad frequency band. Even when the integral intensities of them are comparable, their spectral densities strongly differ. For example, at the center of the narrow line, the relative intensity of the broad component is infinitely small. When the reference phase undergoes some additional unlocked fluctuation with a rate ν_c which is smaller than basic fluctuation rate γ of phase-locked noise, then the central peak acquires a finite small broadening and the ratio of spectral densities of broad and narrow components at their maxima is equal to ν_c/γ. We show that even when this ratio is very small (for example, the broad component is three orders of magnitude less than the narrow component), the locked-phase-noise field cannot be considered to be coherent. We show that TLA saturation by the narrow field component is affected by the broad field component. The latter contributes to the $1/T_1$ and $1/T_2$ relaxation rates. When they become nearly equal, non-Bloch saturation takes place. This increase of relaxation rates also leads to an acceleration of the transient nutation decay. Moreover, we showed that the broad component of the field saturates its own hole in the spectrum. As the width of this hole is comparable with frequency switch interval in photon echo experiments, we surmise that saturation by the broad field component may affect the photon echo decay rate.

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APPENDIX

Let us consider excitation of the cavity by the field

$$E(t)=E_0\cos[\omega t+\alpha(t)] \tag{A1}$$

with the pump rate $\mathcal{P}$. For simplicity we assume that the phase of the field α jumps by the same value θ for each time interval τ_0, i.e., the field phase grows steadily with time. If the damping rate G of the cavity is the smallest parameter and the pump rate is the largest parameter, i.e.,

$$G\ll 1/\tau_0\ll\mathcal{P} \tag{A2}$$

then for each time interval τ_0, a fraction of the field with current phase α_k is stored in the cavity. The amplitude of this stored field fraction is equal to $\mathcal{P}\tau_0E_0$ at the end of the time interval τ_0. In the time scale of the cavity excitation lifetime $T_q=1/G$, we have many changes of the pump field phase since $\tau_0\ll T_q$. For this reason many fractions of the field

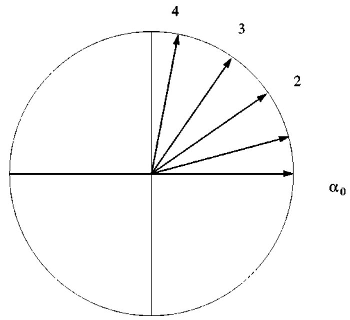

FIG. 6. Phase evolution of the pump field (see text).

with constant, different phases are stored during the lifetime of the cavity excitation. Thus the excited cavity field is the sum of many field fractions with different phases. For further simplification we consider the cavity with $T_q \to \infty$. Then the evolution of the cavity field may be presented as

$$E_n = \mathcal{P}\tau_0 E_0 (e^{i\alpha_0} + e^{i\alpha_1} + e^{i\alpha_2} + \cdots + e^{i\alpha_n}), \qquad \text{(A3)}$$

where $\alpha_n = \alpha_0 + n\theta$ and $n = t/\tau_0$ is considered as integer, i.e., we look for the field evolution in the process of pump addition (fraction by fraction) at the end of each time interval τ_0 and do not consider fast changes within these time intervals. (The field fraction with a particular phase α_k grows in the cavity during the duration τ_0 of the corresponding pump field fraction. The stored fraction is not changed after the phase change of the pump as we assume infinite lifetime for the cavity excitation.) Equation (A3) is simplified as

$$E_n = \mathcal{P}\tau_0 E_0 e^{i\alpha_0} \sum_{k=0}^{n} e^{i\theta k} = \mathcal{P}\tau_0 E_0 e^{i\alpha_0} \frac{1 - e^{i\theta(n+1)}}{1 - e^{i\theta}}. \qquad \text{(A4)}$$

If the phase change θ is small ($\theta \ll 1$), then this equation is reduced to

$$E_n = i \frac{\mathcal{P}\tau_0}{\theta} E_0 e^{i\alpha_0} [1 - e^{i\theta(n+1)}]. \qquad \text{(A5)}$$

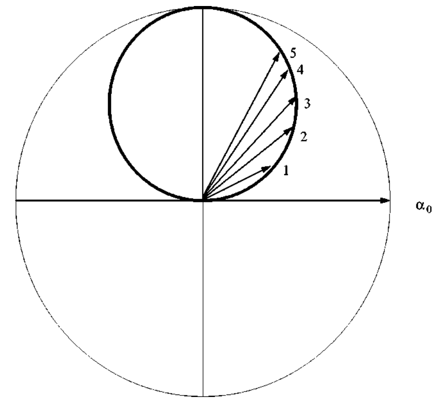

FIG. 7. Amplitude-phase evolution of the cavity field (see text).

In Fig. 6 the evolution of the pump field phase is shown, where numbers 1,2,3,... indicate the current change of the phase (the arrow with subscript α_0 indicates the initial phase of the field in Figs. 6 and 7). In Fig. 7 the evolution of the phase and amplitude of the field stored in the cavity is shown. The bold circle shows the phase-amplitude trajectory of the stored field. Its amplitude changes between zero and $(\mathcal{P}\tau_0/\theta)E_0$. The phase of the stored field changes between α_0 and $\alpha_0 + \pi$. We see that the phase of the pump field grows steadily, whereas the phase of the stored field is locked near phase $\alpha_0 + \pi/2$ within a domain $(\alpha_0, \alpha_0 + \pi)$, i.e., the phase fluctuates by $\pm\pi/2$ near the phase $\alpha_0 + \pi/2$.

Thus we see that a regular phase change of the pump results in phase-locked change of the cavity field. If the pump phase changes randomly according to a diffusion process, then any small phase shift does not essentially change the phase of the cavity field that is locked to the first phase of the pump. Only an accidentally large phase jump (having a small probability) moves the locked phase to a new state. However, this movement takes place gradually within the next period of small phase jumps near the new state of the pump phase.

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[1] M. Lax, in *Fluctuation and Coherence Phenomena in Classical and Quantum Physics*, edited by M. Chretien, E.P. Gross, and S. Deser (Gordon and Breach, New York 1968).

[2] T. J. Jaseja, A. Javan, and C. H. Townes, Phys. Rev. Lett. **10**, 165 (1963).

[3] A. I. Burshtein, Zh. Éksp. Teor. Fiz. **48**, 850 (1965) [Sov. Phys. JETP **21**, 567 (1965)].

[4] A. I. Burshtein, Zh. Éksp. Teor. Fiz. **49**, 1362 (1965) [Sov. Phys. JETP **22**, 939 (1966)].

[5] A. I. Burshtein and Y. S. Oseledchik, Zh. Éksp. Teor. Fiz. **51**, 1071 (1966) [Sov. Phys. JETP **24**, 716 (1967)].

[6] A. I. Burshtein, in *Lectures on Quantum Kinetics* (Novosibirsk State University, Novosibirsk, 1968) (in Russian).

[7] P. A. Apanasevich, G. I. Zhovna, and A. P. Khapalyuk, J. Appl. Spectrosc. **8**, 14 (1968).

[8] B. R. Mollow, Phys. Rev. **175**, 1555 (1968).

[9] G. S. Agarwal, Phys. Rev. A **1**, 1445 (1970).

[10] L. D. Zusman and A. I. Burshtein, Zh. Éksp. Teor. Fiz. **61**, 976 (1971) [Sov. Phys. JETP **34**, 520 (1972)].

[11] S. G. Przhibel'skii and V. A. Khodovoi, Opt. Spektrosk. **32**, 237 (1972) [Opt. Spectrosc. **32**, 125 (1972)].

[12] S. G. Przhibel'skii, Opt. Spektrosk. **35**, 715 (1973) [Opt. Spectrosc. **35**, 415 (1973)].

[13] Y. S. Oseledchik and A. I. Burshtein, Zh. Prikl. Spektrosk. **21**, 1042 (1974).

[14] A. I. Bonch-Bruevich, S. G. Przhibel'skii, V. A. Khodovoi, and N. A. Chigir, Zh. Éksp. Teor. Fiz. **70**, 445 (1976).

[15] Yu. S. Oseledchik, J. Appl. Spectrosc. **25**, 1036 (1976).

[16] A. I. Burshtein and A. G. Kofman, Zh. Eksp. Teor. Fiz. **70**, 840 (1976).

[17] G. S. Agarwal, Phys. Rev. Lett. **37**, 1383 (1976).

[18] J. H. Eberly, Phys. Rev. Lett. **37**, 1387 (1976).

[19] P. V. Elyutin, Opt. Spektrosk. **43**, 542 (1977) [Opt. Spectrosc. **43**, 318 (1977)].

[20] S. G. Przhibelskii, Opt. Spektrosk. **42**, 17 (1977) [Opt. Spectrosc. **42**, 8 (1977)].
[21] P. Zoller and F. Ehlotzky, J. Phys. B **15**, 3023 (1977).
[22] H. J. Kimble and L. Mandel, Phys. Rev. A **15**, 689 (1977).
[23] P. Avan and C. Cohen-Tannoudji, J. Phys. B **10**, 155 (1977).
[24] A. T. Georges and P. Lambropoulos, Phys. Rev. A **18**, 587 (1978).
[25] G. S. Agarwal, Phys. Rev. A **18**, 1490 (1978).
[26] P. B. Hogan, S. J. Smith, A. T. Georges, and P. Lambropoulos, Phys. Rev. Lett. **41**, 229 (1978).
[27] P. Agostini, A. T. Georges, S. E. Wheatley, P. Lambropoulos, and M. D. Levenson, J. Phys. B **11**, 1733 (1978).
[28] B. R. Marx, J. Simons, and L. Allen, J. Phys. B **11**, L273 (1978).
[29] P. Zoller, Phys. Rev. A **19**, 1151 (1979).
[30] K. Wodkiewicz, Phys. Rev. A **19**, 1686 (1979).
[31] A. T. Georges, P. Lambropoulos, and P. Zoller, Phys. Rev. Lett. **42**, 1609 (1979).
[32] P. Zoller and P. Lambropoulos, J. Phys. B **12**, L547 (1979).
[33] A. T. Georges and P. Lambropoulos, Phys. Rev. A **20**, 991 (1979).
[34] P. Zoller, Phys. Rev. A **20**, 1019 (1979).
[35] S. N. Dixit, P. Zoller, and P. Lambropoulos, Phys. Rev. A **21**, 1289 (1980).
[36] A. T. Georges, Phys. Rev. A **21**, 2034 (1980).
[37] K. Wodkiewicz, Z. Phys. B **42**, 95 (1981).
[38] A. T. Georges and S. N. Dixit, Phys. Rev. A **23**, 2580 (1981).
[39] D. E. Nitz, A. V. Smith, M. D. Levenson, and S. J. Smith, Phys. Rev. A **24**, 288 (1981).
[40] A. M. Bonch-Bruevich, S. G. Przhibelskii, and N. A. Chigir, Zh. Éksp. Teor. Fiz. **80**, 565 (1981) [Sov. Phys. JETP **53**, 285 (1981)].
[41] J. J. Yeh and J. H. Eberly, Phys. Rev. A **24**, 888 (1981).
[42] D. S. Elliot, R. Roy, and S. J. Smith, Phys. Rev. A **26**, 12 (1982).
[43] K. Wodkiewicz, J. H. Eberly, and B. W. Shore, J. Opt. Soc. Am. B **1**, 398 (1984).
[44] K. Wodkiewicz, B. W. Shore, and J. H. Eberly, Phys. Rev. A **30**, 2390 (1984).
[45] J. H. Eberly, K. Wodkiewicz, and B. W. Shore, Phys. Rev. A **30**, 2381 (1984).
[46] N. Morita and T. Yajima, Phys. Rev. A **30**, 2525 (1984).
[47] D. S. Elliott, M. W. Hamilton, K. Arnett, and S. J. Smith, Phys. Rev. Lett. **53**, 439 (1984).
[48] K. Wodkiewicz and J. H. Eberly, Phys. Rev. A **31**, 2314 (1985).
[49] R. Beach and S. R. Hartmann, Phys. Rev. Lett. **53**, 663 (1984).
[50] R. Beach, D. DeBeer, and S. R. Hartmann, Phys. Rev. A **32**, 3467 (1985).
[51] M. W. Hamilton, D. S. Elliott, K. Arnett, and S. J. Smith, Phys. Rev. A **33**, 778 (1986).
[52] R. Boscaino and R. N. Mantegna, Phys. Rev. A **40**, 5 (1989); **40**, 13 (1989).
[53] S. R. Hartmann and J. T. Manassah, J. Phys. B **23**, 1363 (1990).
[54] H. Nakatsuka, A. Wakamiya, K. M. Abedin, and T. Hattori, Opt. Lett. **18**, 832 (1993).
[55] U. Elman and S. Kroll, Laser Phys. **6**, 721 (1996).
[56] G. E. Uhlenbeck and L. S. Ornstein, Phys. Rev. **36**, 823 (1930); in *Selected Papers on Noise and Stochastic Processes*, edited by N. Wax (Dover, New York, 1954), pp. 93–111.
[57] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, 2nd ed. (McGraw-Hill, New York, 1984), pp. 205–354.
[58] J. Keilson, J. E. Storer, Q. Appl. Math. **10**, 243 (1952).
[59] W. Feller, in *An Introduction to Probability Theory and Its Applications* (Wiley, New York, 1966).
[60] R. G. DeVoe and R. G. Brewer, Phys. Rev. Lett. **50**, 1269 (1983).
[61] A. Szabo and R. Kaarli, Phys. Rev. B **44**, 12 307 (1991).
[62] M. J. Sellars, R. S. Meltzer, P. T. H Fisk, and N. B. Manson, J. Opt. Soc. Am. B **11**, 1468 (1994).
[63] M. Mitsunaga, N. Uesugi, and K. Sugiyama, Opt. Lett. **18**, 1256 (1993).
[64] A. Szabo and T. Muramoto, Phys. Rev. A **39**, 3992 (1989).
[65] R. Boscaino and V. M. La Bella, Phys. Rev. A **41**, 5171 (1990).
[66] R. N. Shakhmuratov, R. Boscaino, M.Cannas, and F.M. Gelardi (unpublished).
[67] R. Boscaino, F. M. Gelardi, and J. P. Korb, Phys. Rev. B **48**, 7077 (1993).
[68] R. N. Shakhmuratov, F. M. Gelardi, and M. Cannas, Phys. Rev. Lett. **79**, 2963 (1997).
[69] A. Abragam, in *The Principles of Nuclear Magnetism* (Clarendon Press, Oxford, 1961).
[70] A. Losche, in *Kerninduction* (Veb Deutscher Verlag der Wissenschaftten, Berlin, 1957).
[71] A. Szabo, J. Opt. Soc. Am. B **3**, 514 (1986).
[72] N. N. Akhmediev, Opt. Lett. **15**, 1035 (1990).
[73] M. Arend, E. Block, and S. R. Hartmann, Opt. Lett. **18**, 1789 (1993).